Question 1 The equation the income offer curve is \( x_2 = \frac{1}{4} x_1 \).

The optimal consumption is \( x_1 = 20 \quad x_2 = 5 \)

Question 2 The person solves

\[
\max_{x_1, x_2} \ln(x_1) + 4 \ln(x_2) \quad \text{subject to:}\n\]

(i) \( x_1 + 2x_2 \leq 200 \);

(ii) \( x_1 \geq 0 \);

(iii) \( x_2 \geq 0 \).

The Lagrangean is

\[
\mathcal{L} = \ln(x_1) + 4 \ln(x_2) - \lambda (x_1 + 2x_2 - 200) .
\]

The first order conditions are therefore

\[
\frac{\partial \mathcal{L}}{\partial x_1} : \frac{1}{x_1} - \lambda = 0 ; \\
\frac{\partial \mathcal{L}}{\partial x_2} : \frac{4}{x_2} - 2\lambda = 0 ,
\]
which implies \( \frac{2}{x_1} = \frac{4}{x_2} \), i.e., \( x_2 = 2x_1 \). Inserting this in constraint (i) implies 
\[ 5x_1 = 200, \text{ i.e., } x_1 = 40 \text{ and } x_2 = 80. \]

**The optimal consumption is** \( x_1 = 40 \quad x_2 = 80 \)

**Question 3**

| Demand before the price change is \( x_1 = 12 \quad x_2 = 6 \) |
| Demand after the price change is \( x_1 = 24 \quad x_2 = 0 \) |
| The change due to the substitution effect is \( \Delta x_1 = 30 \quad \Delta x_2 = -6 \) |
| The change due to the income effect is \( \Delta x_1 = -18 \quad \Delta x_2 = 0 \) |
Question 4

(a) The company solves

$$\max_{\tilde{h}, F} F - 2\tilde{h} \text{ subject to } -18\tilde{h} + 2\tilde{h}^2 + F \leq 0.$$ 

(b) The Lagrangean is

$$\mathcal{L} = F - 2\tilde{h} - \lambda(-18\tilde{h} + 2\tilde{h}^2 + F).$$ 

The first order conditions are

$$\frac{\partial \mathcal{L}}{\partial F}: 1 - \lambda = 0;$$
$$\frac{\partial \mathcal{L}}{\partial \tilde{h}}: -2 - \lambda(-18 + 4\tilde{h}) = 0.$$ 

Thus, $$-2 + 18 - 4\tilde{h} = 0,$$ which implies $$\tilde{h} = 4.$$ Now insert $$\tilde{h} = 4$$ into the constraint. Thus, $$-72 + 32 + F = 0,$$ i.e., $$F = 40.$$ 

At the optimum, $$F = 40 \quad \tilde{h} = 4$$ 

At the optimum, the company’s profit is $32$

c1 The marginal rate of substitution is $$18 - 4\tilde{h},$$ which must be equal to the price ratio, i.e. $$p/1.$$ Thus, $$p = 18 - 4\tilde{h},$$ which implies:

The person’s demand is given by $$h(p) = 4.5 - 0.25p$$

c2 The firm solves

$$\max_p 4.5p - 0.25p^2 - (9 - 0.5p)$$

The first order condition is $$4.5 - 0.5p + 0.5 = 0.$$ Thus, $$0.5p = 5,$$ i.e., $$p = 10.$$ At this price, demand $$h(10) = 2.$$ Thus, revenue is 20 and costs are 4.

$$p = 10, \pi = 16,$$ which is 50% less than the profit in (b)

With the other numbers: $$p = 6, \pi = 6.4$$ and the decrease is

Question 5 The expected payoff is $$(0.3)4 + (0.2)16 = 4.4.$$ The expected utility is $$(0.3)2 + (0.2)4 = 1.4.$$ The certainty equivalent $$y$$ is given by $$u(y) = 1.4,$$ i.e., $$\sqrt{y} = 1.4$$ which implies $$y = 1.96.$$ 

The expected payoff of the lottery is 4.4
The person’s expected utility from the lottery is 1.4

The lottery’s certainty equivalent is 1.96

Question 6

(a) If the person buys the stock, he will have 14,400 with probability 0.8 or 0 with probability 0.2. Thus,

The expected utility of buying the stock is 96

The expected utility of not buying the stock is 100

The person therefore (mark the correct answer) **Does not buy the stock**

(b) If the person buys the stock then the expected utility is \(0.8\sqrt{T + 4,400} + 0.2\sqrt{T - 10,000}\). This must be the same as the expected utility of not buying the stock, \(\sqrt{T}\). Thus,

\[
0.8\sqrt{T + 4,400} + 0.2\sqrt{T - 10,000} = \sqrt{T}
\]

(The solution is 10,631.84).

Question 7

(a) Jumping left: \((0.1)(0.8)=0.08\). Jumping right \((0.9)(0.4)=0.36\). Then the goal keepers expected utilities from jumping

Left is 0.08, right is 0.36. He will therefore jump to the right.

(b) The probability that team A wins (and B loses) is 1.

(c) \(0.8p = 0.4(1 - p)\). Therefore \(p = 1/3\). If the ball is played left with probability \(1/3\) and the goal keeper jumps left, he will hold with probability \((0.8)/3 = 0.2667\). If he jumps right then he will again hold with probability \((0.4)/2/3 = 0.2667\). Thus, the game ends in a tie with probability 0.2667

The probability \(p = 1/3\).

The probability that team A wins (and B loses) is 0.733.