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What Accounts for the Increase in Single Households and the Stability in Fertility?

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ABSTRACT

Between the mid seventies and the beginning of the nineties the share of single females grew dramatically in the U.S. (from 16% to 29%). So did the share of single mothers (from 12% to 17%). Total fertility remained constant over the same period. At the same time relative wages within and between sexes underwent huge changes. In this paper we measure the contribution that changes in relative wages had in accounting for these and other demographic facts. We construct a model where agents differ in sex, take marital status and fertility decisions and invest in their children's human capital. Our findings show that changes in relative earnings potential account for: *i.* 44% of the observed change in the share of single women in the bottom half of the earnings distribution, *ii.* half of the observed change in the share of single women in the top half, *iii.* 73% of the observed change in the share of single mothers with a sharper increase among women at the bottom of the earnings distribution. This is obtained through a drop in the model economy marriage rate that matches that found in the data. The model economy reproduces a positive inter-generational earnings correlation.

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1 Introduction

Between the mid seventies and the beginning of the nineties the U.S. experienced a dramatic change in the marital status of its population. During this period the share of single female headed households grew from 16% to 29% at the expense of married couple households. Moreover there was a steady increase in out of wedlock childrearing that raised the share of single mothers from 12% to 17% of the total female population. Total fertility remained constant over the same period. The findings of a recent body of empirical literature imply that such a change in household structure might have important consequences on the human capital accumulation process of children attached to those households. Keane and Wolpin (97) show that factors that happen in early stages of life (by age sixteen) are crucial determinants of children's later success. Neal and Johnson (96) find that differences in educational achievements by the time of high-school completion account for almost all the observed black-white wage gap. This empirical evidence suggests that the key determinant of children's future well-being is human capital accumulation during childhood. Moreover children's outcomes are shown to be affected by the type of family children live in. McLanahan and Sandefur (94) documented differences in later achievements, in particular in terms of education, between children raised in single and two parent families. It is important therefore to understand what has determined this shift in marital status of the population. During the last two decades, relative wages within and between sexes underwent significant transformations in the U.S. (see Murphy and Katz (92)), Gottschalk (97)). The wage premia within sexes widened, while the sex wage premium shrank. We claim that demographic trends and relative wage movements are related phenomena. We look at changes in household structure from the point of view of women's "potential" earnings distribution. One of the striking findings is that the shift in marital status among females belonging to the bottom half of the earnings distribution has been double the size of that undergone by the top half. An increase of 90% in the share of single mothers among poor women as also ensued. In the early seventies rich females were more likely to be single or become single mothers. This is no longer the case. The task of the present work is to measure the contribution that changes in relative wages had in accounting for the observed shift in women's marital status

and other related demographic facts like the stability of fertility over the period taken into consideration.
We build on the work of Becker and Tomes (76) and Becker and Tomes(79) and their quality-quantity trade off model of parental fertility decisions. Unlike Knowles (98), and Aiyagari, Greenwood and Guner (97) our model incorporates endogenous household formation and dissolution, endogenous fertility, and intertemporal investment in the dynasty in the form of parents' investment of time and resources in children's education. Agents of opposite sex live for three periods, childhood, adult life and retirement; of these three only the second matters. Agents differ in terms of potential earnings, marital status and number of children attached to them. In each period, if they are single they meet in a marriage market and decide whether to accept a prospective partner as a mate. If married they decide whether to stay together or divorce. Getting or staying married requires agreement of both parties. Once a marital status decision has been taken, fertility and investment in children's education decisions ensue. Upon divorce children go into custody of the mother. For this reason parties within a couple might disagree on the choice of the future number of children. In the absence of transferable utility marriage models pose a problem whenever both parties do not have identical preferences. This is usually dealt with in an ad hoc way; for example with a weighted joint maximization of their utilities like in Cubeddu and Ríos-Rull (97) or with a non-cooperative solution like in Aiyagari, Greenwood and Guner (97). Here we avoid any sort of bargaining between parties, sequencing the fertility and the investment in children's education decisions. First women choose the number of children, then parents completely agree on how much to consume and how much time and resources to invest in childrearing. When children leave the households, then their parents age. Upon becoming adults, children's earning type is a stochastic function of the amount of resources that parents invested in their education during childhood. The method of the paper is to compare equilibrium allocations obtained under different relative wages regimes. The first task is to calibrate the model economy to match the statistics of interest that we compute from the data for the mid seventies. We call the obtained equilibrium allocation the baseline model economy. Then we change relative earnings potential between and within sexes to match the wage patterns observed in the data, without modifying all the other parameters' values obtained in the cal-

ibration of the baseline economy. We assess both the individual and the joint contributions of these wage changes in accounting for the observed shift in marital status of the population from the mid seventies to the early nineties. We find that in our model economy changes in relative earnings potential account for: *i.* 44% of the observed change in the share of single women in the bottom half of the earnings potential distribution, *ii.* half of the change in the share of single women in the top half *iii.* 73% of the observed change in the share of single mothers.*iv.* half of the change in the share of poor single mothers. In the model like in the data the increase in the share of single mothers is sharper among poor women. Moreover, the shift in the marital status composition of the model population is achieved through a 12% reduction in the marriage rate and a stable divorce rate like in the data. In the model economy children who are raised in single parent families have 30% less chance of becoming high earning type adults with respect to children raised in two parent families. When we sort households according to family total income in the model economy for the nineties, single parent households are more likely to be in the bottom tail of the distribution. This result suggests that shifts in the marital status composition of the population might have reinforced the positive inter-generational correlation of earnings that is observed in the U.S. economy. The model economy reproduces a positive inter-generational earnings correlation, but its value of 0.17 is low compared to what found in the empirical literature (Solon(92), Zimmerman(92)).

Section 2 describes how we build the earnings potential indicator. Section 3 lists the main facts that are behind the motivation of the paper. Section 4 describes the model economy. Section 5 outlines the calibration targets, the calibration procedure and the features of the baseline economy. Section 6 describes how changes in relative earnings potential affect the baseline economy equilibrium allocation. Section 7 concludes.

2 The data

We build a measure of permanent earnings for males, aged 35-49 and females aged 30-44, for the years 1972-1975 and 1989-1992, using the PSID data on yearly average hourly earnings expressed in 1992 U.S. dollars. We choose an age lower bound of 30 for women and 35 for men because looking across ages the proportion of married females over the total female population tends to stabilize after the age of 30. The same happens for males after the age of 35. Moreover we want to avoid issues related to delaying of marriage or childrearing decisions. The age gap between males and females is justified because, within a couple, males tend to be older than females. In fact almost 85% of males age 35-49 are married to females age 30-44. We divide the sample of individuals in twenty four groups according to age, work experience, education and race. If individuals are employed for at least two years during the period taken into consideration, their measure of potential earnings is an average of their yearly hourly earnings across those years. Otherwise we impute their earnings as function of the average potential earnings of the members of the group they belong to. We adjust observed individual earnings and imputed earnings to take care of differences in age among individuals. We sort men and women by this measure of earnings and label them rich (top half of the distribution) or poor (bottom half) accordingly. The average age of poor and rich women of our sample is 36.2 and 36.0 years respectively.

3 Facts

The statistics computed from the PSID show the dramatic change in the share of single females and single mothers that took place between the mid seventies and the beginning of the nineties in the U.S.. It used to be that single females were more concentrated at the top of the earnings distribution. However by the beginning of the nineties there are almost as many poor single females as rich single females. The share of poor single mothers almost doubled. These variations in household composition have been accompanied by large changes in relative wages within and between sexes. When we refer to sex wage premium we mean the ratio between average potential earnings of men and average potential earnings of women. When we refer to wage premia within sexes we mean the ratio between average potential earnings of the people in the top 50% of the distribution and in the bottom 50%. While the divorce rate has remained constant over the period taken into consideration, the marriage rate has declined by 18%. Another regularity shown by the data is a high degree of positive assortative matching of couples that has increased over the period.

Table 1 shows the main trends.

Table 1: Computed statistics from the PSID

	1974	1991	Change
Share sing. females rich	.204	.290	42%
Share sing. females poor	.136	.280	106%
Share sing. mothers	.12	.17	41%
Share sing. mothers poor	.11	.21	90%
Females' wage premium	2.12	2.56	21%
Males' wage premium	2.27	2.63	16%
Sex wage premium	1.89	1.56	-17%
Share of married rich females to rich males	.60	.62	3%
Share of married poor females to poor males	.59	.62	5%
from Statistical Abstract of the U.S.			
Marriage rate	.11	.09	-18%
Divorce rate	.02	.02	0%

4 Key Model Objects

- Exponential population model: agents do not take economic decisions in the first stage of their lives. They leave their parents' household with probability $(1-\pi)$. Upon becoming adults, agents meet each period with a different sex agent (if married before this agent will be the spouse). A decision about whether to be married ensues. Both parties have to agree in order for marriage to occur. Agents face a constant probability of aging. In this event their children leave the household and become adults and no more children are possible. Agents move to limbo.

- Agents differ in sex which never changes and we use f for females and m for males.

- Agents are also indexed by the individual states $\epsilon, n, q, \xi, \eta$ that change over time, where $\epsilon \in \{\epsilon_1, \cdots, \epsilon_{n_\epsilon}\}$ is a composite of own education and luck that reports the wage. Sometimes we use the notation ϵ^h to denote the subset of the ϵ's that are associated with higher education. Its complement is ϵ^ℓ. We refer to $n \in \{0, 1, 2, \cdots\}$ as the number of children that are associated with the household, to $q \in \{0, 1\}$ as whether the person starts the period being married or not, to η as the quality of the match within a couple (abusing notation we also use it to index the quality of the match with a prospective spouse), and $\xi \in \{\xi_1, \cdots, \xi_{n_\xi}\}$ indicates the type of person of the opposite sex (if any) to which the person is associated at the beginning of the period. If married, $q = 1$, ξ is the education-wage indicator of the spouse, if single, $q = 0$, ξ indicates the education-wage type of a prospective spouse that has shown up. As we will see there are dramatic differences between prospective spouses. In the absence of a couple children are associated with the mother. A single man who meets a single woman with children does not care for the well-being of those children when making the decision of whether to marry. Once he is married, those children will be treated as if they were the man's own offspring. This has two nice properties: one we do not have to keep track of the descendents of men, and two it avoids having a bargaining problem between the two members of the couple since they agree (once they are married).

- The evolution of these variables is as follows: the person's own wage evolves according to a Markov process conditional on education which is constant throughout the life of the person (but not of the dynasty). Let $\Gamma_g[\epsilon'|\epsilon]$ denote the relevant Markov process. The number of children conditional on not aging can only be either the previous number or the previous number plus one and is chosen by the mother alone. If the partner is a new one, the quality of match shock is drawn from some γ_η distribution; if the partner is the spouse, then η follows a Markov process with matrix $\Gamma_\eta[\eta'|\eta]$. Finally ξ evolves according to the same rules as ϵ does, conditional on remaining married. New prospective partners are drawn from the single population, whatever this may be (an equilibrium object: $\{x_f(\xi, n, 0, ., .), x_m(\xi, 0, 0, ., .)\}$. Note that the children go into custody of the mother upon divorce. Since parents care about their children's future well-being, this latest feature of the model implies that prospective partners' effective preferences over the number of children differ.

- Use $x_m(\epsilon, n, q, \xi, \eta)$ to denote the measure of males of type $(\epsilon, n, q, \xi, \eta)$ and use $x_f(\epsilon, n, q, \xi, \eta)$ to denote the corresponding measure of females. Recall that we are only considering steady states for now so x is not an individual state variable.

- The functions $G_{g,s}(\epsilon, n, q, \xi, \eta)$ for $g \in \{f, m\}$ represent the value of being single today given type $\{\epsilon, n, q, \xi, \eta\}$, while functions $G_{g,m}(\epsilon, n, q, \xi, \eta)$ for $g \in \{f, m\}$ represent the value of being married today to the $\{\xi, \eta\}$ type that the person is associate with. These functions assume that future behavior is optimal.

- The functions $V_g(\epsilon, n, q, \xi, \eta)$ for $g \in \{f, m\}$ are the value functions and they give the highest possible value of being single or being married if the latter option is available (it requires agreement of the prospective spouse).

- The functions $\Omega_{g,m}(\epsilon, \xi, \eta)$ represent the expected value of being retired for females and males that are currently married. $\Omega_{g,s}(\epsilon)$ represent the expected value of being retired for females and males that are currently single. The pair $\{\epsilon, \xi\}$ for married people and $\{\epsilon\}$ for singles are assumed to be the same from now on (no more changing of partners for retirees).

- We still have to describe the evolution of the wage-education variable ϵ across members of the same dynasty. Conditional on being of some education group the offspring draw their wage from some distribution γ^h or γ^ℓ. The offspring become educated with certain probability. This probability increases with the time and money that the parents allocate to the children's education and decreases with the number of children. Children's human capital totally depreciates between two periods.

- The current utility function has as inputs the consumption enjoyed by each household member, which is a function of total household consumption c and the number of people in the household. The current utility function also has as an input whether there are changes in the marital status of the person (this helps to account for separation costs). The consumption enjoyed by each household member is denoted by $\bar{c}$ where $\bar{c} = \frac{c}{\varphi(1,n')}$ in the single female household and by $\tilde{c} = \frac{c}{\varphi(2,n')}$ in the two parent household. Given our timing, the number of children is the new number of children that we denote n'. The presence of a spouse is detected by both the type of spouse ξ and whether the couple remains together, while q indicates whether the person was married or not the previous period. We are using short intervals of time, so we do not allow for consecutive, but different, partners. As we will see later on, we use the indicator function $\chi_g(\epsilon, n, q, \xi, \eta)$ to describe whether a partnership effectively forms. Necessarily, $\chi_f(\epsilon, n, q, \xi, \eta) = \chi_m(\xi, n, q, \epsilon, \eta)$. Of course current utility of married people also depends on how much they like each other η. Therefore, we write $u_g(\bar{c}, q, q', \eta) = u_g(\bar{c}, q, 0, 0)$ for single households and $u_g(\tilde{c}, q, 1, \eta)$ for married.

- There is another weird assumption that we are making. Parents do not know the sex of their children until they leave home. We abstract from gender bias in parents' investment in children's education.

- To introduce alimony and child support all we will have to do is to change the conditional distribution of ϵ' explicitly incorporating the difference between q and q'. Obviously, when this difference is positive the conditional distribution of ϵ' improves for the woman and deteriorates for the man.

- We use g^* to refer to the opposite sex to g.

4.1 Problem of a female conditional on being single (under steady state)

We first define an intermediate object $\hat{G}_{f,s}$ that takes the new number of children as given, and then the relevant object $G_{f,s}$, where the new number of children has been decided.

$$\begin{aligned}\hat{G}_{f,s}(\epsilon,n,q,z,\eta,n') &= \max_{c,y,l_f>0} u_f\left(\bar{c},q,0,0\right) + \ \pi\beta\ E\left\{V_f\left(\epsilon',n',0,z',\eta'\right)|\epsilon\right\} + \\ & (1-\pi)\beta\Omega_{f,s}(\epsilon) + \\ & b(n')(1-\pi)\frac{1}{2}\ E\left\{V_f\left(\epsilon'_f,0,0,z'_f,\eta'_f\right) + V_m\left(\epsilon'_m,n'_m,0,z'_m,\eta'_m\right)|l_f,y,\epsilon\right\}\end{aligned}$$

where $\bar{c} = \frac{c}{\varphi(1,n')}$

Subject to the budget constraint:

$$c + y = (1-l_f)\epsilon \tag{1}$$

and to the conditional probability of the children's educational success which is given by:

$$\text{Prob}(\epsilon'_g \in \epsilon^h_g) \quad = \quad \rho^h_g(\epsilon,n',y,l_f,0)$$

with ρ satisfying the Inada conditions in the last three arguments (the last one is time of the father that in this case is zero). Now we can rewrite this maximization problem without the hard to interpret conditional expectation operator. It amounts to

$$\begin{aligned}\hat{G}_{f,s}(\epsilon,n,q,\xi,\eta,n') &= \max_{c,y,l_f>0} u_f\left(\bar{c},q,0,0\right) + (1-\pi)\beta\Omega_{f,s}(\epsilon) + \pi\beta\sum_{\epsilon'}\ \Gamma_\epsilon[\epsilon'|\epsilon] \\ & \left[\sum_{\xi'}\ \sum_{\eta'} V_f\left(\epsilon',n',0,\xi',\eta'\right)\ \frac{x_m(\xi',0,0,.,.)}{x_m(.,0,0,.,.)}\ \gamma_\eta[\eta']\ \right] + b(n')\frac{1-\pi}{2} \\ & \left\{\sum_{\epsilon'_f}\ \rho_f^{\epsilon'_f}(\epsilon,n',y,l_f,0)\ \ \left[\sum_{\xi'_f}\ \sum_{\eta'_f}\ V_f\left(\epsilon'_f,0,0,\xi'_f,\eta'_f\right)\ \frac{x_m\left(\xi'_f,0,0,.,.\right)}{x_m\left(.,0,0,.,.\right)}\ \gamma_\eta\left[\eta'\right]\right]\right. \\ & \left. +\sum_{\epsilon'_m}\ \rho_m^{\epsilon'_m}(\epsilon,n',y,l_f,0)\ \left[\sum_{n'_m}\ \sum_{\xi'_m}\ \sum_{\eta'_m}\ V_m\left(\epsilon'_m,n'_m,0,\xi'_m,\eta'_m\right)\ \frac{x_f\left(\xi'_m,n'_m,0,.,.\right)}{x_f\left(.,.,0,0,.\right)}\ \gamma_\eta\left[\eta'\right]\right]\right\}\end{aligned}$$

Note that this problem has special features. The value function has a finite domain.

The action space does not. So given a vector of values for tomorrow, we iterate in this problem by solving finitely many (the size of the state space) maximization problems that have the features that its FOC completely characterize the solution. We denote the solutions of this problem by functions $\hat{y}_{f,s}(\epsilon, n, q, \xi, \eta, n')$, $\hat{c}_{f,s}(\epsilon, n, q, \xi, \eta, n')$, $\hat{l}_{f,s}(\epsilon, n, q, \xi, \eta, n')$. We now write the decision of how many children to have as:

$$n^*_{f,s}(\epsilon, n, q, \xi, \eta) \equiv \underset{n' \in \{n, n+1\}}{\text{Argmax}} \; \hat{G}_{f,s}(\epsilon, n, q, \xi, \eta, n') \tag{2}$$

So we write for reasons of convenience

$$\begin{aligned}
G_{f,s}(\epsilon, n, q, \xi, \eta) &= \hat{G}_{f,s}(\epsilon, n, q, \xi, \eta, n^*_{f,s}(\epsilon, n, q, \xi, \eta)) \\
c_{f,s}(\epsilon, n, q, \xi, \eta) &= \hat{c}_{f,s}(\epsilon, n, q, \xi, \eta, n^*_{f,s}(\epsilon, n, q, \xi, \eta)) \\
l_{f,s}(\epsilon, n, q, \xi, \eta) &= \hat{l}_{f,s}(\epsilon, n, q, \xi, \eta, n^*_{f,s}(\epsilon, n, q, z, \eta)) \\
y_{f,s}(\epsilon, n, q, \xi, \eta) &= \hat{y}_{f,s}(\epsilon, n, q, \xi, \eta, n^*_{f,s}(\epsilon, n, q, \xi, \eta))
\end{aligned}$$

And from now on we will drop the arguments of the function $n^*_{f,s}$ that returns the optimal number of children that a woman chooses to have given the state.

4.2 Situation of a female conditional on being married

Now there is no explicit optimization for the female as such. We proceed in two steps: first we describe how to update the female function $G_{f,m}$ given joint decisions, second we will describe which procedure is used to arrive at these joint decisions. To this end, define the function $\hat{G}_{f,m}(\epsilon, n, q, \xi, \eta, n')$ as the value for a type $\{\epsilon, n, q\}$ woman who has a type ξ partner with compatibility η keeps the partner and has n' children this period.

$$\begin{aligned}
\hat{G}_{f,m}(\epsilon, n, q, \xi, \eta, n') &= u_f(\tilde{c}, q, 1, \eta) + \pi\beta \, E\{V_f(\epsilon', n', 1, \xi', \eta') \,|\epsilon, \xi, \eta\} + \\
&\qquad (1-\pi)\beta\Omega_{f,m}(\epsilon, \xi, \eta) + \\
&\qquad b(n')(1-\pi)\frac{1}{2} \, E\left\{V_f\left(\epsilon'_f, 0, 0, \xi'_f, \eta'_f\right) + V_m(\epsilon'_m, n'_m, 0, \xi'_m, \eta'_m) \,|l_f, l_m, y, \epsilon\right\}
\end{aligned}$$

where $\tilde{c} = \frac{c}{\varphi(2,n')}$

Take the decisions that are made to be denominated $\hat{y}_{f,m}(\epsilon, n, q, \xi, \eta, n')$, $\hat{c}_{f,m}(\epsilon, n, q, \xi, \eta, n')$, $\hat{l}_{f,m}(\epsilon, n, q, \xi, \eta, n')$, $\hat{l}_{m,m}(\epsilon, n, q, \xi, \eta, n')$. Now since we assume that it is the woman who chooses the number of children, we can define the value with the woman's preferred number of children by solving the problem

$$n^*_{f,m}(\epsilon, n, q, \xi, \eta) = \underset{n' \in \{n, n+1\}}{\text{Argmax}} \; \hat{G}_{f,m}(\epsilon, n, q, \xi, \eta, n') \tag{3}$$

So we write, again for reasons of convenience

$$\begin{aligned}
G_{f,m}(\epsilon, n, q, \xi, \eta) &= \hat{G}_{f,m}(\epsilon, n, q, \xi, \eta, n^*_{f,m}) \\
c_{f,m}(\epsilon, n, q, \xi, \eta) &= \hat{c}_{f,m}(\epsilon, n, q, \xi, \eta, n^*_{f,m}) \\
l_{f,m}(\epsilon, n, q, \xi, \eta) &= \hat{l}_{f,m}(\epsilon, n, q, \xi, \eta, n^*_{f,m}) \\
y_{f,m}(\epsilon, n, q, \xi, \eta) &= \hat{y}_{f,m}(\epsilon, n, q, \xi, \eta, n^*_{f,m}) \\
l_{m,m}(\epsilon, n, q, \xi, \eta) &= \hat{l}_{m,m}(\epsilon, n, q, \xi, \eta, n^*_{f,m})
\end{aligned}$$

4.3 Situation for males (under steady state)

To describe the situation for males, recall that they do not participate in the decision of whether to have children. Males who are single have zero children. Males who are married have the number of children that their wives want.

4.3.1 Single Males

In fact, conditional on being single, males make no relevant economic decisions, (their home education time is zero and they consume what they earn) and their values are given by

$$G_{m,s}(\epsilon, n, q, \xi, \eta) \;=\; u_m\left(c, q, 0, 0, 0\right) + \;\pi\beta\; E\left\{V_m\left(\epsilon', n', 0, \xi', \eta'\right)|\epsilon\right\} + (1-\pi)\beta\Omega_{m,s}(\epsilon, 0, 0)$$

Subject to the budget constraint:

$$c = \epsilon \tag{4}$$

Note that in this case, n refers to the number of children of the prospective spouse. Also note that divorce costs and alimony can be introduced by having $u_m(c,1,0,0,0) < u_m(c,0,0,0,0)$.

4.3.2 Married Males

Recall that males who are not yet married have no concern about their prospective spouses' children. They do have concern about their own children. We therefore write two problems depending on whether they were married the previous period or not. Males who remain married to women of type ξ with shock η have the following value:

$$\begin{aligned}G_{m,m}(\epsilon,n,1,\xi,\eta,n^*) \quad &= \quad u_m(\tilde{c},1,1,\eta) + \ \pi\beta E\{V_m(\epsilon',n^*,1,\xi',\eta')\,|\epsilon,\xi,\eta\} + (1-\pi)\beta \\ &\Omega_{m,m}(\epsilon,\xi,\eta) \ + \ b(n^*)\frac{1-\pi}{2}\,E\left\{V_f\left(\epsilon'_f,0,0,\xi'_f,\eta'_f\right)\right. \\ &\left.+V_m\left(\epsilon'_m,n'_m,0,\xi'_m,\eta'_m\right)|l_f,l_m,y,\epsilon\right\}\end{aligned}$$

where $\tilde{c} = \frac{c}{\varphi(2,n^*)}$

Note that in this case, n refers to the number of children of the prospective spouse, and n^* to the new number of children that the wife will choose.

Note that the maximization of the right hand side by means of the choice of consumption, education investment and father and mother allocation time to the children is the same as that which we obtained in maximizing the right hand side of the married female's problem. Therefore in this model there is no battle of the sexes nor any other couple disagreement, as the number of children is chosen by the mother (she takes into account how certain choices might affect the odds that the spouse leaves or the odds of getting a new spouse). We write the decision functions that arise from solving the married couple's problem as $c(\epsilon,n^*,q,\xi,\eta)$, $l_f(\epsilon,n^*,q,\xi,\eta)$, $l_m(\epsilon,n^*,q,\xi,\eta)$, $y(\epsilon,n^*,q,\xi,\eta)$.

Males who were single the previous period and this period marry a woman of type ξ with shock η assess their future according to the following value function:

$$G_{m,m}(\epsilon, n, 0, \xi, \eta, n^*) = u_m(\tilde{c}, 0, 1, \eta) + \pi\beta E\{V_m(\epsilon', n^*, 1, \xi', \eta')|\epsilon, \xi, \eta\}$$
$$+(1-\pi)\beta\Omega_{m,m}(\epsilon, \xi, \eta)$$

where $\tilde{c} = \frac{c}{\varphi(2,n^*)}$

Recall that the male in this situation understands that once he is married he will accept (and care for) the children of his partner as his own children. Now, before the marriage, however he does not care for them.

4.4 Should we stay or should we go?

Each person then has to decide whether to form a couple or not to do so. Of course, it takes agreement of both parties to form a couple.

A female at the beginning of the period assesses her options in the following way:

$$\max \quad \{G_{f,s}(\epsilon, n, q, \xi, \eta), G_{f,m}(\epsilon, n, q, \xi, \eta)\}.$$

Her prospective partner faces the following decision himself

$$\max \quad \{G_{m,s}(\epsilon, n, q, \xi, \eta),\ G_{m,m}(\epsilon, n, q, \xi, \eta)\}.$$

Whether they remain together depends on both people wanting to do so.

Therefore, we write

$$V_g(\epsilon, n, q, \xi, \eta) \equiv \begin{cases} G_{g,m}(\epsilon, n, q, \xi, \eta) & \text{if } \begin{cases} G_{f,m}(\epsilon, n, q, \xi, \eta) > G_{f,s}(\epsilon, n, q, \xi, \eta) & \text{and} \\ G_{m,m}(\epsilon, n, q, \xi, \eta) > G_{m,s}(\epsilon, n, q, \xi, \eta) \end{cases} \\ G_{g,s}(\epsilon, n, q, \xi, \eta) & \text{otherwise.} \end{cases} \tag{5}$$

4.5 A few remarks on the equilibrium properties

- Married people or prospective partners are matched to each other:

$$x_m(\epsilon, n, q, \xi, \eta) = x_f(\xi, n, q, \epsilon, \eta). \tag{6}$$

- It would be easy to incorporate divorce costs as either a loss of current consumption or a degradation of the people's own types.

- In the same fashion, child support could be thought of as a transfer from the man to the woman of goodness of their current types.

4.6 Competitive equilibrium

Definition. *A stationary equilibrium is a set of decision rules for consumption* $\{c_g(\epsilon, n, q, \xi, \eta)\}_{g \in f,m}$*, resources* $\{y_g(\epsilon, n, q, \xi, \eta)\}_{g \in f,m}$*, and time* $\{l_g(\epsilon, n, q, \xi, \eta)\}_{g \in f,m}$ *investment in children's education, number of children chosen by women* $n'_f(\epsilon, n, q, \xi, \eta)$*, value functions for males and females* $\{V_g(\epsilon, n, q, \xi, \eta)\}_{g \in f,m}$*, functions that yield the value for marrying and for remaining single,* $\{G_{g,s}(\epsilon, n, q, \xi, \eta), G_{g,m}(\epsilon, n, q, \xi, \eta)\}_{g \in f,m}$*, indicator functions that determine who gets or remains married and who does not,* $\{\chi_g(\epsilon, n, q, \xi, \eta)\}_{g \in \{f,m\}}$*, stationary distributions of males* $x_m(\epsilon, n, q, \xi, \eta)$*, and females* $x_f(\epsilon, n, q, \xi, \eta)$*, and a rate of population growth* λ *such that:*[1]

1. Functions V and G are constructed as above (i.e. agents maximize).

$$V_g(\epsilon, n, q, \xi, \eta) = \chi_g(\epsilon, n, q, \xi, \eta) G_{g,m}(\epsilon, n, q, \xi, \eta) + [1 - \chi_g(\epsilon, n, q, \xi, \eta)] G_{g,s}(\epsilon, n, q, \xi, \eta)$$

2. Individual decisions do in fact generate the indicator functions that they take as given

[1] Note that we are indexing the consumption and investment functions by sex. This facilitates the record keeping and simplifies notation. Of course, a new equilibrium condition is required.

when assessing their future prospects.

$$\chi_f(\epsilon,n,q,\xi,\eta) = \chi_m(\xi,n,q,\epsilon,\eta) \equiv \begin{cases} 1 & \text{if} \begin{cases} G_{f,m}(\epsilon,n,q,\xi,\eta) > G_{f,s}(\epsilon,n,q,\xi,\eta) \\ G_{m,m}(\xi,n,q,\epsilon,\eta) > G_{m,s}(\xi,n,q,\epsilon,\eta) \end{cases} \\ 0 & \text{otherwise.} \end{cases}$$

3. Consistency between male and female equilibrium objects (the χ's) and decision rules (the $c's$ and the y's).

$$\chi_f(\epsilon,n,q,\xi,\eta) = \chi_m(\xi,n,q,\epsilon,\eta)$$

$$y_f(\epsilon,n,q,\xi,\eta) = y_m(\xi,n,q,\epsilon,\eta) \qquad \text{if} \quad \chi_f(\epsilon,n,q,\xi,\eta) = 1$$

$$c_f(\epsilon,n,q,\xi,\eta) = c_m(\xi,n,q,\epsilon,\eta) \qquad \text{if} \quad \chi_f(\epsilon,n,q,z\xi,\eta) = 1$$

4. Individual and aggregate behavior are consistent. This implies that the law of motion of the population yields a stationary distribution after normalizing by aggregate population growth. The latter, as is easy to see, is in steady state given by the following expression:

$$\lambda = \pi + \frac{1}{2}(1-\pi)\sum_{\epsilon}\sum_{n}\sum_{q}\sum_{\xi}\sum_{\eta} x_f(\epsilon,n,q,\xi,\eta)n^*(\epsilon,n,q,\xi,\eta) \tag{7}$$

For ease of exposition we write the law of motion of the population in three parts, distinguishing between those females and males who chose to be married last period, and those who did not. We start with the former

$$\lambda x_g(\epsilon',n',1,\xi',\eta') = \pi \sum_{\epsilon} \Gamma_g[\epsilon'|\epsilon] \sum_{\xi} \Gamma_{g^*}[\xi'|\xi] \sum_{\eta} \Gamma_{\eta}[\eta'|\eta] \sum_{q}\sum_{n} 1_{n'=n^*(\epsilon,n,q,\xi,\eta)}\, \chi_g\left(\epsilon,n,q,\xi,\eta\right) x_g\left(\epsilon,n,q,\xi,\eta\right) \tag{8}$$

where $1_{n'=n^*(\epsilon,n,q,\xi,\eta)}$ denotes the indicator function that takes the value of one if the statement is true and zero otherwise. We now turn to those who start the period being single and who include the newly aged, and for whom the new prospective partners are different than in last periods. To account for this group it is easier if we distinguish by sex, so we start with the women.[2]

$$\begin{aligned}\lambda x_f(\epsilon', n', 0, \xi', \eta') &= \pi\gamma_\eta[\eta'] \sum_\epsilon \Gamma_f[\epsilon'|\epsilon] \sum_n \sum_q \sum_\xi \sum_\eta 1_{n'=n^*(\epsilon,n,q,\xi,\eta)} \Big[1 - \chi_f(\epsilon, n, q, \xi, \eta)\Big] \\ &\qquad x_f\,(\epsilon, n, q, \xi, \eta) \quad \frac{x_m(\xi', 0, 0, ., .)}{x_m(., 0, 0, ., .)} \\ &\qquad + 1_{n'=0}\, \frac{1-\pi}{2} \sum_\epsilon \sum_n \sum_q \sum_\xi \sum_\eta \rho_f^{\epsilon'}(\epsilon, n^*, y^*, l_f^*, l_m^*) \;\; n^* \\ &\qquad x_f\,(\epsilon, n, q, \xi, \eta) \quad \frac{x_m(\xi', 0, 0, ., .)}{x_m(., 0, 0, ., .)} \end{aligned} \tag{9}$$

where the starred variables refer to the decision rules described above, and whose arguments are $(\epsilon, n, q, \xi, \eta)$ (again we do this for economy of notation). Finally we turn to the males who start the period being single and who include the newly aged, and for whom the new prospective partners are different than in last period.

$$\begin{aligned}\lambda x_m(\epsilon', n', 0, \xi', \eta') &= \pi\gamma_\eta[\eta'] \sum_\epsilon \Gamma_m[\epsilon'|\epsilon] \sum_n \sum_q \sum_\xi \sum_\eta [1 - \chi_m(\epsilon, n, q, \xi, \eta)] \\ &\qquad x_m\,(\epsilon, n, q, \xi, \eta) \; \frac{x_f(\xi', n', 0, ., .)}{x_f(., ., 0, ., .)} \\ &\qquad + \frac{1-\pi}{2} \sum_\epsilon \sum_n \sum_q \sum_\xi \sum_\eta \rho_f^{\epsilon'}(\epsilon, n^*, y^*, l_f^*, l_m^*) \;\; n^* \\ &\qquad x_f\,(\epsilon, n, q, \xi, \eta) \; \frac{x_f(\xi', n', 0, ., .)}{x_f(., ., 0, ., .)} \end{aligned} \tag{10}$$

[2]It will also be easy to add assertive sorting by educational-wage group. Note that we left the term of the other sex inside the summation signs, when it could be pulled out. The reason is to leave open the possibility of adding a multiplicative term $\iota(\epsilon', \xi')$ that is bigger when the arguments are similar.

In the two last expressions we are using tomorrow's distribution to determine tomorrow's distribution. However, given today's distribution and the set of decision rules, the measure of males of type ξ' who start tomorrow's period being single is given by

$$\lambda x_m\left(\xi', 0, 0, ., .\right) = \pi \sum_{\xi} \Gamma_m\left[\xi'|\xi\right] \sum_{n} \sum_{q} \sum_{\epsilon} \sum_{\eta} \left[1 - \chi_m(\xi, n, q, \epsilon, \eta)\right] \tag{11}$$

$$x_m\left(\xi, n, q, \epsilon, \eta\right)$$

$$+ \frac{1-\pi}{2} \sum_{\epsilon} \sum_{n} \sum_{q} \sum_{\xi} \sum_{\eta} \rho_m^{\xi'}(\epsilon, n^*, y^*, l_f^*, l_m^*) \;\; n^*$$

$$x_f\left(\epsilon, n, q, \xi, \eta\right)$$

The sum of the last expression over ξ' gives the measure of males who start tomorrow's period being single, i.e. $\lambda x_m\left(., 0, 0, ., .\right)$. In a similar way, the measure of females of type $\{\xi', n'\}$ who start tomorrow's period being single is given by

$$\lambda x_f\left(\xi', n', 0, ., .\right) = \pi \sum_{\xi} \Gamma_f\left[\xi'|\xi\right] \sum_{n} \sum_{q} \sum_{\epsilon} \sum_{\eta} 1_{n'=n^*(\xi,n,q,\epsilon,\eta)} \left[1 - \chi_f(\xi, n, q, \epsilon, \eta)\right] \tag{12}$$

$$x_f\left(\xi, n, q, \epsilon, \eta\right)$$

$$+1_{n'=0} \frac{1-\pi}{2} \sum_{\epsilon} \sum_{n} \sum_{q} \sum_{\xi} \sum_{\eta} \rho_f^{\xi'}(\epsilon, n^*, y^*, l_f^*, l_m^*) \;\; n^*$$

$$x_f\left(\epsilon, n, q, \xi, \eta\right)$$

The sum of the previous expression over ξ' and n' gives the measure of females who start tomorrow's period being single $\lambda x_f\left(., ., 0, ., .\right)$

4.7 Technical Issues

Finding an equilibrium requires value function iteration.

$$\{V_m^1, V_f^1\} = T(V_m^0, V_f^0)$$

In standard models this mapping is continuous. In this model it is not. The reason is the required agreement of the two parties to be married and the absence of transferable utility. We solve this problem by introducing an effort variable that determines the probability of success in achieving what is desired (children, marriage). Moreover, this mapping is not monotonic which implies that iterations need not converge. Effectively this is solved by slow updating.

5 Calibration of the Baseline Economy (the Seventies)

For most of the parameters of the model we have a priori no insights on what would be a reasonable range of values. A comprehensive calibration strategy is needed. We calibrate all the parameters with a method of moments procedure with no over-identifying restrictions. We pick as many statistics computed from the data as the parameters we have in the model. We then search over all parameters in order to minimize a weighted sum of squared errors resulting from the difference between the statistics computed from the data and the corresponding statistics computed from the model equilibrium allocations. We use the following set of statistics to perform the calibration of the baseline model economy.

- Wage dispersion within sexes.
- Wage gender gap.
- Ratio between average earnings of "bottom" women and men.
- Ratio between average earnings of "top" women and men.
- Distribution of women by marital status in the different earnings groups.

- Distribution of women by parental status in the different earnings groups. More precisely
  - distribution of women by number of children in the different earnings groups.
  - share of mothers by different earnings groups.
- Marriage and divorce rates.
- Assortative mating that is how people from different earnings groups are paired with each other.

5.1 Demographics

We assume that when children become adults they are eighteen and that the average 'fertile' life, that correponds to adulthood, is thirty two years. This implies that on average women's childrearing period is from eighteen to fifty. A value of .911 is assigned to π, the probability of not aging, in the calibration. This implies that the length of the period is three years.

5.2 Preferences

We assume that current utility is separable in consumption, effort and quality of the match. Utility of consumption exhibits constant relative risk aversion; disutility of effort is quadratic in effort and the utility derived from the quality match indicator is linearly additive. Since only women make effort $\bar{ef}_f$ to determine the future number of children, current utility for women has an extra term with respect to current utility for men. Current utility for married women is for example given by:

$$u_f\left(\tilde{c}, q, q', ef_f, \bar{ef}_f, \eta_f\right) = \frac{\tilde{c}^{1-\sigma}}{1-\sigma} + \alpha\left(ef_f\right)^2 + \varsigma\left(\bar{ef}_f\right)^2 + \eta_f \tag{13}$$

where σ is the coefficient of relative risk aversion and it is assigned a value of 2.95. ef_f is the effort that women make in order to reduce the uncertainty of achieving the desired marital status. α is not sex-specific. Depending on previous marital status α differs. For those who start the period being married $\alpha \equiv \alpha_m$, for those who are single $\alpha \equiv \alpha_s$. α_m is set to -0.93 and α_s is set to -0.82. The third term is the quadratic disutility from effort made to determine the future number of children. ς is set to 4.32. Current utility for single women is obtained simply by replacing $\tilde{c}$ with $\bar{c}$. The quality match indicator η_g is sex-specific and symmetric. η_f for females takes the following values

$$\{-0.377 \quad -0.0001 \quad 0 \quad 0.0001 \quad 0.377\}$$

η_m for males takes the following values

$$\{-0.098 \quad -0.0002 \quad 0 \quad 0.0002 \quad 0.098\}$$

People discount their own future well-being at a rate β and their children's future welfare at a rate $b(n') \equiv \beta_c (n')^{1-\delta}$ that is increasing with n' but at a decreasing rate. β is set to .65. β_c is calibrated to be .99 and δ is set to .84.

5.3 Potential earning types

In the baseline economy men's potential earning type ξ can take one of the following values

$$\{1.0 \quad 1.0 \quad 1.962 \quad 4.169\}$$

Women's potential earning type ϵ can take one of four values

$$\{.483 \quad .483 \quad 1.140 \quad 2.089\}$$

They have been calibrated together with the Markov processes $\Gamma_\epsilon[\epsilon'|\epsilon]$ and $\Gamma_\xi[\xi'|\xi]$ in order to set females' wage premium, males' wage premium and sex-wage premium to 2.12, 2.27 and 1.89 in the baseline economy and to match the ratio between average earnings of top males and top females and the ratio between average earnings between bottom males and bottom females which in the data are respectively 1.93 and 1.81.

5.4 Unconditional distributions

We assume that new prospective partners draw a quality match indicator η_g from the unconditional distribution γ_η that is uniform with zero mean.

5.5 Markov Processes

Each person's own earning type evolves according to a Markov Process that is conditional on the amount of "education" received during childhood which is constant throughout the life of the person. $\Gamma_\xi[\xi'|\xi]$ is the relevant Markov process for ξ and is characterized by the following transition matrix:

$$\begin{vmatrix} .85 & .0 & .15 & .0 \\ .0 & .85 & .0 & .15 \\ .0 & .0 & 1.0 & .0 \\ .0 & .0 & .0 & 1.0 \end{vmatrix}$$

We assume an identical transition matrix for the Markov process for ϵ, $\Gamma_\epsilon[\epsilon'|\epsilon]$. Conditional on being married the quality match indicator for the couple evolves according to the Markov process $\Gamma_\eta[\eta'|\eta]$ characterized by the following transition matrix

$$\begin{vmatrix} .93 & .07 & .0 & .0 & .0 \\ .47 & .06 & .47 & .0 & .0 \\ .0 & .02 & .96 & .02 & .0 \\ .0 & .0 & .47 & .06 & .47 \\ .0 & .0 & .0 & .07 & .93 \end{vmatrix}$$

5.6 Conditional probability of children's wage-educational success

When children become adults they draw one of the two lowest earning types $\epsilon(j)$ and $\xi(j)$ with $j \in 1, 2$ respectively for females and males. The probability of drawing $\epsilon(1)$ for new adult females or $\xi(1)$ for new adult males is given by

$$\frac{1}{e^{\left(\gamma_1 l_f^{\mu_1} + \gamma_2 \left(\frac{y}{n'}\right)^{\mu_2} + \gamma_3 l_m^{\mu_3}\right)}}$$

if children have been raised in a two parent household. If children grew up in a single parent household then this probability is given by

$$\frac{1}{e^{\left(\gamma_1 l_f^{\mu_1}+\gamma_2\left(\frac{y}{n'}\right)^{\mu_2}\right)}}$$

These probabilities should be interpreted as conditional probabilities of children's wage-educational success. The first draw splits the sample of those who have become adults into high and low education types. Low education-earning type females are those who draw $\epsilon(1)$, high education-earning type are those who draw $\epsilon(2)$ upon becoming adults. It is the same for males. Given the structure of the Markov matrix for females' and males' earnings low education-earning types have lower expected life time earnings than high education-earning types. We calibrate μ_1 to .17, μ_2 to .16, μ_3 to .57 and γ_1 to 1.12 γ_2 to .87 and γ_3 to 1.18

5.7 Technology to achieve desired marriage, divorce and childbirth outcomes

As we said, for technical reasons we need to make marital status and childbirth outcomes uncertain. The probability of not increasing the number of children in the current period is given by

$$\frac{e^{\bar{ef}}}{e^{\bar{ef}}+\kappa_1 e^{-\bar{ef}}}$$

where $\bar{ef}$ is the effort made by the woman to achieve her desired number of children. κ_1 is calibrated to be .31 . The probability of getting married conditional on being single at the beginning of the period is given by

$$\frac{e^{\left(ef_f+ef_m\right)}}{e^{\left(ef_f+ef_m\right)}+\kappa_2 e^{-\left(ef_f+ef_m\right)}}$$

where ef_f and ef_m are the efforts made by the two prospective partners to achieve the desired marital status outcome. κ_2 is set equal to 4.92. The probability of staying married conditional on being married at the beginning of the period is given by

$$\frac{e^{\left(ef_f+ef_m\right)}}{e^{\left(ef_f+ef_m\right)}+\kappa_2 e^{-\left(ef_f+ef_m\right)}}$$

κ_3 is set equal to 0.06.

5.8 Value of being retired

With probability $1 - \pi$ agents age and retire. The function $\Omega_{g,s}(\epsilon)$ that represents the expected value of being retired and single assumes the same values for all ϵ and for $g \in f, m$. $\Omega_{g,s}$ is calibrated to be 150.63. The expected value of being married and retired $\Omega_{g,m}(\epsilon, \xi, \eta)$ is calibrated to 138.19 for all ϵ, ξ, η.

5.9 Properties of the Baseline Economy

In this section we describe the properties of the baseline economy and we compare them with those found in the PSID data for the mid seventies. Table 2 shows that the baseline economy closely matches the share of single females at the top and at the bottom of the female potential earnings distribution and also the aggregate share of single mothers that we found in the data. Moreover the share of poor single mothers is almost .10 in the baseline economy and .11 in the data. This is considerably less than what we find in the top half of the distribution, a pattern that replicates what we observe in the PSID data. The wage premia within and between sexes are also equal to those we find in the data. The ratios between average earnings of top males and females and between average earnings of bottom males and females are not shown in Table 2. In the baseline model economy they are respectively equal to 1.94 and 1.81, in the data they are 1.93 and 1.82 . In the baseline economy there is a certain degree of positive assortative matching of couples, i.e. high potential earning type tend to marry among each other, but not as much as we find in the data. This feature should be seen as a strength and not a weakness of the model. In the real world in fact part of the sorting is due to the fact that single people do not match randomly like in the model. It is more likely that single people with certain earning characteristics meet other singles with similar earning characteristics because they have similar social circles. As concerns flows, the baseline economy matches the marriage rate that we find in the data quite closely while the divorce rate is lower than that found in the data. The asymmetry in the flows that we observe in the data is also obtained in the model. We attain this by allowing for a different calibration of the technology to achieve desired marriage conditional on agents being previously married or single, as we said in section 5.7. If the marital status outcome were not uncertain a similar asymmetry in the flows could be obtained introducing divorce costs into the set up.

Table 2: Properties of the Baseline Model Economy and the U.S. Data

	1974 PSID	Model
Share sing. fem. rich	.204	.202
Share sing. fem. poor	.136	.138
Share sing. mothers	.121	.136
Share sing. mothers poor	.110	.098
Females' wage premium	2.12	2.12
Males' wage premium	2.27	2.27
Sex wage premium	1.89	1.89
Marriage rate	.110	.103
Divorce rate	.021	.015
Share of married rich females to rich males	.601	.514
Share of married poor females to poor males	.592	.526

6 Changing wage premia

In this section we look at the effect that changes in relative wage premia have on the initial baseline economy steady state allocation. We change wage premia within and between sexes in accordance with the pattern observed in the data between the mid seventies and the early

nineties. We then compare the before and after change equilibrium allocations.
We change relative wage premia in this order

1. Sex wage premium alone

2. Male wage premium alone

3. Female wage premium alone

4. Sex, Male, Female wage premia together

In this way we can assess the individual contribution of each change. Notice that while we implement this change in relative wages we hold constant all the remaining parameters calibrated in the baseline economy.

6.1 Change in the sex wage premium

Given the initial equilibrium allocation of the baseline economy we reduce the average sex wage premia to match the drop observed in the data from 1.89 to 1.56, keeping males' and females' wage premia constant. Reducing the sex wage premia has important effects on the main statistics of interest. It increases the share of single females both at the top (30%) and at the bottom (71%) of the females' potential earnings distribution. This is achieved mainly through a drop of 17% in the marriage rate while the divorce rate is almost unchanged. After the change single males are generally more willing to get married because the "quality" of prospective partners has improved. On the other hand both "rich" and "poor" single females become pickier at the moment they have to take marriage decisions and less willing to marry. They are also more reluctant in accepting a match with a low earning male. The value of being a single woman has increased relative to that of being a married woman for each earning type single female, in particular for low earning females. Married females are more willing to divorce males, while married males are less willing to damp their partners. The fact that the shift in household structure takes place through a reduction in the marriage rate rather than through an increase in the divorce rate is a result of the calibration of the technology to achieve the desired marriage outcome of section 5.7. Given the value of being married and single, if the second is greater than the first less effort is required to achieve the desired outcome when agents are already single. The net effect of this change in relative wages between sexes is an increase in the share of single females and of single mothers.

Table 3: Change in the sex wage premium

	new alloc	model change	74-91 data
Sex wage premium	1.56	-17%	-17%
Share sing. fem. rich	.243	30%	42%
Share sing. fem. poor	.237	71%	106%
Share sing. mothers	.198	46%	41%
Share sing. mothers poor	.172	75%	90%
Females' wage premium	2.12	0%	21%
Males' wage premium	2.27	0%	16%
Marriage rate	.085	-17%	-18%
Divorce rate	.016	6%	0%
Share of married rich females to rich males	.506	-1.4%	3%
Share of married poor females to poor males	.508	-3.3%	5%

In fact the share of single mothers increases by 46% while the share of mothers in the economy is left unchanged. The increase in childrearing out of wedlock is higher among "poor" women.

6.2 Change in males' wage premium

Next, we change males' wage premium keeping constant both sex wage premium and females' wage premium. When we change males' wage premium to match the sharp rise observed in the data we obtain an increase in the share of single females both at the top, by 1.9%, and at the bottom, by 5.6%, of their earnings potential distribution. Females are less willing to marry or stay married with "poor" males because the drop in earnings potential of bottom males make them much worse partners than before.

Table 4: Change in males' wage premium

	new alloc	model change	data 74-91
Males' wage premium	2.63	16%	16%
Share sing. fem. rich	.206	1.9%	42%
Share sing. fem. poor	.147	5.7%	106%
Share sing. mothers	.141	3.5%	41%
Share sing. mothers poor	.103	5%	90%
Females' wage premium	2.12	0%	21%
Sex wage premium	1.89	0%	-17%
Marriage rate	.101	-1.9%	-18%
Divorce rate	.015	0%	0%
Share of married rich females to rich males	.523	1.7%	3%
Share of married poor females to poor males	.512	-2.6%	5%

6.3 Change in females' wage premium

In fact the increase in the share of single males is concentrated among the "poor". The result is that we observe a reduction in the share of poor women married to poor men. On the other hand top males, whose earnings have improved with respect to females' wage are now pickier than before on the marriage market, but they are also better prospective partners than before. The second effect partially outweighs the first. Therefore neither the share of rich married men nor the proportion of rich married woman show important modifications. In particular more rich women are successful in marrying top men.

Table 5: Change in females' wage premium

	new alloc	model change	74-91 data
Females' wage premium	2.56	21%	21%
Share sing. fem. rich	.186	-8.5%	42%
Share sing. fem. poor	.105	-24.4%	106%
Share sing. mothers	.118	-11.7%	41%
Share sing. mothers poor	.076	-22.1%	90%
Males' wage premium	2.27	0%	16%
Sex wage premium	1.89	0%	-17%
Marriage rate	.108	4.8%	-18%
Divorce rate	.015	0%	0%
Share of married rich females to rich males	.513	0%	3%
Share of married poor females to poor males	.512	-2.6%	5%

We increase females' wage premium from 2.12 to 2.56 as observed in the data for the U.S. economy, keeping males' relative wages and sex wage premium constant to the baseline economy values. In the model economy we have a reduction in the share of single females with respect to the baseline economy. These effects mirror those we obtained increasing males' wage premium, but with an opposite sign and a bigger magnitude. On one hand the drop in potential earnings of poor women who are at the bottom of the distribution make them less picky at the moment of taking marital decisions. On the other hand they become less attractive partners for single males. The first effect outweighs the second and we observe a conspicuous reduction in the share of poor single women and poor single mothers.

Women at the top of the earnings distribution see their relative earnings improve with respect to men's potential earnings. These single women are now willing to wait longer for the right mate. On the other hand all men are more willing to accept a match with a prospective top earnings female partner because their quality has improved. The latter effect dominates the former and the result is also a drop in the number of single women at the top of the females' earnings distribution. It is again the change in the marriage rate which is responsible for the observed dynamics.

6.4 Change all wage premia

Once we have assessed the individual effect of each of the changes in relative wages we consider what happens to the model economy when we increase wage-premia within sexes and decrease wage-premia between sexes simultaneously. Table 6 reports the results. We change relative wages within and between sexes also matching the ratios between average wages of top males and females, and of bottom males and females that in the data drop respectively from 1.93 to 1.57 and from 1.82 to 1.53. The share of rich single females increases by 21%. This is less than what we obtained by simply reducing the sex wage premium. The effect induced by the widening of the males' wage premium that increases the share of "rich" single females is dominated by the drop in the share of "rich" single females obtained with the increase in females' wage premium. The same happens at the bottom of the females' earnings distribution.

Table 6: Change in all wage premia together

	new alloc	model change	data 74-91
Females' wage premium	2.56	21%	21%
Males' wage premium	2.63	16%	16%
Sex wage premium	1.56	-17%	-17%
Share sing. fem. rich	.244	21%	42%
Share sing. fem. poor	.204	47%	106%
Share sing. mothers	.177	30%	41%
Share sing. mothers poor	.144	47%	90%
Marriage rate	.090	-12%	-18%
Divorce rate	.016	6%	0%
Share of married rich females to rich males	.506	-1.4%	3%
Share of married poor females to poor males	.506	-3.5%	5%

The net effect of the widening in the wage-premia within sexes is to reduce the rise in the share of single females and single mothers induced by the drop in the sex wage premia. In the model economy we obtain a dramatic 30% increase in the share of single mothers. This rise is more significant among "poor" women in the model economy like in the data. In the data the increase in the share of single mother is accompanied by a reduction in the total amount of mothers. In the model economy the total share of mothers instead is left virtually unchanged. Changing simultaneously all relative wages accounts for:

- half of the change in the share of rich single women

- 44% of the change in the share of poor single women
- 73% of the change in the share of single mothers
- half of the change in the share of “poor“ single mothers

This is achieved through a reduction in the marriage rate and a stable divorce rate in the model economy like in the data. We compute the inter-generational earnings correlation both in the baseline economy and in the model economy where all three relative wages have changed. What we observe is an increase in this correlation that can be attributed to the shift in the marital status composition of the population towards a higher share of single parent headed households. In fact if we sort households according to total household income single headed households are more likely to be in the bottom tail of the distribution. Moreover in our model children who grow up in a single female headed family have 30% less chance of becoming a high earning type when adults. This is comparable with the results of McLanahan and Sandefur (94) who find that children raised in one parent families are 13% more likely to drop out of high school and even less likely to enroll in college. Nevertheless the value of the inter-generational correlation of earnings is too low in the model economy of the nineties. The value of this correlation is .17 which is positive but still far from what has been found by some empirical studies. For example a value of .41 is found by Solon (92) using the PSID, and an inter-generational correlation of earnings of .68 is estimated by Zimmerman (92). The low intergenerational correlation of income that characterizes the model economy is due to the reduced difference in per child investment between rich and poor households, caused by the high fertility behaviour of rich mothers.

6.5 What happens to fertility ?

In the model economy the average number of children per mother does not change significantly when we move relative wages, nor does the share of mothers. Birth rates of poor and rich women are not affected by the change either. This result is in line with the stable fertility pattern observed for the U.S. during the last thirty years. The model economy does not match the negative relationship between mothers' earnings and average number of children that we observe in the data:

	PSID74
Kids per rich mother	2.39
Kids per poor mother	2.72

In the model economy the relationship between mother's earning type and average number of children is relatively flat, due to the high fertility of rich mothers. High earning type females substitute time with resources investment in children's education. The substitution effect plus the wealth effect generated by high earnings cause rich women's fertility to be close to poor women's fertility. To increase the cost of childrearing respectively more for high earnings females, a fixed time cost of childrearing is usually introduced. In this set up wealth effects are so strong that a reasonable parametrization for this time cost is unsuccesful in achieving the negative relationship females' earnings average number of kids. A suitable alternative would be to make a female's earnings transition probabilities dependent on the total number of children she had in the past. In this case a high fertility behaviour in early stages of adult life would be associated with a reduced chance of moving from a low to a higher earning type conditional on the education received. This would reflect the loss of work experience associated with the fixed amount of time women spent childrearing or the loss of some experience premium not directly related to market hours.

7 Conclusion

In this paper we measure the contribution of changes in relative wages within and between sexes in accounting for the shift in household structure observed during the last two decades in the U.S. We build a general equilibrium model where agents differ by sex and take fertility, marriage-divorce and investment in children's education decisions. We calibrate the model economy to match the statistics we computed from the data for the mid seventies and then we change relative wages and compare the changes induced in the model equilibrium allocations with those observed in the data. We find that changes in relative wages account for: half of the change in the share of rich single women, 44% of the change in the share of poor single women, 73% of the change in the share of single mothers and half of the change in the share of poor single mothers. Much of the change in the share of single females and single mothers is produced by the shrinking in the sex wage premium. The effects produced by changing relative wages between sexes partially offset each other. The rise in the share of "poor" single mothers, that we also find in the data, is also drastic in the model economy. The model economy is able to reproduce the drop in the marriage rate that we observe in the data. In our model children who grow up in a single female headed family have way less chance of becoming a high earning type. Empirical studies support this result. The model up to now fails in two related dimensions: It cannot closely reproduce either the high inter-generational persistence in earnings found in the U.S. (in the model economy is just .17) or the earnings-number of children negative relationship that we observe in the data. Introducing fixed cost of childbearing or childrearing and making females' earnings transition probabilities dependent on the number of children attached to the family could probably help to improve the model along these two dimensions. In addition the framework developed here should allow us to address issues related to the consequences of these changes in the marital status of the population on inter-generational persistence of earnings, and also to address policy issues concerning the effect that marital status contingent income taxes or welfare measures might have on the patterns of marriage and remarriage.

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A Computational procedure

A calibrated version of the model is solved applying numerical methods. The state space is divided into a finite number of points (discrete grid). The marital status variable q and the total number of children in the household are naturally discrete. ϵ and ξ are approximated with 4 grid points respectively and η with 5. The procedure is as follows:

(i) Start with initial guesses for the measure of males $x0_m$ and $x0_f$ females and for the value functions $V0_m$ and $V0_f$

(ii) Given $x0_m$ and $x0_f$ solve the households' problem by iterating on the value functions. At each step of the value function iteration:

(ii.1) Solve the single female and the married females' problems in terms of consumption and resources allocated to childrens' education for given number of children attached to the households, using FOCs.

(ii.2) Solve females' effort decision to determine the number of children, again using FOCs.

(ii.3) Solve females' and males' effort decision to determine the current marital status outcome. Use FOCs.

(iii) Using the resulting decision rules, update the initial guesses of the measures $x0_m$ and $x0_m$; if at the first iteration on the the $x's$ the difference between $x0_m$ and $x1_m$ and between $x0_f$ and $x1_f$ is in both cases less than some tolerance value we reached the stationary distribution. Otherwise we iterate on the $x's$ until convergence. In the second case call the updated guesses for the measures of types xn_m and xn_f. Set $x0_m = xn_m$ and $x0_f = xn_f$ and go back to step (ii).