Problem Set 3
Simultaneity, Causality, Cointegration, and Unit-Roots

This problem set combines two topics, the first part of the problem set is based on the celebrated paper by Thurman and Fisher (1988) which resolved the longstanding scientific dispute over “which came first: the chicken or the egg?” The second part deals with classical simultaneous equation models.

As a prelude to doing the problem set please use either the R or Stata versions of the Granger-Newbold simulation code to generate 100 realizations and record the proportion of them that give a significant t-statistic at the conventional 5 percent level.

The data for the first part of the problem set was kindly provided by Thurman, consists of annual time series 1930-1983 for U.S. egg production in millions of dozens and the December 1 USDA estimate of the US chicken population, (excluding broilers). Unfortunately, as provided, the data seems to be slightly different than that analyzed by Thurman and Fisher. As a result your results based on the problem set data can be expected to vary somewhat from those reported in the Thurman and Fisher paper. The data is provided on the class web page as `eggs.txt`. Also provided are two R functions `granger()` and `adf()` to help analyze the data. We also provide a `johansen()` function as well.

1. Using `granger()` try to reproduce the results of Thurman and Fisher’s Table 1. Try to suggest some graphical technique that might help to explain the nature of the rather striking results. Make sure that you take a close look at the `granger()` function and understand how it works.

2. Test each series for I(1) behavior using the augmented Dickey-Fuller test. You may use the `adf()` function for this purpose, but again look closely to see how it works. Note that your results here may depend on the length of the lag you specify in the ADF test. How do your results here influence your interpretation of the findings in question 1?

3. Test for cointegration of the chicken-egg process, using both the Engle-Granger and Johansen approaches. Contrast your results and reconsider findings in question 1.

The second half of this problem set deals with two simple cobweb models of supply and demand. Data for Questions 1-4 appears as `system1` on the course webpage. and data for Questions 5-6, appears as `system2`. 
The model for the first part of the problem is:

(Supply) \[ Q_t = \alpha_1 + \alpha_2 p_{t-1} + \alpha_3 z_t + u_t \]

(Demand) \[ p_t = \beta_1 + \beta_2 Q_t + \beta_3 w_t + v_t \]

Last periods price determines current period supply while current period demand determines the market clearing price. The variables \( z_t \) and \( w_t \) may be regarded as exogenous influences on supply and demand, respectively.

1. Estimate the model and illustrate the dynamic behavior of the model by drawing a picture of the supply and demand functions for fixed values of the exogenous variables \( z \) and \( w \), say at \( z = z_T, w = w_T \). You may assume that \( u_t \) and \( v_t \) are independent so the model is recursive.

2. Make a point forecast of the price variable for the next 3 periods assuming that the exogenous variables remain constant at their end of sample values. Suppose that they (the exogenous variables) remained fixed at these values indefinitely; on average, what value would \( p \) take in equilibrium?

3. Now suppose that \( u_t \) were autocorrelated. Explain briefly why \( p_{t-1} \) can no longer be considered exogenous in this case. Provide a test of autocorrelation and devise a strategy for estimating the model, and reestimate.

4. Now suppose that a disagreeable referee criticized your specification of the model arguing that \( z \) should be treated as endogenous, briefly describe how this would alter your approach to estimating the model. In particular, if possible, provide a test of the referee’s hypothesis.

Now consider the following simultaneous dynamic supply and demand model of the “cobweb” form:

(Supply) \[ Q_t = \alpha_1 + \alpha_2 p_t + \alpha_3 p_{t-1} + \alpha_4 z_t + u_t \]

(Demand) \[ p_t = \beta_1 + \beta_2 Q_t + \beta_3 w_t + v_t \]

The current period’s price influences current period supply while current period demand determines the market clearing price.

5. Estimate the model by two stage least squares and compare your estimates with those obtained by ordinary least squares for this model. Interpret the differences.

6. Test the hypothesis that the long run supply response to a change in the price is unity: i.e., that \( \alpha_2 + \alpha_3 = 1 \), and the hypothesis that the first period and second period price effects are the same, i.e., \( \alpha_2 = \alpha_3 \).