Question 1: Let $S = \{s_1, \ldots, s_{11}\}$ be the set of states. Suppose that at $t=0$ the state is unknown. At $t=1$ the person learns whether the state is in $\{s_1, \ldots, s_5\}$ or $\{s_6, \ldots, s_{11}\}$. At $t=2$ the person learns whether the state is in $\{s_1, \ldots, s_3\}$, $\{s_4, s_5\}$, $\{s_6, \ldots, s_{10}\}$ or $\{s_{11}\}$. Finally, at $t=3$ all remaining uncertainty is revealed. *Graph the event tree.*
Question 2: For each of the following Bernoulli utility functions determine whether or not the person is risk averse (in all cases $x \geq 0$). To get credit you need to provide a proof, don’t just say yes or no.

A) $u(x) = 10x - 4x^4$

B) $u(x) = 4x^2 - 10x^4$

C) $u(x) = 4\sqrt{x} - 10x$
**Question 3:** Suppose there are four states $S=\{s_1, \ldots, s_4\}$. The probabilities of the four states are given by 0.2, 0.6, 0.1, and 0.1, respectively. Suppose there are two investments: Investment A results in payoffs 10, 20, 40, 30, while investment B results in payoffs of 20, 10, 100, and 80, respectively, for each of the four states.

A) Suppose that the person’s Bernoulli utility function is $u(x) = x$. Then

Therefore the investor will select *(mark the correct answer)*


6 points

B) Now suppose that the Bernoulli utility function is $u(x) = -1/x$. Then

Therefore the investor will select *(mark the correct answer)*

6 points
**Question 4:** Suppose that asset A has a return of 20% with probability 0.5, 10% with probability 0.3 and -10% with probability 0.2.
Suppose a person’s utility is $U(\mu, \sigma) = \mu - 4\sigma^2$. Then

the person’s utility from asset A is

Determine the return of a riskless asset that gives the person exactly the same utility.

the return of the riskless asset must be
Question 5: Asset A has a mean return of 20% and a standard deviation of 15% (i.e., $\mu=0.2$ and $\sigma=0.15$). In addition, there is a riskless asset that has a return of 10%. Graph the efficient frontier in the grid below:

12 points
Question 6: Suppose there are two risky assets, A, and B. Suppose that $\mu_A=0.1$ and $\sigma_A=0.1$, while $\mu_B=0.2$ and $\sigma_A=0.3$. Suppose that the correlation between the returns of assets A and B is -1. Graph the set of feasible portfolios in the grid below (clearly indicate the set by shading it).
Question 7: Suppose there is a risky asset, with return 0.4 and standard deviation 0.2 and a riskless asset with return 0.1. The investor has mean variance preferences given by $G(\mu, \sigma) = \mu - 0.6\sigma^2$. The person wants to find the optimal portfolio $(a, 1-a)$ where $a$ is the fraction of wealth invested in the risky asset and $(1-a)$ the fraction invested in the riskless asset.

The optimal value of $a$ is

The mean return of the optimal portfolio is
Question 8: Suppose there are three risky assets A, B, and C. Their returns are \( \mu_A = 0.2 \), \( \mu_B = 0.3 \), and \( \mu_C = 0.5 \). Their standard deviations are \( \sigma_A = 0.1 \), \( \sigma_B = 0.1 \), and \( \sigma_C = 0.2 \). The correlation between the assets is zero. Determine the portfolio \((a_1, a_2, a_3)\) that has the lowest standard deviation (i.e., the MRP).

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<th>( a_1 = )</th>
<th>( a_2 = )</th>
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The portfolio’s mean return is

The portfolio’s standard deviation is