Question 1

\[ \frac{\partial x_1(p, p, e)}{\partial p_1} \] evaluated at \( \bar{p} \) is 0.8

Therefore demand for good 1 (circle the correct answer) increases decreases if the price of good 1 is increased.

\[ \frac{\partial x_1(p, p, e)}{\partial p_1} = \frac{\partial x_1(p, w)}{\partial p_1} + \frac{\partial x_1(p, w)}{\partial w} e_1 \]
\[ = -0.2 + 0.1(10) = 0.8 \]

Question 2 Consider a choice structure \((C(.), B)\) where \(B = \{\{x_1, x_2\}, \{x_1, x_2, x_3\}\}\). Suppose that \(C(\{x_1, x_2\}) = x_1\). Then which of the following violate the weak Axiom (Circle all that apply)

1. \( C(\{x_1, x_2, x_3\}) = \{x_1\} \)
2. \( C(\{x_1, x_2, x_3\}) = \{x_2\} \)
3. \( C(\{x_1, x_2, x_3\}) = \{x_3\} \)
4. \( C(\{x_1, x_2, x_3\}) = \{x_2, x_4\} \)
5. \( C(\{x_1, x_2, x_3\}) = \{x_1, x_3\} \)
6. \( C(\{x_1, x_2, x_3\}) = \{x_1, x_2, x_3\} \)

Question 3 Consider a choice structure \((C(.), B)\) where \(B = \{\{x_1, x_2\}, \{x_1, x_2, x_4\}, \{x_1, x_3\}\}\). Suppose that \(C(\{x_1, x_2\}) = x_2, C(\{x_1, x_2, x_4\}) = x_4, \text{ and } C(\{x_1, x_3\}) = x_3\). Then the following preferences rationalize the choice structure (In each of the following insert either \(\succeq\) or \(\preceq\)).

\[
\begin{align*}
&x_1 \preceq x_2 \\
&x_1 \preceq x_3 \\
&x_1 \preceq x_4 \\
&x_2 \preceq x_3 \\
&x_2 \preceq x_4 \\
&x_3 \preceq x_4
\end{align*}
\]

Now find different preferences that rationalize the above choice structure:
Note that \( x_2 \geq x_3 \) also works. No other preferences rationalize the choice structure.

**Question 4** At prices \( P = (2, 2) \) a consumer chooses \( x = (4, 6) \). At prices \( P' = (4, P'_2) \) the consumer chooses \( x' = (6, 2) \). Suppose that the weak Axiom is satisfied. Then

(Insert the lower and upper bounds on \( P'_2 \). Note that these bounds can also be 0 and \( \infty \).)

\[ 2 < P'_2 < \infty \]

\( x' \) is affordable at prices \( P \). Thus, the weak Axiom is violated if \( x \) is affordable at prices \( P' \), i.e., if \((6, 2) \geq (4, 6) P' \). This implies \( 24 + 2P'_2 \geq 16 + 6P' \). Therefore, the weak Axiom is violated if \( P'_2 \leq 2 \)

**Question 5** Suppose that demand is given by \((w/(p_1 + p_2), w/(p_1 + p_2))\). We want to prove directly (i.e., not by computing the substitution matrix) that the weak Axiom is satisfied.

(a) Suppose that \( x(p_1, p_2, w) \neq x(p'_1, p'_2, w') \). Assume by way of contradiction that the weak Axiom is violated for \((p_1, p_2, w) \) and \((p'_1, p'_2, w') \). Then the following two inequalities must hold:

\[ w' \geq \frac{w(p'_1 + p'_2)}{p_1 + p_2} \]

\[ w \geq \frac{w'(p_1 + p_2)}{p'_1 + p'_2} \]

(b) Without loss of generality we can assume that wealth is 1 in both cases, i.e., \( w = w' = 1 \). Thus,

\[ 1 \geq \frac{p'_1 + p'_2}{p_1 + p_2} \]

\[ 1 \geq \frac{p_1 + p_2}{p'_1 + p'_2} \]

\( p'_1 + p'_2 = p_1 + p_2 \). However, since \( w = w' = 1 \) this implies that \( x(p_1, p_2, w) = x(p'_1, p'_2, w') \), a contradiction. Therefore, the weak Axiom is satisfied.
Question 6  Suppose that there are only two commodities, i.e., \( X = \mathbb{R}_+^2 \). At prices \( \bar{p} = (4, 1) \) and wealth \( \bar{w} \) it is optimal to consume 2 units of good 1. Suppose that \( \frac{\partial x_1(\bar{p}, \bar{w})}{\partial p_1} = -1 \) and \( \frac{\partial x_1(\bar{p}, \bar{w})}{\partial w} = 0.4 \).

(a) The substitution matrix is given by (Fill in the missing numbers.)

\[
D_p h(p, u) = \begin{pmatrix}
-0.2 & 0.8 \\
0.8 & -3.2
\end{pmatrix}
\]

The Slutsky equation implies

\[
\frac{\partial h_1(\bar{p}, \bar{u})}{\partial p_1} = \frac{\partial x_1(\bar{p}, \bar{w})}{\partial x_1} + x_1 \frac{\partial x_1(\bar{p}, \bar{w})}{\partial w} = -1 + 2(0.4) = -0.2.
\]

(b) You can conclude that (do not use the the above substitution matrix to find the answer)

\[
\frac{\partial x_2(\bar{p}, \bar{w})}{\partial p_1} = 2, \quad \frac{\partial x_2(\bar{p}, \bar{w})}{\partial w} = -0.6
\]

Differentiate the budget line equation with respect to \( p_1 \) to get

\[
x_1(\bar{p}, \bar{w}) + p_1 \frac{\partial x_1(\bar{p}, \bar{w})}{\partial p_1} + p_2 \frac{\partial x_2(\bar{p}, \bar{w})}{\partial p_1} = 0.
\]

Therefore,

\[
2 + 4(-1) + \frac{\partial x_2(\bar{p}, \bar{w})}{\partial p_1} = 0.
\]

Therefore, \( \frac{\partial x_2(\bar{p}, \bar{w})}{\partial p_1} = 2 \). If we differentiate the budget line equation with respect to \( w \) then we get

\[
p_1 \frac{\partial x_1(\bar{p}, \bar{w})}{\partial w} + p_2 \frac{\partial x_2(\bar{p}, \bar{w})}{\partial w} = 1.
\]

Therefore,

\[
4(0.4) + \frac{\partial x_2(\bar{p}, \bar{w})}{\partial w} = 1,
\]

which implies \( \frac{\partial x_2(\bar{p}, \bar{w})}{\partial w} = -0.6 \).

Question 7  See page 59 of the textbook.

Question 8  Suppose that “\( \succeq \)” are continuous and locally non-satiated preferences on \( \mathbb{R}_+^L \).

We now want to construct a utility function that represents these preferences. We define
\[ u(x) = \min_{y \in \mathbb{R}^+_L} \sum_{i=1}^{L} y_i \text{ subject to } y \succeq x. \]

(a) Suppose that \( x \succeq x' \). We want to prove that \( u(x) \succeq u(x') \). Suppose by way of contradiction that \( u(x) < u(x') \). Let \( y' \) be the solution of the minimization problem for \( x' \), i.e., \( \sum_{i=1}^{L} y'_i = u(x') \) and \( y' \succeq x' \). Complete the proof in the box below ...

Let \( y \) be the solution to the minimization problem for \( x \). Then \( y \succeq x \) which implies \( y \succeq x' \). But \( \sum_{i=1}^{L} y_i = u(x) < u(x') = \sum_{i=1}^{L} y'_i \). Thus, \( y' \) does not solve the minimization problem, a contradiction.

(b) We now want to prove that \( u \) is continuous. Thus, consider a sequence of consumption bundles \( x^n, n \in \mathbb{N} \) with \( \lim_{n \to \infty} x^n = x \). Let \( y^n \) be the solution to the above minimization problem given \( x^n \), i.e., \( y^n \succeq x^n, y^n \in \mathbb{R}^+_L \) and \( \sum_{i=1}^{L} y^n_i = u(x^n) \). Assume that we have proven that the sequence \( y^n \) converges, i.e., \( \lim_{n \to \infty} y^n = y \) (one can prove that \( y^n \) must contain convergent subsequences by compactness arguments). Now prove that \( y \) fulfills the constraints of the minimization problem.

Since \( y^n \) fulfills the constraint of the minimization problem, we have \( y^n \succeq x_n \). By continuity we get \( y \succeq x \).

(c) It remains to prove that \( u(x) = \sum_{i=1}^{L} y_i \), i.e., that \( y \) solves the optimization problem. Suppose by contradiction that there exist \( y' \) with \( y' \succeq x \) and \( \sum_{i=1}^{L} y'_i < \sum_{i=1}^{L} y_i \).

Let \( \epsilon > 0 \) such that \( \sum_{i=1}^{L} y'_i + L \epsilon < \sum_{i=1}^{L} y_i \). By local non-satiation, there exists \( \hat{y} \) with \( ||\hat{y} - y'|| < \epsilon \) and \( \hat{y} \succ y' \). Because \( ||\hat{y} - y'|| < \epsilon \) and \( \sum_{i=1}^{L} y'_i + L \epsilon < \sum_{i=1}^{L} y_i \), it follows that \( \sum_{i=1}^{L} \hat{y}_i < \sum_{i=1}^{L} y_i \). Because \( y' \succeq x \) we get \( \hat{y} \succ x \). By continuity of preferences, \( \hat{y} \succ x^n \) for all sufficiently large \( n \).

Since \( y^n \to y \) follows that \( \sum_{i=1}^{L} \hat{y}_i < \sum_{i=1}^{L} y^n_i \) for all sufficiently large \( n \). This, however, implies that \( y^n \) does not solve the minimization problem, a contradiction.