Question 1 The equation of a person’s income offer curve is

\[ x_2 = \frac{p_1^2}{2p_2} x_1, \]

where \( p_1 \) is the price of good 1 and \( p_2 \) is the price of good 2. Suppose that prices are \( p_1 = 2 \), \( p_2 = 1 \). Suppose the person’s income is \( I = 30 \). Solve graphically for the optimal consumption choice:

The optimal consumption is \( x_1 = \) \( x_2 = \) 12 points
**Question 2** A utility function is given by \( u(x_1, x_2) = 2 \ln(x_1) + \ln(x_2) \), where \( \ln(x) \) is the logarithm of \( x \). Recall that the derivative of \( \ln(x) = 1/x \), e.g., the partial derivative of \( u(x_1, x_2) \) with respect to \( x_2 \) is \( 1/x_2 \). Suppose that the person’s income is \( I = 600 \) and that prices are \( p_1 = 8, p_2 = 1 \).

(a) Specify the person’s utility maximization problem (don’t forget any of the constraints)!  

(b) Solve the optimization problem by using the Lagrangean. Assume that only the budget constraint binds.

The optimal consumption is \( x_1 = x_2 = \)  

*5 points*
Question 3 A person’s indifference curves are depicted below.

Suppose that originally prices are $p_1 = 2$, $p_2 = 2$ and income $I = 32$. Then the price of $p_2$ increases to $p'_2 = 7$. Then

Demand before the price change is $x_1 = \quad x_2 =$

Demand after the price change is $x_1 = \quad x_2 =$

The change due to the substitution effect is $\Delta x_1 = \quad \Delta x_2 =$

Note: Determine the Slutsky substitution effect, i.e., where the person is compensated with just enough income to afford the original consumption choice at the new prices, $p_1 = 2$, $p'_2 = 7$.

The change due to the income effect is $\Delta x_1 = \quad \Delta x_2 =$
Question 4 A person’s utility function over hours of cell phone calls, $h$, and consumption of other goods, $c$, is given by $u(h, c) = 24h - 2h^2 + c$. A calling plan is characterized by a fixed fee $F$ and the number of allowed minutes, $\bar{h}$. The cell phone company wants to choose a calling plan that maximizes profits. The company’s cost per hour of calls is 4. Thus, the firm’s profit is $F - 4\bar{h}$. The consumer will select the calling plan if the net benefit is positive, i.e., $u(\bar{h}, c - F) \geq u(0, c)$, i.e., $24\bar{h} - 2\bar{h}^2 - F \geq 0$ (note that the variable $c$ cancels).

(a) Specify the profit maximization problem (ignore the constraints $\bar{h} \geq 0$ and $F \geq 0$).  

(b) Solve the maximization problem by using the Lagrangean.  

At the optimum, $F = \bar{h} =$  

At the optimum, the company’s profit is
(c) Now suppose that the company wants to offer instead a calling plan, where the consumer pays a price $p$ for each hour of calling, i.e., there is no fixed fee and no limit on the hours. In order to determine the price that the company should charge, proceed as follows.

(c1) Suppose that the consumer’s income is $I$ and that the price of a unit of consumption $c$ is 1, i.e., $c$ is simply the amount of money spent on other goods. Determine the equation of the income offer curve, and solve it for $h$, i.e.,

$\text{The person’s demand is given by } h(p) =$

(c2) The company will maximize profits. Note that profits is revenue $h(p)p$ minus costs $4h$, i.e., profit is $h(p)p - 4h(p)$. Determine $p$ and the maximum profit, $\pi$.

$p = \ldots, \pi = \ldots$, which is $\%$ less than the profit in (b)

Note: If you are unable to solve (c1), use the demand function $h(p) = 5 - 0.3p$. 

5
Question 5 A person’s Bernoulli utility function is given by $u(x) = \sqrt{x}$. Consider the following lottery: With probability 0.5 the payoff is 0, with probability 0.4 the payoff is 16, and with probability 0.1 the payoff is 100. Then

10 points

The expected payoff of the lottery is

The person’s expected utility from the lottery is

The lottery’s certainty equivalent is

Note: Recall that the certainty equivalent is a payment $y$ that the person receives with certainty such that expected utility of $y$ is the same as that of the lottery.
**Question 6** A person’s income is $I$. The person consider whether or not to buy a stock at a current price of 10,000 Dollars. The stock will have a value of either 16,900 with probability 0.6 or 0 with probability 0.4. The person’s Bernoulli utility function is $u(x) = \sqrt{x}$.

(a) Suppose the person’s income is $I = 10,000$. Then

The expected utility of buying the stock is

The expected utility of not buying the stock is

The person therefore (mark the correct answer)  Buys the stock  Does not buy the stock

(b) Let $\tilde{I}$ be the level of wealth at which the person is indifferent between buying and not buying the stock. Then the following equation determines $\tilde{I}$:
Question 7  In the 90th minute of a Soccer game between teams A and B, the referee awards a penalty (11m) kick to team A. The game is currently tied—so the penalty will decide whether the game ends in a tie, or whether A wins and B loses.

The goal keeper of team B must decide whether to jump left or right. His opponent must decide whether to shoot the ball to the left or to the right (The goal keeper must choose the corner at the same time when the player decides into which corner he should shoot the ball).

1. If the keeper jumps to the left, and the ball is shot to the left, then the keeper will hold with probability 0.2 (Hence the game remains tied with probability 0.2, and A wins with probability 0.8)
2. If the keeper jumps to the right, and the ball is shot to the right, then the keeper will hold with probability 0.6. (Hence the game remains tied with probability 0.6, and A wins with probability 0.4)
3. If the keepers moves to the wrong side, then team A wins.

The goal keeper’s Bernoulli utility is given by $u(\text{tie}) = 1, u(\text{loss}) = 0$.

(a) Suppose that goal keeper believes that his opponent will take advantage of the goal keeper’s weaker side. In particular the goal keeper believes that the ball will be played to the right with probability 0.1 and to the left with probability 0.9.

Then the goal keepers expected utilities from jumping

Left is , right is . He will therefore jump to the .

(b) Now suppose that the goal keeper always jumps to the same side, and the player who shoots the penalty knows the side. Then (quick answer, no algebra needed):

The probability that team A wins (and B loses) is .

5 points

3 points
(c) As we see in (b), it is better for the goal keeper to randomize between jumping left and right. In order for him to do so, he must be indifferent between jumping left and right, i.e., his expected utility from left and right must be the same. Thus, let \( p \) be the probability that goal keeper believes that his opponent will play to the left, and \( 1 - p \) the probability that his opponent will be play to right. Determine the value of \( p \) at which the goal keeper is indifferent between jumping left and right.  

5 points

The probability \( p = \) .

The probability that team \( A \) wins (and \( B \) loses) is .

(Note: If you determined \( p \) correctly, then the probability of winning should be the same if the goal keeper jumps to the left or to the right. This lets you determine the probability that team \( A \) wins.)