

Large Economies and Two-Player Games

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Abstract. We characterize the competitive allocations of a continuum economy as the Nash equilibria of an associated game with only two players.

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1 Introduction

In this paper we consider a pure exchange economy with a continuum of agents and finitely many commodities (Aumann 1964, 1966). We associate a game with only two players to each Aumann's economy. Our aim is to characterize the competitive allocations of the economy as Nash equilibria of the associated game.

The strategies of the first player are feasible allocations of the economy. Each strategy of the second player consists in a coalition of agents and a feasible allocation for this coalition. If all the agents in the coalition proposed by player 2 are better off with the allocation proposed by player 1, then player 1 payoff depends on the difference of utilities that the agents in the coalition obtain with the allocations proposed by the two players. Otherwise, her payoff is the essential infimum of the difference of utilities in that coalition. The payoff of the second player is symmetric to the payoff of first player.

Given any strategy of player 1, player 2 can get zero payoff by choosing the strategy given by the coalition of all agents and the same allocation as player 1. She could obtain a positive payoff if and only if the allocation proposed by player 1 can be blocked by a coalition. We show that in any Nash equilibrium both payoffs are zero.

Our main result proves that any allocation in the core of the continuum economy is a Nash equilibrium of the two players game and, reciprocally, any Nash equilibrium strategy can be identified with a core allocation in the continuum economy. Then, under the assumption of Aumann (1964, 1966) the competitive allocations of the continuum economy are characterized as Nash equilibria of the associated two players game.

In a previous work (see Hervés-Beloso and Moreno-García, 2008, we have characterized the walrasian allocations of a n -agents economy as the Nash equilibrium of the an associated two players game. Thus, in this paper we extend to the continuous case our previous result highlighting the power of the veto mechanism in atomless economies.

The power of the veto mechanism in the continuum economy can be stressed by considering the game associated with an economy with a finite number of types of agents. For it, we consider a finite pure exchange economy with n agents that can be identified as a continuum economy with n types of agents. Assuming convexity of preferences, for each competitive allocation in the n types economy, there is also a competitive allocation which is constant in each type. This step function corresponds to a Walrasian allocation in the finite economy (see García-Cutrín and Hervés-Beloso, 1993). The attempt to apply our main result to a finite economy, via an n -types continuum economy, does not longer allow to exploit the veto power of the coalitions as we do in the continuum case. This is due to the fact that in the n agents economy the information about the coalition proposed by the player two which affect to both agents' payoff becomes insufficient. In fact, the size of the agents of each type forming the coalition is the only data that can be collected in the payoff functions. Further, we highlight with an example that this is not sufficient to conclude that any Nash equilibrium underlies a Walrasian allocation. Precisely, in our example there is a Nash equilibrium of the game associated to the finite economy with positive payoff for player two. Therefore, the allocation proposed by player one can not be Walrasian.

In our study we have only considered a finite number of commodities. This assumption is not essential. Our main result is actually a characterization of the core. Then, the same result holds for a general commodity space under the assumptions that guarantee the core-Walras equivalence.

The remainder of this paper is organized as follows. In Section 2 we define the continuum economy, we state the assumptions that guarantee the core-Walras equivalence and we also define the associated two-players game. In Section 3 we present the properties of the game, we prove our main result and we state some remarks regarding the size of the coalition selected by player 2 for which the main result does still hold. In Section 4, we consider the particular case of a continuum economy with n types of agents in order to recast the associated game for an Arrow-Debreu pure exchange economy. A final example points out that the discrete version of the game does not allow to characterize Nash equilibria as Walrasian allocations.

2 The economy and the game

Consider a pure exchange economy \mathcal{E} with ℓ commodities. The space of consumers is represented by an atomless measure space (I, \mathcal{A}, μ) .

Each agent $t \in I$ is characterized by her consumption set \mathbb{R}_+^ℓ , her initial endowment $\omega(t) \in \mathbb{R}_+^\ell$, and her preference relation \succeq_t which is represented by the utility function U_t .

An allocation is a μ -integrable function $f : I \rightarrow \mathbb{R}_+^\ell$. An allocation f is feasible in the economy \mathcal{E} if $\int_I f(t) d\mu(t) \leq \int_I \omega(t) d\mu(t)$.

A price system is an element of Δ , where Δ denotes the $(\ell - 1)$ -dimensional simplex of \mathbb{R}_+^ℓ , that is, $\Delta = \{p \in \mathbb{R}_+^\ell \text{ such that } \sum_{h=1}^{\ell} p_h = 1\}$.

A competitive equilibrium for \mathcal{E} is a pair (p, f) , where p is a price system and f is a feasible allocation such that, for almost every agent t , the bundle $f(t)$ maximizes the utility function U_t in the budget set $B_t(p) = \{y \in \mathbb{R}_+^\ell \text{ such that } p \cdot y \leq p \cdot \omega(t)\}$.

A coalition is any measurable set S with $\mu(S) > 0$. A coalition S blocks an allocation f via another allocation g in the economy \mathcal{E} if:

- (i) $\int_S g(t) d\mu(t) \leq \int_S \omega(t) d\mu(t)$ and
- (ii) $U_t(g(t)) > U_t(f(t))$ for almost all $t \in S$.

A feasible allocation belongs to the core of the economy if it is not blocked by any coalition of agents.

We suppose that the economy \mathcal{E} has competitive equilibrium and the Core-Walras equivalence holds, i.e., the core coincides with the set of competitive allocations (see Aumann, 1964, 1966 and Hildenbrand, 1968, 1974).

We define a game \mathcal{G} associated to the economy \mathcal{E} in order to analyze the relation between the non-cooperative solution of Nash equilibrium and the decentralized solution of competitive

equilibrium.

There are two players. The strategy set for the player 1 is denoted by Θ_1 and is given by

$$\Theta_1 = \{f : I \rightarrow \mathbb{R}_+^\ell : \int_I f(t) d\mu(t) = \int_I \omega(t) d\mu(t)\}$$

That is, a strategy for player 1 is a feasible allocation f . Observe that $\omega \in \Theta_1$.

The strategy set for the player 2 is denoted by Θ_2 and is defined as follows:

$$\Theta_2 = \{(S, g) : \int_S g(t) d\mu(t) = \int_S \omega(t) d\mu(t)\}$$

That is, the strategy set for player 2 is the set of pairs which specify a coalition of agents and a trade for the coalition. Observe that if f is a feasible allocation then $(I, f) \in \Theta_2$. Further, $(S, \omega) \in \Theta_2$ whatever coalition S may be.

Let Θ denote the product set $\Theta_1 \times \Theta_2$. A strategy profile is any $\theta = (f, S, h) \in \Theta$, that is, a strategy profile is a strategy $\theta_1 = f \in \Theta_1$ for player 1 and a strategy $\theta_2 = (S, g) \in \Theta_2$ for player 2.

In order to define the payoff functions, given a function $F : I \rightarrow \mathbb{R}$ and a coalition of agents $S \subset I$ let

$$\text{ess inf}\{F(t), t \in S\} = \sup\{c \in \mathbb{R} \mid F(t) \geq c \text{ for almost all } t \in S\}.$$

Given a strategy profile (f, S, g) , we define the following real valued functions

$$\alpha(f, S, g) = \text{ess inf}\{U_t(f(t)) - U_t(g(t)), t \in S\}$$

$$\beta(f, S, g) = \text{ess inf}\{U_t(g(t)) - U_t(f(t)), t \in S\}$$

Now for every $(f, S, g) \in \Theta$, the payoff functions Π_1 and Π_2 for player 1 and 2, respectively, are defined as follows

$$\Pi_1(f, S, g) = \begin{cases} \int_S (U_t(f(t)) - U_t(g(t))) d\mu(t) & \text{if } \alpha(f, S, g) \geq 0 \\ \alpha(f, S, g) & \text{otherwise} \end{cases}$$

$$\Pi_2(f, S, g) = \begin{cases} \int_S (U_t(g(t)) - U_t(f(t))) d\mu(t) & \text{if } \beta(f, S, g) \geq 0 \\ \beta(f, S, g) & \text{otherwise} \end{cases}$$

3 Main Results

In this section we analyze some properties of the game \mathcal{G} , which we have previously associated to the continuum economy, and we also present our main result which characterizes the competitive equilibrium allocations as Nash equilibria.

A strategy profile

Given the associated game \mathcal{G} , note that if $\Pi_1(f, S, g) > 0$ (resp. $\Pi_2(f, S, g) > 0$) then $\Pi_2(f, S, g) < 0$ (resp. $\Pi_1(f, S, g) < 0$). That is, both payoff can be negative for some strategy profiles but can not be strictly positive at the same time. Observe also that $\Pi_1(\theta) = 0$ if and only if $\Pi_2(\theta) = 0$.

Proposition 3.1 *The set of Nash equilibria for the game \mathcal{G} is nonempty.*

Proof. This is a consequence of the existence of competitive equilibrium for the continuum economy \mathcal{E} . In fact, if f is a competitive allocation for the economy \mathcal{E} then (f, I, f) is a Nash equilibrium for the game \mathcal{G} . To see this, note that $\Pi_1(f, I, f) = \Pi_2(f, I, f) = 0$. If player 1 has an incentive to deviate then there exists $g \in \Theta_1$ such that $\Pi_1(g, I, f) > 0$ and therefore f is not efficient. If $\Pi_2(f, S, g) > 0$ for some $(S, g) \in \Theta_2$, then f is blocked by the coalition S . By the core-Walras equivalence this is a contradiction with the fact that f is a competitive allocation.

Q.E.D.

Lemma 3.1 *If θ^* is a Nash equilibrium then $\Pi_1(\theta^*) = \Pi_2(\theta^*) = 0$.*

Proof. Let $\theta^* = (f^*, S^*, g^*)$ be a Nash equilibrium. Since (I, f^*) is a possible strategy for player 2, we have $\Pi_2(\theta^*) \geq 0$. Assume $\Pi_2(\theta^*) > 0$. Then $\Pi_1(\theta^*) < 0$. Consider the allocation f given by

$$f(t) = \begin{cases} g^*(t) & \text{if } t \in S^* \\ \omega(t) & \text{otherwise} \end{cases}$$

Note that $f \in \Theta_1$ and $\Pi_1(f, S^*, g^*) = 0$, which is a contradiction.

Q.E.D.

Proposition 3.2 *Any Nash equilibrium of the game \mathcal{G} is a strong Nash equilibrium.*

Proof. Let θ^* be a Nash equilibrium. Then, $\Pi_1(\theta^*) = \Pi_2(\theta^*) = 0$. By definition of the payoff function, both Π_1 and Π_2 can not be strictly positive at the same time and $\Pi_1(\theta) = 0$ if and only if $\Pi_2(\theta) = 0$. This implies that the coalition formed by the two players have no incentive to deviate from the profile θ^* .

Q.E.D.

Theorem 3.1 *If $\theta^* = (f^*, S^*, h^*)$ is a Nash equilibrium for the game \mathcal{G} , then f^* is a competitive equilibrium allocation for the economy \mathcal{E} .*

Reciprocally, if f^ is a competitive equilibrium allocation for the continuum economy \mathcal{E} , then any strategy profile $(f^*, I, h^*) \in \Theta$, with $U_t(f^*(t)) = U_t(h^*(t))$, for almost all $t \in I$, is a Nash equilibrium for the game \mathcal{G} .*

In particular, f^ is a Walrasian equilibrium allocation for the economy \mathcal{E} , if and only if (f^*, I, f^*) is a Nash equilibrium for the game \mathcal{G} .*

Proof. Let $\theta^* = (f^*, S^*, h^*)$ be a Nash equilibrium. By Lemma 3.1 $\Pi_1(\theta^*) = \Pi_2(\theta^*) = 0$. Assume that f^* is not a competitive allocation. Then, f^* does not belong to the core of the economy \mathcal{E} which implies that there exists $(S, g) \in \Theta_2$ such that $\Pi_2(f^*, S, g) > 0 = \Pi_2(\theta^*)$.

Reciprocally, let f^* be a competitive equilibrium allocation and $(I, h^*) \in \Theta_2$ such that $U_t(f^*(t)) = U_t(h^*(t))$, for almost all $t \in I$. Assume that $(f^*, I, h^*) \in \Theta$ is not a Nash equilibrium. Then, either player 1 or player 2 has an incentive to modify her strategy. If player 1 has an incentive to deviate f^* is not efficient and then it is not competitive. If there exists $(S, g) \in \Theta_2$ such that $\Pi_2(f^*, S, g) > 0 = \Pi_2(f^*, I, h^*)$ then f^* is blocked by S via g which, by core-Walras equivalence, is a contradiction.

Q.E.D.

Remark 1. We stress that from the proof of the previous Theorem we can deduce that it is not essential the existence of a finite number of commodities in the economy. The result is still true for atomless economies with infinitely many commodities whenever the core-Walras equivalence holds (see for instance Bewley (1973)).

Remark 2. Let us consider a number $\varepsilon \in (0, 1)$. Let $\mathcal{G}(\varepsilon)$ the game which coincides with \mathcal{G} except for the strategy set of player 2 that is restricted to coalitions S , with $\mu(S) \geq 1 - \varepsilon$, and feasible allocations for such coalitions. For the case of finitely many commodities, Vind (1972) showed that in atomless economies it is enough to consider the blocking power of coalitions with measure ε in order to get the core (see Hervés-Beloso et al. 2000, 2005 for economies with an infinite dimensional commodity space). Then, we can conclude that our main result holds for any game $\mathcal{G}(\varepsilon)$.

4 Economies with n types of consumers

Let us consider the particular case of a continuum economy \mathcal{E}_c with only n different types of agents. The set of agents is represented by the real interval $[0, 1]$, with the Lebesgue measure μ . We write $I = [0, 1] = \bigcup_{i=1}^n I_i$, where $I_i = [\frac{i-1}{n}, \frac{i}{n})$, if $i \neq n$, and $I_n = [\frac{n-1}{n}, 1]$. Each consumer $t \in I_i$ is characterized by her consumption set \mathbb{R}_+^ℓ , her utility function $U_t = U_i$ and her initial endowment $\omega(t) = \omega_i \in \mathbb{R}_+^\ell$. We will refer to I_i as the set of agents of type i in the atomless economy \mathcal{E}_c .

This particular economy \mathcal{E}_c can be considered as a representation of a finite economy \mathcal{E}_n , with n consumers and ℓ commodities, where each consumer i is characterized by her utility function U_i and her initial endowments $\omega_i \in \mathbb{R}_+^\ell$.

In this Section, we assume convexity of preferences and the hypotheses that guarantee the core-Walras equivalence for the n -types continuum economy \mathcal{E}_c .

Observe that an allocation x in \mathcal{E}_n can be interpreted as an allocation f_x in \mathcal{E}_c , where f_x is the step function given by $f_x(t) = x_i$, if $t \in I_i$. Reciprocally, an allocation f in \mathcal{E}_c can be interpreted as an allocation $x^f = (x_1^f, \dots, x_n^f)$ in \mathcal{E}_n , where $x_i^f = \frac{1}{\mu(I_i)} \int_{I_i} f(t) d\mu(t)$. Observe

also that (x, p) is an equilibrium for the economy \mathcal{E}_n if and only if (f, p) is an equilibrium for the continuum economy \mathcal{E}_c , where $f(t) = x_i$ if $t \in I_i$.

Let \mathcal{G}_c denote the two players game associated to the n -types continuum economy \mathcal{E}_c . A discrete approach of the game \mathcal{G}_c to an associated game \mathcal{G}_n for the finite economy \mathcal{E}_n is related with the equal treatment property of allocations. Note that if in \mathcal{G}_c the strategy profiles are required to satisfy the equal treatment property, then each player selects the same bundle for agents of the same type and, therefore, the payoff functions depend on the weight of the types in the coalitions selected by player 2 and on the corresponding step functions but do not reflect the possibility of any other different distribution of resources among members of the same type. On the other hand, under convexity of preferences, applying our main result we can deduce that any Nash equilibrium for the game \mathcal{G}_c , defines a Nash equilibrium for which the equal treatment property holds.

The game \mathcal{G}_c provides a two-players game \mathcal{G}_n associated to the economy with n consumers. Observe that, without loss of generality, we can assume that the strategy set of player 2 can be restricted to those strategies (S, g) such that g is feasible for the coalition S and $\mu(S) > 1 - \frac{1}{n}$ (see Remark 2 in the previous Section). This guarantees that all types are represented in the coalition selected by player 2.

Thus, in the game \mathcal{G}_c the strategy set for the player 1 is given by

$$\{x \in \mathbb{R}_+^{\ell n} : \sum_{i=1}^n x_i = \sum_{i=1}^n \omega_i\}$$

The strategy set for the player 2 is as follows:

$$\{(a, y) \in [\delta, 1]^n \times \mathbb{R}_+^{\ell n} : \sum_{i=1}^n a_i y_i = \sum_{i=1}^n a_i \omega_i\},$$

where δ is any real number in the interval $(0, 1)$ and $a_i = n\mu(S \cap I_i)$.

Given $a \in [\delta, 1]^n$ let us denote by $\mathcal{S}(a)$ the set of coalitions S_a in the continuum economy \mathcal{E}_c such that $n\mu(S_a \cap I_i) = a_i$ for every $i = 1, \dots, n$. Observe that the game \mathcal{G}_n does not distinguish between the coalitions in $\mathcal{S}(a)$.

Then, in practice, the strategy set for player 2 is the set of pairs which specify a parameter (a rate of participation) and a bundle for each agent such that the resulting allocation is feasible in the sense of Aubin (1979).

Observe that if x is a feasible allocation then $(1, x)$ is a possible strategy for player 2. Further, (a, ω) is also a strategy that player 2 can choose whatever $a \in [\delta, 1]^n$ may be.

Now for every profile (x, a, y) the payoff functions Φ_1 and Φ_2 for player 1 and 2, respectively, are defined as follows

$$\Phi_1(x, a, y) = \begin{cases} \sum_{i=1}^n a_i (U_i(x_i) - U_i(y_i)) & \text{if } U_i(x_i) \geq U_i(y_i) \text{ for every } i \\ \min\{U_i(x_i) - U_i(y_i)\} & \text{otherwise} \end{cases}$$

$$\Phi_2(x, a, y) = \begin{cases} \sum_{i=1}^n a_i (U_i(y_i) - U_i(x_i)) & \text{if } U_i(y_i) \geq U_i(x_i) \text{ for every } i \\ \min\{U_i(y_i) - U_i(x_i)\} & \text{otherwise} \end{cases}$$

Theorem 4.1 *If $(x^*, 1, x^*)$ is a Nash equilibrium for the game \mathcal{G}_n , then x^* is a Walrasian equilibrium allocation for the economy \mathcal{E}_n .*

Reciprocally, if x^ is a Walrasian equilibrium allocation for the economy \mathcal{E}_n , then any strategic profile (x^*, a^*, y^*) with $U_i(y_i^*) = U_i(x_i^*)$ for every $i = 1, \dots, n$, is a Nash equilibrium for the game \mathcal{G}_n .*

In particular, x^ is a Walrasian equilibrium allocation for the economy \mathcal{E}_n , if and only if (x^*, \mathbf{b}, x^*) with $\mathbf{b}_i = b$, for every $i = 1, \dots, n$, (for instance $(x^*, \mathbf{1}, x^*)$) is a Nash equilibrium for the game \mathcal{G}_n .*

Proof. Let $s^* = (x^*, 1, x^*)$ be a Nash equilibrium for the game \mathcal{G}_n . If x^* is not a Walrasian allocation, then x^* is blocked in the sense of Aubin with weights a_i as closed as one as one wants for every $i = 1, \dots, n$ (see Hervés-Beloso and Moreno-García, 2001, 2005, for details). That is there exists a strategy (a, y) for player 2 such that $\Phi_2(x^*, a, y) > 0 = \Phi_2(s^*)$.

Reciprocally, let x^* be a Walrasian allocation and let (x^*, a^*, y^*) , with $U_i(y_i^*) = U_i(x_i^*)$ for every $i = 1, \dots, n$, a strategy profile. If player 1 has an incentive to deviate, then x^* is not efficient. If there is a strategy (a, y) for player 2 such that $\Phi_2(x^*, a, y) > 0$ then x^* is blocked by the grand coalition in the sense of Aubin which is in contradiction with the fact that x^* is Walrasian (see again Hervés-Beloso and Moreno-García, 2001, 2005, for details in the infinite dimensional case).

Q.E.D.

5 Some Remarks

Let us consider the finite economy \mathcal{E}_n and the continuum n -types economy \mathcal{E}_c with their associated games \mathcal{G}_n and \mathcal{G}_c , respectively.

Let (x, a, y) be a strategy profile in the game \mathcal{G}_n . Note that if $a_i < 1$ for every i , then player 2 has an incentive to deviate by selecting the strategy (b, y) where $b_i = \frac{a_i}{\max_i a_i}$. Therefore if (x, a, y) is a Nash equilibrium in the game \mathcal{G}_n then $a_i = 1$ for some i .

Let x be a feasible allocation in the economy \mathcal{E}_n . Recall that if (f_x, S, g) is a Nash equilibrium for the game \mathcal{G}_c then both players get a null payoff. This is so because if player 2 obtains a payoff strictly positive, then player 1 can select the strategy which assigns g to the coalition S and ω to the individuals who do not belong to S . However, this strategy is not possible for the player 1 in the game \mathcal{G}_n . The reason is the following; the discrete approach which allows us to recast the game \mathcal{G}_c as the game \mathcal{G}_n takes only into account the size of the members of a coalition belonging to each type and does not reflect any distribution among agents with

the same type. Further, as we have already remarked, in the continuum case, the strategy set for player 2 can be restricted to coalitions with any size and, therefore, to arbitrarily big coalitions. This implies that, in the particular case of an atomless n -type economy we can consider, without loss of generality, that player 2 only selects coalitions where all types are represented. Then, when passing to the finite case \mathcal{G}_n the player 1 is restricted to select equal treatment allocations, provided that the distribution properties among agents of the same type are not contemplated as strategies. That is, when we recast the game \mathcal{G}_c as the game \mathcal{G}_n we drop possibilities of distribution among agents of the same type. This implies a reduction of strategies (basically for player 1) which results in the existence of Nash equilibria where player 2 can obtain a strictly positive payoff and, therefore, the allocation proposed by player 1 is not Walrasian. The next example shows our claim:

An Example. Consider an economy with two agents and one commodity. Both agents have the same preference relation represented by the utility function $U(x) = x$. Let $\omega_1 = \omega_2 = \omega > 0$ be the initial endowments. Let us consider the associated game \mathcal{G}_2 where, without of generality, the parameter $a \in [1/2, 1]^2$. Consider that player 1 chooses feasible allocation $x^* = (2\omega, 0)$ which is efficient but it is not a Walrasian allocation. The best response for player 2 is obtained by maximizing $\alpha(y_1 - 2\omega) + \beta y_2$ subject to $\alpha y_1 + \beta y_2 = (\alpha + \beta)\omega$, $y_1 \geq 2\omega$ and $y_2 \geq 0$. Then $\beta = 1$ and $\alpha < 1$ (see remarks above). Further, the payoff function for player 2, taking into account the restrictions, takes the value $(1 - \alpha)\omega$. Therefore, the player 2's best response is given by the weights $a^* = (1/2, 1)$ and the allocation $y^* = (5\omega/2, \omega/4)$. Observe that $\Phi_2(x^*, a^*, y^*) = \omega/2$ whereas $\Phi_1(x^*, a^*, y^*) = -\omega/2$. Note also that when player 2 selects (a^*, y^*) player 1 is not able to get a positive payoff and then $\Phi_1(x, a^*, y^*) = \min\{x_1 - 5\omega/2, x_2 - \omega/4\}$. This implies that, when (x^*, a^*, y^*) is the strategy profile, player 1 has no incentive to deviate because there is no feasible allocation which improve upon x^* for both consumers. Therefore, we conclude that (x^*, a^*, y^*) is a Nash equilibrium but x^* is not a Walrasian allocation.

Consider now the associated continuum economy with two types of agents. Let the strategy profile $(f_{x^*}, S_{a^*}, f_{y^*})$, where S_{a^*} is any coalition S such that $\mu(S \cap I_1) = 1/4$ and $\mu(S \cap I_2) = \mu(I_2) = 1/2$. Note that $\Pi_2(f_{x^*}, S_{a^*}, f_{y^*}) = \Phi_2(x^*, a^*, y^*) > 0$ which allows us to conclude that $(f_{x^*}, S_{a^*}, f_{y^*})$ is not a Nash equilibrium for the two players game associated to the continuum economy (see Lemma 3.1). Actually, player 1 has an incentive to deviate by selecting the feasible allocation f given by $f(t) = f_{y^*}(t)$ if $t \in S_{a^*}$ and $f(t) = \omega(t)$ otherwise.

We refer the reader to Hervés-Beloso, C., Moreno-García, E., (2007) where we completely characterize the Walrasian allocations of a finite economy as the Nash equilibria of a two-players game.

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