The Impact of Owners and Policy on Small Firms

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Abstract

How important are differences in owner personal characteristics (risk tolerance or optimism) versus the environment in which a firm operates (bankruptcy institutions or access to credit) for firm financial structure, size, owner net-worth and welfare? To answer this question we construct a dynamic, computable model with heterogeneous agents and endogenous default in which entrepreneurs weigh the firm's current and future returns. We find that modest differences in risk aversion match SSBF data, and the environment in which a firm operates matters greatly. The option to declare bankruptcy insures an owner against extreme current loss and the ability to bail out the firm with personal funds preserves the potential for high future gains. We find that welfare gains from bankruptcy reform or improved access to credit are substantial, especially for agents most willing to bear risk. Risk aversion also affects firm legal status: we show that incorporation always leads to higher welfare for less risk averse entrepreneurs while the more risk averse may have higher welfare by remaining unincorporated. The more risk averse cannot credibly commit ex-ante to refrain from default ex post; having some personal assets that would be seized in bankruptcy mitigates this problem. In contrast, the firms of the less risk averse are larger and have higher future value, which credibly limits their incentive to default.

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1 Introduction

Small firms are a vital part of the macroeconomy, producing more than 50 percent of non-farm private U.S. GDP, employing half of all private sector employees and paying 45 percent of total private payroll. They are a source of "good jobs," generating 60 to 80 percent of net new jobs annually over the last decade, employing 41 percent of high tech workers (scientists, engineers and computer workers) and producing almost 14 times more patents per employee than larger patenting firms. Among all U.S. employer firms, 89 percent have less than 20 employees.[1] Unlike large firms, they are closely associated with their owners. We quantify the effects of two owner traits, risk aversion and optimism, and policies regarding bankruptcy and credit access, on firm size, capital structure and default. The distinction between policies and owner traits is important because policies can be changed but innate characteristics cannot.

We construct a model economy with a risk neutral representative lender and many long-lived agents who differ in willingness to bear risk. Each period, agents choose consumption and whether to run a firm with risky returns. If they run a firm, they choose its size, capital structure (mix of personal funds and outside loans), and whether to default ex-post.[2] Firm risk is non-tradable (i.e., the owner runs a single firm, not a portfolio of firms) and firms may be credit constrained. Default occurs in equilibrium, with the lender recovering only a fraction of the loan and the firm unable to obtain credit for several periods. Firms weigh the effect of default today against access to future credit. We show that modest differences in risk interact with institutions to generate significant welfare effects that affect firm legal status. The less risk-averse run larger firms with higher future value and this limits their incentive to default, hence they incorporate to protect current personal assets. In contrast, if the more risk averse run firms they are small with lower future value. It may be optimal for such owners to leave some personal assets at risk in bankruptcy by remaining unincorporated. The fact that these assets would be seized credibly limits their default ex post.

In both England and the U.S. early bankruptcy law applied only to merchants, not consumers. Supporters of the U.S. Bankruptcy Act of 1800 argued that "unforeseen accidents" were ruining respectable merchants and there was substantial social value in returning these merchants to active business (see Mann (2003), note 11, pp. 57 and 73). The fundamental role of corporate law was to limit liability (see Hovenkamp (1991), pp. 49-55). We model these unforeseen accidents and

[1] See http://www.sba.gov/advo/stats/sbfaq.pdf. We use the 1993 and 1998 Survey of Small Business Finance, which contain data on firms and owners. The median number of employees is 7 and median assets are $270,000.

[2] Models with representative agents are aggregated by multiplying the optimal decision rules from the individual's problem by the number of (identical) agents. This is not possible in our setting because differences in willingness to bear risk (i.e., heterogeneous risk aversion parameters) are central to the debate on entrepreneurship. As in Krusell and Smith (1998), we construct distributions to account for heterogeneity.

limited liability, and show that bankruptcy insures owners against poor firm returns but permits upside gain, even after accounting for the impact of default on loan interest rates. This insurance induces risk-averse entrepreneurs to operate larger firms, which leads to higher output and welfare, while the option value of maintaining the firm to realize future value limits default.[3]

Because the goal of bankruptcy is to limit risk, an agent's attitude towards uncertain returns is crucial (e.g., the same bankruptcy rule will have different implications for owners with different degrees of risk aversion). Thus, we allow for heterogeneity and use the model to derive a distribution of risk aversion for those who choose to become entrepreneurs. Apart from bankruptcy, firms can also manage risk through decisions such as financial structure, scale of production and the amount of personal net-worth to invest. We show how these optimal decisions depend on differences in owner risk aversion. Finally, accounting for heterogeneous risk aversion and uncertain firm returns requires us to derive cumulative probability distribution functions for firm decisions. We compare model predictions to distributions constructed from data. The discipline imposed by this check for consistency between model predictions and data is the analog of matching moments in quantitative macroeconomic models (cf., Prescott (2006)).

Our analysis requires technical innovation and high performance computing. The ex-post default decision introduces a non-convexity,[4] heterogeneity requires distributions, and the return distribution cannot be captured by the first two moments or a few states. We prove that when an agent's value function is scalable in net worth, complexity is reduced. The problem is computationally intensive because non-convex optimization requires care to find an appropriate start value. Also, constructing distributions for firm size, capital structure and personal net worth invested in the firm requires the fixed point problem to be computed for a sufficiently large number of risk-aversion values to account for agent heterogeneity.[5] Although calibration exercises can typically use small discrete approximations of uncertainty and match moments, we cannot approximate the return distribution by a few states without introducing large errors because, as we show in section 7.5, the non-normal shape of the distribution matters, particularly the thick tails. Thus, given continuation values, when computing an agent's utility maximization problem we must use numerical integration in every step of the optimization.

Our paper is related to a large literature on entrepreneurship and risk aversion.[6] For exam-

[3]Owners will "bail out" a firm today with personal funds if they expect sufficient future returns – firm option value.

[4]The non-convexity arises because an agent cannot commit to refrain from bankruptcy. See section 3.

[5]The distribution of risk aversion is constructed by repeatedly computing the distribution of net-worth invested for different distributions of risk-aversion to minimize the distance to the empirical cdf.

[6]In a model with homogeneous risk preferences, Hopenhayn and Vereshchagina (2006) find that borrowing constraints, outside opportunities and endogenous risk choice are important for explaining entrepreneurship.

ple, Kihlstrom and Laffont (1979) focuses on differences in risk aversion and formalizes ideas in Knight (1921). Our paper also complements recent analyses of the quantitative effects of consumer bankruptcy rules in dynamic models with limited commitment and incomplete markets begun in Athreya (2002). Chatterjee, Corbae, Nakajima, and Rios-Rull (2007) and Livshits, MacGee, and Tertilt (2007) show that consumer bankruptcy provides partial insurance against bad luck due to health, job, divorce or family shocks, but it drives up interest rates, which impedes intertemporal smoothing. In the latter paper the insurance effect generally dominates the interest rate effect for U.S. consumers and the former finds the reverse, but both find modest net welfare effects. Meh and Terajima (2008) extend the model to study the effect of consumer bankruptcy on unincorporated entrepreneurs and find larger welfare gains from eliminating the personal bankruptcy exemption but losses from eliminating consumer bankruptcy entirely. In contrast, our baseline model focuses on bankruptcy by incorporated firms with heterogeneity in owner willingness to bear risk. Bankruptcy provides insurance against poor firm returns, but default is tempered by potentially high future gains due to kurtosis in firm returns. We find welfare effects that are much greater than in consumer studies, especially for those most willing to bear risk.[7] We also show that less risk-averse owners incorporate, while more risk-averse owners may remain unincorporated.[8]

Our model differs from the previous literature in several ways: (i) agents differ in willingness to bear risk; (ii) we study bankruptcy by incorporated firms with risky returns; (iii) firm size, financial structure and default are endogenous and can be used to manage risk; (iv) default occurs in equilibrium;[9] and (iv) we link firm legal status (incorporated or unincorporated) to risk aversion and a commitment problem. We find that changes in bankruptcy rules, credit constraints and firm legal rules can have vastly different impacts on agents with only small differences in risk aversion. Thus, agent heterogeneity is important for policy analysis. We also find that credit constraints bind for many but not all entrepreneurs and our results are consistent with mild entrepreneur optimism.[10]

The paper proceeds as follows. Section 2 contains the model. Section 3 uses theory to derive a computable problem and constructs distributions predicted by the model. The model is mapped into U.S. data in Section 4. Section 5 shows that the model is quantitatively plausible along a number of dimensions, including firm size, capital structure and default rates. Section 6 examines firm legal status (incorporation). Section 7 reports policy experiments. Section 8 concludes.

[7] The insurance effect is more important than the interest rate effect.

[8] The less risk-averse run larger firms, default less and use personal funds to "bail out" their firms. More risk-averse owners have a commitment problem which the seizure of personal assets in bankruptcy mitigates.

[9] As in Chatterjee, Corbae, Nakajima, and Rios-Rull (2007), this differs from models in which default does not occur in equilibrium, e.g., Kehoe and Levine (1993).

[10] Although initial net-worth and the return distribution are identical across firms ex-ante, net-worth and consumption evolve differently over time due to differences in risk aversion and return realizations.

2 Model with Incorporated Firms

The economy has $t = 0, 1, \ldots$ time periods. A risk-neutral competitive lender with an elastic supply of funds makes one-period loans.[11] Many infinitely lived risk averse agents discount the future at common rate β, each with a CRRA utility function over consumption. Preferences are heterogeneous with respect to risk aversion, with parameter $\rho \sim N(\mu, \sigma^2)$ and

$$u(c) = \frac{c^{1-\rho}}{1-\rho}.$$

Agents have an initial endowment w_0 and access to an ex-ante identical constant returns to scale technology. If operated, the technology produces output x per unit of assets invested A. The firm's return is given by random variable X with cumulative distribution function $F(x)$ and probability density function $f(x)$, which is strictly positive on support $[\underline{x}, \overline{x}]$, $\underline{x} \leq 0$, $\overline{x} > 0$ and iid. The firm's output is then Ax. A negative realization means that firm losses in a year exceed its current assets; the owner must either use personal funds to stay solvent or default. Net-worth w_t is derived from the return on investment in all periods $t \geq 1$, known at the beginning of the period.[12] Agents also have access to an outside investment opportunity with return r.

Entrepreneurs are agents who choose to operate a firm, which means $A > 0$; agents who do not set $A = 0$. Entrepreneurs raise assets to invest in their firm at time t in two ways:

Equity: Use personal net-worth w_t to self-finance at real opportunity cost r.
Debt: Take a loan, secured by business assets, which gives the lender reservation return $1 + r_L$.

The interest rate on the loan is determined endogenously for each entrepreneur by the model. Agents are long-lived and hence can invest long-term, and the opportunity cost r of using personal net-worth to fund the firm will generally be higher than the lender's opportunity cost, $r > r_L$.[13] Given a level of business assets A in a period, an entrepreneur determines the optimal financial structure by choosing the percentage of self-finance ϵ. Thus, total equity is ϵA and debt is $(1-\epsilon)A$ at the beginning of the period. At the end of each period assets are Ax and the firm owes $A\overline{v}$.

The firm faces a borrowing constraint, $(1-\epsilon)A \leq bw$, which limits business loans to percentage b of entrepreneur net-worth. Note that the constraint depends on agent total net worth w, which

[11] We consider a composite lender that supplies all liabilities – bank loans, trade credit and other liabilities. Small firms lack access to long-term loans because they do not have payment or profit histories, audited financial statements, or verifiable contracts with workers, input suppliers or customers. The Federal Reserve *Survey of Terms of Business Lending* shows that the average maturity on such loans is less than one year.

[12] The risky technology and w_0 are ex-ante identical, but net-worth and consumption evolve stochastically over time.

[13] In section 4 we consider a business loan financed by home equity with $r = 4.5\%$ and lender opportunity cost, $r_L = 1.2\%$, given by the 6-month T-bill rate.

includes both firm and personal assets. Of course firm assets can be seized in bankruptcy, but the constraint indicates that the bank also takes account of the fact that the entrepreneur can use personal assets to “bail out the firm.” Because the project’s expected return exceeds the lender’s opportunity cost of funds (r_L) plus expected default costs, the risk neutral lender would like the projects to be highly levered and run at a very large scale. The constraint, which imposes lending limits for entrepreneurs of each risk type, ensures the lender will have a diversified portfolio.

Ex post, the entrepreneur chooses whether to repay loan $A\overline{v}$ or default.[14] When default occurs, bankruptcy follows immediately and is described by two parameters, δ and T. The court determines the total value of firm assets and transfers $1-\delta$ percent to the lender, where δ is a deadweight bankruptcy loss (e.g., firm assets are sold at a loss). The entrepreneur is protected by limited liability (only firm assets can be seized), but has the option to pay firm debt with personal funds if this is optimal. If bankruptcy occurs, the entrepreneur does not have access to the firm’s returns for T periods, which has two interpretations. First, corresponding to Chapter 7 in the U.S. Bankruptcy Code, the firm may be liquidated. Because bankruptcy remains on a credit record for a period of time, creditors and customers would be unwilling to do business with the entrepreneur during this period.[15] Second, corresponding to Chapter 11, the firm may continue to operate, but is owned by the debt-holders who make investments and receive payments, or shut it down. After T periods, when the credit record is clean, the entrepreneur can either restart a new firm or regain control of the original firm, in Chapter 7 or 11 respectively.

The timing of events for incorporated firms is as follows:

1. Beginning of period t (*ex-ante*) entrepreneur net-worth is w. There are two cases:

   (a) *The entrepreneur has not declared bankruptcy in any of the previous T periods.* The entrepreneur chooses consumption c, firm assets A, self-finance ϵ (debt is $1-\epsilon$), and amount $\overline{v}$ to repay per unit A, subject to the lender receiving an ex-ante expected payoff of at least $(1-\epsilon)(1+r_L)$.

   (b) *The entrepreneur declared bankruptcy k periods ago.* The owner cannot operate the firm for the next $T-k$ periods. Hence, only current consumption is chosen.

2. At the end of period t (*ex-post*) the firm’s return on assets, x, is realized. Total end-of-period firm assets are Ax. The entrepreneur must decide whether or not to default. If

[14] A firm may default if it is unable to repay $A\overline{v}$ (firm plus personal assets are less than A) and unwilling to repay otherwise. Owners can “bail out the firm” with personal assets to forestall bankruptcy, but cannot be forced to do so.

[15] In our model personal credit histories affect business loans, causing a credit interruption. Mester (1997) p. 7 finds that in small business loan scoring models, “the owner’s credit history was more predictive than net worth or profitability of the business” and “owners’ and businesses’ finances are often commingled”.

(a) *Default:* Only firm assets are seized; the entrepreneur is left with personal net-worth $(1+r)(w-\epsilon A-c)$, personal assets invested at outside interest rate r.

(b) *No Default:* Entrepreneur net-worth is $A(x-\overline{v})+(1+r)(w-\epsilon A-c)$, which includes both net-equity in the firm and the return on personal assets.

3 An Individual Agent's Problem

Consider the optimization problem of an agent, with a given coefficient of risk aversion ρ. The goal is to determine the structure of the value function. We state the problem recursively, with beginning of period entrepreneur net-worth w. If bankruptcy occurred in the previous T periods, then the state is given by (B,k,w) where k is the number of periods since default. Otherwise, the state is given by (S,w). Denote the value functions by $V_{B,k}(w)$ and $V_S(w)$, respectively. After T periods the firm can restart, thus $V_{B,T}(w)=V_S(w)$. Let $\mathfrak{B}$ denote the set of asset return realizations x for which bankruptcy occurs, with complement $\mathfrak{B}^c$.

If the firm did not default in the previous T periods, the agent solves:

Problem 1 $V_S(w)=\max_{c,A,\epsilon,\overline{v}} u(c)+\beta\Big[\int_{\mathfrak{B}} V_{B,1}((1+r)(w-\epsilon A-c))\,dF(x)$

$$+\int_{\mathfrak{B}^c} V_S(A(x-\overline{v})+(1+r)(w-\epsilon A-c))\,dF(x)\Big]$$

Subject to:

$$\int_{\mathfrak{B}\cap\mathbb{R}_-} x\,dF(x)+\int_{\mathfrak{B}\cap\mathbb{R}_+}(1-\delta)x\,dF(x)+\int_{\mathfrak{B}^c}\overline{v}\,dF(x)\geq(1-\epsilon)(1+r_L) \tag{1}$$

$$x\in\mathfrak{B} \textit{ if and only if } V_{B,1}((1+r)(w-\epsilon A-c))>V_S(A(x-\overline{v})+(1+r)(w-\epsilon A-c)) \tag{2}$$

$$(1-\epsilon)A\leq bw \tag{3}$$

$$c\geq 0,\ \ A\geq 0,\ \ 0\leq\epsilon\leq 1. \tag{4}$$

The objective is an agent's utility of current consumption plus the discounted continuation value of end of period net-worth. Constraint (1) ensures that the lender is willing to supply funds. The right-hand-side indicates that the $1-\epsilon$ percent of funds the lender invests in the firm earns at least reservation return $1+r_L$. The left-hand side is the lender's expected return from the loan: the first term accounts for the fact that the lender may absorb some losses when the firm's return is negative,[16] the second term is the net amount recovered from firm assets in bankruptcy states with positive net returns (deadweight default loss δ arises only if x is positive and the firm has not lost more than the value of its assets in the period), the third term is the net amount recovered from personal assets and the fourth term is the fixed debt repayment in solvency states. Constraint (2) specifies ex-post optimality of the default decision: An entrepreneur will default if and only if the expected continuation payoff after default exceeds that from solvency.[17] Constraint (3) is a standard borrowing constraint, see, for example, Evans and Jovanovic (1989). Finally, (4) ensures consumption and assets are non-negative, and ϵ is a percentage.[18]

Now consider the problem of a firm that defaulted $k \leq T$ periods ago. After T periods the firm can operate again, thus $V_{B,T}(\cdot) = V_S(\cdot)$. Let w' denote net-worth next period.

Problem 2 $V_{B,k}(w) = \max_{c,w'} u(c) + \beta V_{B,k+1}(w')$

Subject to:

$$c(1+r) + w' \leq w(1+r); \tag{5}$$

$$c, w' \geq 0. \tag{6}$$

The objective of problem 2 is expected ex-ante utility. If default occurred, the agent cannot operate the firm for T periods and chooses only consumption and saving, consistent with budget constraint (5) and non-negativity constraint (6).

We now use the fact that CRRA utility is scalable in wealth to determine the structure of the value function. Proposition 1 permits value functions $V_{B,1}$ and V_S to be replaced with a number v_S. The problem can be restated as a 1-dimensional fixed point problem in v_S, simplifying the analysis.[19] The proof is in Appendix B.

[16]This can occur if the loan has an overdraft provision or the firm has trade credit. In the data, this corresponds to the case where the firm has negative equity and defaults.

[17]Bailing out the firm with personal funds means that the entrepreneur continues to operate the firm even if $x < \overline{v}$. In a one period model (instead of the dynamic model) both $V_{B,1}$ and V_S would be the identity mapping, and (2) would reduce to $x \in \mathfrak{B}$ if and only if $(1+r)(w - \epsilon A - c) > A(x - \overline{v}) + (1+r)(w - \epsilon A - c)$, which implies $x \in \mathfrak{B}$ if and only if $x < \overline{v}$ (bankruptcy only if the return is less than debt plus interest).

[18]Ex ante ϵ is a percentage, but ex post negative equity may occur. This distinction arises because the non-negativity constraint on equity only applies ex-ante. Ex post, if the project realization is low, assets are low and end-of-period equity will be negative due to the accounting identity: assets = debt + equity.

[19]We need only $v_{B,1}$, the continuation utility given that default was just announced, and v_S. To simplify notation, write v_B for $v_{B,1}$.

Proposition 1 *Suppose that the entrepreneur has constant relative risk aversion. Let* $v_S = V_S(1)$ *and* $v_{B,k} = V_{B,k}(1)$*. Then* $V_S(w) = w^{1-\rho} v_S$ *and* $V_{B,k}(w) = w^{1-\rho} v_{B,k}$*.*

Applying Proposition 1 to Problem 2 it is straightforward to compute $v_{B,k}$ as a function of v_S. Further, Lemma 1 and Lemma 2 in Appendix B prove that the investor's constraint binds and bankruptcy set $\mathfrak{B}$ is a lower interval, with cutoff x^*. Thus, the optimization problem can be rewritten as follows, where all endogenous variables are expressed as a percentage of net-worth w:

Problem 3 $v_S = \max_{c,A,\epsilon,\overline{v}} u(c) + \beta v_B \int_{\underline{x}}^{x^*} \left[(1+r)\Big(1-\epsilon A - c\Big)\right]^{1-\rho} dF(x)$

$$+\beta v_S \int_{x^*}^{\overline{x}} \left[A(x-\overline{v}) + (1+r)\Big(1-\epsilon A - c\Big)\right]^{1-\rho} dF(x)\Big]$$

Subject to:

$$\int_{\underline{x}}^{x^*} \min\{(1-\delta)x, x\}\, dF(x) + \int_{x^*}^{\overline{x}} \overline{v}\, dF(x) = (1-\epsilon)(1+r_L) \tag{7}$$

$$x^* = \max\left\{\overline{v} - \left[1 - \left(\frac{v_B}{v_S}\right)^{\frac{1}{1-\rho}}\right] \frac{(1+r)(1-\epsilon A - c)}{A}, \underline{x}\right\} \tag{8}$$

$$c + \epsilon A \leq 1 \tag{9}$$

$$(1-\epsilon)A \leq b \tag{10}$$

$$c \geq 0, A \geq 0, 0 \leq \epsilon \leq 1. \tag{11}$$

The objective is to maximize the utility of current consumption and the discounted value of end of period net-worth in firm bankruptcy and solvency states. Constraint (7) corresponds to lender individual rationality constraint (1), and binds by Lemma 1 in Appendix B. Constraint (8) is the optimal default cutoff and follows from (2) by Lemma 2. (9) ensures feasibility and (10) is the borrowing constraint. (11) is obvious.

Problem 3 is non-convex because the timing of decisions leads to a commitment problem: c, A, ϵ, $\overline{v}$ are chosen ex-ante, but the bankruptcy decision is made ex-post and the firm cannot commit to refrain from bankruptcy. This implies that default set cutoff x^* is determined by (8). Lotteries cannot be used to convexify the problem because independent randomization over A, ϵ, c, $\overline{v}$ and x^* is not possible. See Krasa and Villamil (2000), Krasa and Villamil (2003) for an analysis of randomization and commitment.

3.1 Existence and Uniqueness

Proposition 2 *There exist $\underline{\rho} < 1$ and $\overline{r} > \frac{1}{\beta} - 1$ such that Problem 3 has a solution for all $\rho \geq \underline{\rho}$ and for all $r \leq \overline{r}$.*

Let $\Gamma(v_S)$ be the expected utility given continuation value v_S. In general $\Gamma'(v_S) > 1$ for all v_S close to 0. Thus, Γ is not a contraction mapping because net-worth is unbounded. In the proof of Proposition 2 in Appendix B, we show that $\Gamma(0) \leq 0$ and that there exists v_S such that $\Gamma(v_S) \geq 0$ for risk aversion $\rho > 1$. As a consequence of the intermediate value theorem, continuity of Γ implies that Γ has a fixed point. By continuity, the result extends for some $\rho < 1$.

If there is more than one solution to the recursive problem, then the solution with the maximal v_S corresponds to the solution of the infinite horizon problem where agents select sequences for consumption, assets, debt-equity and default.

3.2 Heterogeneous Entrepreneurs & Model Predictions

Agents are heterogeneous with respect to risk aversion. This requires matching model predictions and data in terms of *distributions* (see Krusell and Smith (1998)). We now specify the distributions predicted by the model for end-of-period firm assets, personal net-worth invested in the firm, and the ratio of equity over assets (firm capital structure). Given firm return pdf $f(x)$ and risk aversion pdf $g_{\mu,\sigma}(\rho)$, the cdfs predicted by the model are:[20]

Cdf of Net-Worth: After realization x, firm assets are $A(\rho)x$ and debt is $A(\rho)\overline{v}$. Equity in the firm is $A(\rho)(x - \overline{v}(\rho))$, which is positive if $x \geq \overline{v}(\rho)$. Owner personal net-worth outside the firm is $(1 + r)(1 - c(\rho) - \epsilon(\rho)A(\rho))$. The percent of total net-worth invested is

$$\mathfrak{w} = \frac{A(\rho)(x - \overline{v}(\rho))}{A(\rho)(x - \overline{v}(\rho)) + (1 + r)(1 - c(\rho) - \epsilon(\rho)A(\rho))}. \tag{12}$$

It follows immediately that $\mathfrak{w}$ is strictly increasing in x. We can solve this equation for $x = x(\mathfrak{w}, \rho)$. The percent of net-worth invested is less than or equal to $\mathfrak{w}$ for all $x \leq x(\mathfrak{w}, \rho)$. For firms with positive equity, net worth is therefore given by[21]

$$W^m_{\mu,\sigma}(\mathfrak{w}) = \frac{\int_{-\infty}^{\underline{\rho}} \int_{\overline{v}(\rho)}^{x(\mathfrak{w},\underline{\rho})} f(x) g_{\mu,\sigma}(\rho)\, dx\, d\rho + \int_{\underline{\rho}}^{\infty} \int_{\overline{v}(\rho)}^{x(\mathfrak{w},\rho)} f(x) g_{\mu,\sigma}(\rho)\, dx\, d\rho}{\int_{\overline{v}(\rho)}^{\infty} f(x)\, dx}. \tag{13}$$

[20] We will construct $f(x)$ and $g_{\mu,\sigma}(\rho)$ in the quantitative analysis.

[21] The denominator is the probability that the entrepreneur has positive equity, where $\underline{\rho}$ is the lowest parameter for which a model solution exists. For all $\rho < \underline{\rho}$ we assign the model solution as explained in section 5.

Cdf of Equity/Assets: The percent of equity is given by

$$\mathfrak{e} = \frac{A(\rho)(x - \overline{v}(\rho))}{A(\rho)x}.$$

Solve this equation for $x = x(\mathfrak{e}, \rho)$. For firms with positive equity, the cdf of equity/assets is therefore

$$E^m_{\mu,\sigma}(\mathfrak{e}) = \frac{\int_{-\infty}^{\underline{\rho}} \int_{\overline{v}(\rho)}^{x(\mathfrak{e},\underline{\rho})} f(x) g_{\mu,\sigma}(\rho)\, dx\, d\rho + \int_{\underline{\rho}}^{\infty} \int_{\overline{v}(\rho)}^{x(\mathfrak{e},\rho)} f(x) g_{\mu,\sigma}(\rho)\, dx\, d\rho}{\int_{\overline{v}(\rho)}^{\infty} f(x)\, dx}. \tag{14}$$

Cdf of End of Period Assets: The current realization of end of period assets as a percent of net-worth outside the firm is

$$\mathfrak{a} = \frac{A(\rho)x}{(1 + r)(1 - c(\rho) - \epsilon(\rho)A(\rho))} \tag{15}$$

Solve this equation for $x = x(\mathfrak{a}, \rho)$ to get the cdf of end of period assets

$$A^m_{\mu,\sigma}(\mathfrak{a}) = \int_{-\infty}^{\underline{\rho}} \int_{\underline{x}}^{x(\mathfrak{a},\underline{\rho})} f(x) g_{\mu,\sigma}(\rho)\, dx\, d\rho + \int_{\underline{\rho}}^{\infty} \int_{\underline{x}}^{x(\mathfrak{a},\rho)} f(x) g_{\mu,\sigma}(\rho)\, dx\, d\rho. \tag{16}$$

4 Mapping the Model to U.S. Data

Table 1: Exogenous Parameters

Parameter	Interpretation	Value	Comment/ Observations
r_L	lender opportunity cost	1.2%	real rate, 6 mo T-Bill, 1992-2006
r	entrepreneur opportunity cost	4.5%	real rate, 30 year mortgage, 1992-2006
β	discount factor	0.97	determined from r and r_L
T	default exclusion period	11	U.S. credit record
δ	default deadweight loss	0.10	Boyd-Smith (1994)

We use U.S. data to assign values to five model parameters and to construct the distribution of firm returns. We jointly calibrate three remaining parameters. In table 1, we identify r_L, the lender's opportunity cost of short-term funds, with the average real return on 6 month Treasury bills between 1992 and 2006.[22] The interest rate charged by the lender will be strictly higher than r_L because of bankruptcy costs. We identify the owner's opportunity cost of funds r with the real rate on 30 year mortgages over the period; the cost of using home equity to finance a business loan will also be strictly higher. $\beta = 0.97$ is approximated by $1/(1 + 0.5r_L + 0.5r)$, with r and r_L weighed equally (firm risk cannot be diversified since a portfolio of small firms does not exist).

[22] We use monthly data for T-Bill rates and deduct for each month the CPI reported by the BLS.

The bankruptcy parameters are $T = 11$, because in the U.S. after 10 years past default is removed from a credit record, and $\delta = 0.1$, the bankruptcy deadweight loss in Boyd and Smith (1994) and the midpoint of costs of 0-20% of assets in Bris, Welch, and Zhu (2006).

Herranz, Krasa, and Villamil (2008) use data from the Survey of Small Business Finance (SSBF) on incorporated firms to compute firm return distribution $f(x)$.[23] They assume firms have access to a common constant returns to scale "blue print" technology. The return per unit of asset for a particular firm is a sample point from this distribution (see section 10.1). Table 2 shows that $f(x)$ is risky, with rightward skew and a long upper tail.[24]

Table 2: Real Firm Return on Assets for Incorporated Firms: Summary Statistics

Moment:	median	mean	standard dev.	skewness	kurtosis
1993 SSBF	1.094	1.30	1.57	13.2	290
95% conf.	[1.08, 1.11]	[1.22, 1.38]	[0.95, 2.13]	[2.3, 17.3]	[29, 488]

The remaining parameters are jointly calibrated by choosing b, μ, σ to minimize the distance between model predictions and data. We first construct two empirical cumulative density functions from the SSBF data. The empirical cdf of net-worth invested is, $W^e(\mathfrak{w})$:[25]

$$\frac{\text{owners' share} * \text{equity}}{\text{net-worth outside the firm} + \text{owners' share} * \text{equity}}.$$

The empirical cdf of end-of-period assets per unit of net-worth, $A^e(\mathfrak{a})$, is:

$$\frac{\text{owners' share} * \text{asset}}{\text{net-worth outside the firm}}.$$

The model-predicted median assets are $\mathfrak{a}_{\mu,\sigma}$ such that $A^m_{\mu,\sigma}(\mathfrak{a}_{\mu,\sigma}) = 0.5$.

Parameters b, μ, σ are chosen to minimize the supnorm distance between the cdf implied by the model and the cdf from the SSBF data:

$$\min_{b,\mu,\sigma\geq 0} \|W^m_{\mu,\sigma}(\mathfrak{w}) - W^e(\mathfrak{w})\|_\infty + (0.431 - \mathfrak{a}_{\mu,\sigma})^+ + (\mathfrak{a}_{\mu,\sigma} - 0.519)^+ \tag{17}$$

[23] The SSBF is a survey administered by The Board of Governors of the Federal Reserve System and the U.S. Small Business Administration in 1987, 1993, 1998 and 2003. Each survey is a cross section of about 4000 non-farm, non-financial, non-real estate small businesses that represents about 5 million firms. All surveys are available at *http://www.federalreserve.gov.* The surveys contain information on the characteristics of small firms and the primary owner (e.g., owner age, gender, industry, type of business organization), firm income statements and balance sheets, details on the use and source of financial services, and recent firm borrowing experience (including trade credit and capital injections such as equity). We consider only incorporated firms with assets of at least $50,000.

[24] 95% confidence bands are computed for each moment using bootstrap sampling, except the interquartile range is reported for the median. Only the 1993 SSBF has interest payments, required to compute return on assets.

[25] $W^e(\mathfrak{w})$ is the number of observations, accounting for sample weights, at which the percent of net-worth invested is less than or equal to $\mathfrak{w}$.

The cdf of net-worth invested implied by the model, $W^m_{\mu,\sigma}(\mathfrak{w})$, is given by (13). The supremum norm $\|.\|_\infty$ is taken over all non-negative percentages of net-worth.[26] The second and third terms impose penalties only for asset values outside the 95% confidence interval for firm assets, which Herranz, Krasa, and Villamil (2008) find is [43.1,51.9]. Since we exclude firms with negative equity when determining W^e, net-worth invested is between 0% and 100%, but assets are unbounded.[27] The lack of a well defined upper bound for assets is a problem because tail behavior would greatly impact model prediction; requiring the median asset level to lie in its 95% confidence interval solves this problem.

Table 3: Calibrated Parameters

Parameter	**Interpretation**	**Est. Value**
b%	borrowing constraint: loan $\leq bw$	21.5
μ	median of distribution of risk aversion	1.55
σ	standard deviation of distribution of risk aversion	0.83

Table 3 reports the calibrated parameters. The model predicts a maximal ex-ante loan size of 21.5% of entrepreneur net-worth. These loans are secured by risky business assets because the firm is incorporated; the lender cannot seize personal assets in default. The median risk aversion of the owner of an incorporated firm is 1.55, with a standard deviation of 0.83. Thus, about 75% of all such entrepreneurs have a coefficient of risk aversion between 1 and 3, the range in real business cycle models. Using the Consumer Expenditure Survey, Mazzocco (2006) estimates a median coefficient of risk aversion of 1.7 for men. We would expect entrepreneurs to be somewhat less risk averse than the general population; our estimate for ρ is in line with this.[28] Parameters μ and σ are used to construct the distribution of risk aversion for incorporated firms, $g_{\mu,\sigma}(\rho)$, the final object in the model that must be mapped into data.

Appendix A shows that the values of the calibrated parameters do not vary significantly with δ and T. The insensitivity to changes in δ is due to the low equilibrium default rate. Table 12 shows that the best model fit is obtained at a value of $T = 13$. Thus, if we had calibrated T instead of

[26]To compute the supremum norm we evaluate $|W^m_{\mu,\sigma}(\mathfrak{w}) - W^e(\mathfrak{w})|$ at 1,000 equidistant points between 0 and 1, and take the maximum. Appendix C shows the estimates are not affected by using square distance

$$\sqrt{\int \left(W^m_{\mu,\sigma}(\mathfrak{w}) - W^e(\mathfrak{w})\right)^2 d\mathfrak{w} + ((0.431 - \mathfrak{a}_{\mu,\sigma})^+)^2 + ((\mathfrak{a}_{\mu,\sigma} - 0.519)^+)^2}$$

[27]For example, 5% of firms had assets over ownership share that exceeded owner net-worth by 500%.

[28]Since Mazzocco (2006) does not estimate the distribution of risk aversion, his estimate of the standard deviation of 0.96 is close, but not directly comparable to ours. We discuss gender differences in section 7.2.

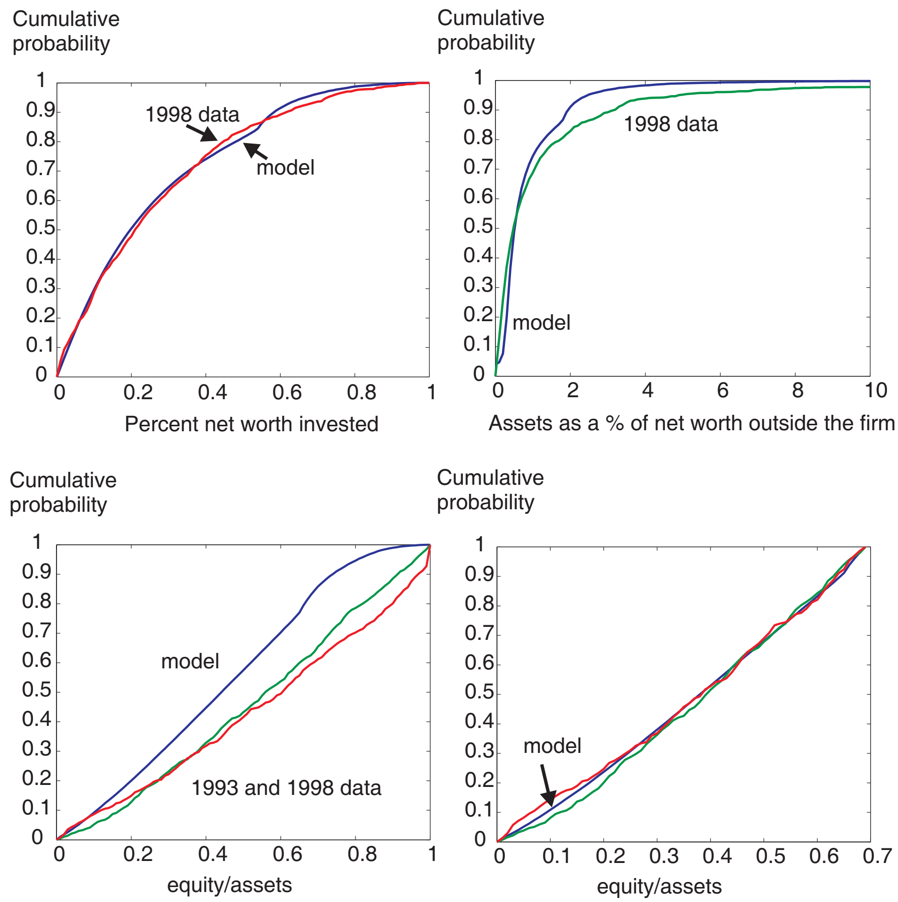

Figure 1: Model Predictions and SSBF Data for incorporated firms: cdfs

choosing it to be consistent with U.S. institutions, the numbers for the calibrated parameters and model results would not have changed significantly.

5 Matching Model Predictions and Data

Our model is quantitatively plausible along a number of dimensions. Figure 1 compares the cdfs predicted by the model (computed as explained in section 3.2) with SSBF data.[29] The first panel shows the model-predicted and empirical cdfs for the percent of net-worth an owner invests in the firm. Since we fit to this empirical cdf one would expect to see a match, but the match is surprisingly good given there are only three fitting parameters. The data show that owners invest substantial personal net-worth in their firms: the median is 21% and the mean is 27%. The data

[29]Owner net worth, personal net-worth plus home equity, is only in the 1998 SSBF. The data cdf for the percent of net-worth invested is for firms with positive net-worth outside the firm and non-negative equity. Only firms with at least $50,000 in assets are included.

also show a surprising lack of diversification: 3% invest more than 80%, 11% invest more than 60%, 25% invest more than 40% and 52% invest more than 20%. The model replicates these facts.

The next panel compares the predicted cdf of firm assets to its empirical counterpart. The match between these cdfs is also good, but the model under predicts a few large firms. This occurs because model solutions do not exist below $\underline{\rho} = 0.74$, and we assign point mass of $\mu(\{\rho \leq \overline{\rho}\})$ to $\underline{\rho}$.[30] At $\underline{\rho}$, the ex-ante level of ϵ and A are 0.720 and 0.766, respectively. Thus, end of period net-worth outside the firm, $(1 - \epsilon A - c)(1 + r)$ is about 0.470. Using median return $\hat{x} = 1.094$ from table 2, the ex-post level of assets as a percentage of net-worth for risk aversion level $\underline{\rho}$ is $A\hat{x}/(1 - \epsilon A - c)(1 + r)) = 1.786$. In the graph, this is the range where the model predicted curve moves away from the data. The model predicted median asset level of 48.1% in table 4 below is well within the 95% confidence interval of [43.1, 51.9]. This also shows that the penalty term in (17) is not relevant in the neighborhood of the optimal parameters.

The bottom panels of figure 1 compare the model prediction for firm capital structure to the empirical cdfs for 1993 and 1998. The left panel shows that the model somewhat over predicts equity/assets. This again occurs because no model solutions exist below $\underline{\rho}$ and (14) assigns point mass to these values. At $\underline{\rho} = 0.74$ the associated value of $\overline{v}$ is 0.335. At median return level $\hat{x} = 1.094$, this gives an ex post value of equity/assets of $(\hat{x} - \overline{v})/\hat{x}$ is about 0.7, which is where the kink in the left panel occurs. If the cdf of ϵ is computed conditional on $\epsilon < 0.7$, the model does an excellent job of replicating the empirical distribution of equity/assets among firms – see the right panel. By definition total assets are debt plus equity, thus equity/assets is a measure of firm capital structure. The approximately uniform cdf indicates that all capital structures are equally likely and this suggests agent heterogeneity, if individual firm capital structure is optimal.[31]

Table 4: Model Point Estimates

Parameter	**Interpretation**	**Model**	**Data**
median A%	median firm assets (size)	48.1	[43.1, 51.9]
consumption %	consumption as a percent of net worth	3.6	3-5
default %	small firm default rate	4.4	3.5-4.5
neg. equity %	negative equity in the firm	10.6	15.7, 21.0

Table 4 shows that the model replicates successfully other targets. Median firm assets match well (as discussed above) and consumption is in the standard range.[32] The default prediction is

[30] Model solutions do not exist because if ρ is too low current consumption goes to zero and future consumption goes to infinity, as implied by the standard intertemporal MRS condition.

[31] A uniform distribution for all firms is consistent with a determinate capital structure for each firm.

[32] Point estimates for expected percent of net-worth spent on consumption and the default probability are

slightly higher than the average annual default rate of 3.5% on small business loans guaranteed by the Small Business Administration reported by Glennon and Nigro (2005) and close to the default rate on trade credit of 4.5% Boissay and Gropp (2007), table 2.4 estimate for small French firms.[33]

Negative equity, accounted for in the model in constraint (1), indicates that non-business assets are used to cover business losses (e.g., personal funds or unpaid bills absorbed by creditors). The model value of 10.6% is below the SSBF empirical values of 15.7% in 1993 and 21.0% in 1998 reported by Herranz, Krasa, and Villamil (2008). The use of personal funds to "bail out" a firm may seem puzzling since we consider only incorporated firms, which are protected by limited liability in bankruptcy. Why do these entrepreneurs not simply default on their loans? In a dynamic model an entrepreneur will not default, and hence will continue to operate a poorly performing firm, if the firm's expected discounted continuation value is sufficiently high. While the benchmark model's predicted level of negative equity falls short of the values observed in the SSBF, section 7.4 will show the model can match the data if entrepreneurs are slightly optimistic.

Table 5: Entrepreneur's Ex-Ante Optimal Choice and Risk Aversion

ρ	**0.9**	**1.2**	**1.5**	**1.8**	**2.1**	**2.5**	**3.0**	**3.5**	**4.0**
$(1-\epsilon)A$ %	21.5	21.5	21.5	21.5	21.5	18.7	15.2	12.8	11.1
A %	61.0	44.2	35.3	30.0	27.0	22.7	18.3	15.4	13.3
ϵ %	64.8	51.5	39.1	28.5	20.4	17.6	17.2	16.8	16.5
$\overline{v}$	0.409	0.550	0.682	0.798	0.891	0.921	0.925	0.928	0.930
default %	3.6	3.7	4.0	4.6	5.4	5.6	5.4	5.2	5.1
c %	2.2	4.0	4.6	4.9	5.0	5.0	5.0	4.9	4.9

Finally, parameters μ and σ were used to construct the distribution of risk aversion, $g_{\mu,\sigma}(\rho)$, with mean risk aversion parameter $\mu = 1.55$. In order to better understand the effect of risk aversion on endogenous parameters, table 5 shows how the loan size, firm size, financial structure, debt burden and default vary as risk aversion increases. The percentage of net-worth an entrepreneur borrows, $(1-\epsilon)A$, is constant when borrowing constraint (3) binds and falls as the risk aversion parameter increases because the borrowing constraint becomes slack. More risk averse agents also run smaller firms, A, and use less of their own money, ϵ. As a consequence, firms become more leveraged and their debt burden rises, $\overline{v}$, which increases the incentive to default. Consumption is roughly constant except for the agents most willing to bear risk, where current consumption (as a percentage of net worth) is lower because they invest more now to consume more in the future.

$\int_{-\infty}^{\underline{\rho}} c(\rho) g_{\mu,\sigma}(\rho)\, d\rho + \int_{\underline{\rho}}^{\infty} c(\rho) g_{\mu,\sigma}(\rho)\, d\rho$ and $\int_{-\infty}^{\underline{\rho}} \int_{\underline{x}}^{x^*(\epsilon,\underline{\rho})} f(x) g_{\mu,\sigma}(\rho)\, dx\, d\rho + \int_{\underline{\rho}}^{\infty} \int_{\underline{x}}^{x^*(\epsilon,\rho)} f(x) g_{\mu,\sigma}(\rho)\, dx\, d\rho$.

[33] They report that trade credit is a third of all firms' total liabilities in most OECD countries.

6 Welfare Effects of Limited Liability

Up to this point we have focused on incorporated firms.[34] However, in the SSBF roughly half the firms are incorporated and half are unincorporated. We now consider the welfare effects of incorporation, assuming the return distributions of both types of firms are similar. The main benefit of incorporation is limited liability: an owner's personal assets are held separately from business assets and cannot be seized by creditors. A secondary benefit is taxation (e.g., firm owners may lower self employment taxes by organizing as S-corporations) and disadvantages include small legal costs and information disclosure requirements. In order to understand why incorporated and unincorporated small firms co-exist, we focus on the effect of limiting personal liability on ex-ante welfare. Our main finding is that the less risk-averse receive higher welfare from incorporation and the more risk-averse tend to be better off remaining unincorporated. This occurs because less risk-averse owners run bigger firms with higher intertemporal value, which tempers their incentive to default. In contrast, the more risk averse are unable to credibly commit ex-ante to refrain from defaulting ex post; putting some personal assets at risk by forgoing incorporation mitigates this commitment problem.

In the baseline model, limited liability corresponds to $\gamma = 0$. We now relax this assumption by considering a legal system in which unincorporated agents can be forced to pay a percentage $\gamma > 0$ of personal assets $(w - \epsilon A - c)(1 + r)$ to investors. That is, unincorporated owners are personally liable for firm debt, and in the extreme case of $\gamma = 1$ all personal net-worth can be seized if the firm defaults.[35] Appendix E modifies Problem 1 to account for γ. In the objective, expression $(w - \epsilon A - c)(1 + r)$ in the default integral is replaced by $(1 - \gamma)(w - \epsilon A - c)(1 + r)$. The investor receives $\gamma(w - \epsilon A - c)(1 + r)$, after deadweight loss δ is deducted. Clearly, default cutoff x^* is also affected and is decreasing in γ. Appendix E shows that Problem 1 is equivalent to a slightly modified version of Problem 3 in which the integrand over default states is $(1-\gamma)(1-\epsilon A-c)(1+r)$ and the investor receives $(1-\delta)\gamma\left(\frac{1}{A} - \epsilon - \frac{c}{A}\right)(1+r)$, which is divided by assets A since the investor's constraint specifies the return per unit of assets.

Figure 2 shows that for agents with low (below median) risk aversion, any positive γ decreases welfare.[36] The loss is substantial, especially for those most willing to bear risk. Clearly, these

[34] SSBF data do not contain sufficient information to compute ROA for unincorporated firms (owner wage is not reported). See Herranz, Krasa, and Villamil (2008) for discussion of this issue.

[35] In the U.S., sole proprietors and partners are personally and jointly responsible for business liabilities. In practice, $\gamma = 1$ does not occur for unincorporated firms because some private assets are exempt from seizure (e.g. some equity in a home, retirement assets, and personal assets). Thus even if a firm is unincorporated, the effective level of γ is significantly less than 100% and varies across individuals (with different portfolios and asset class exemptions).

[36] Welfare is computed using the equivalent variation, which for given ρ and γ is the λ such that $V_S^{\gamma}(\lambda w) = V_S^{\gamma'}(w)$;

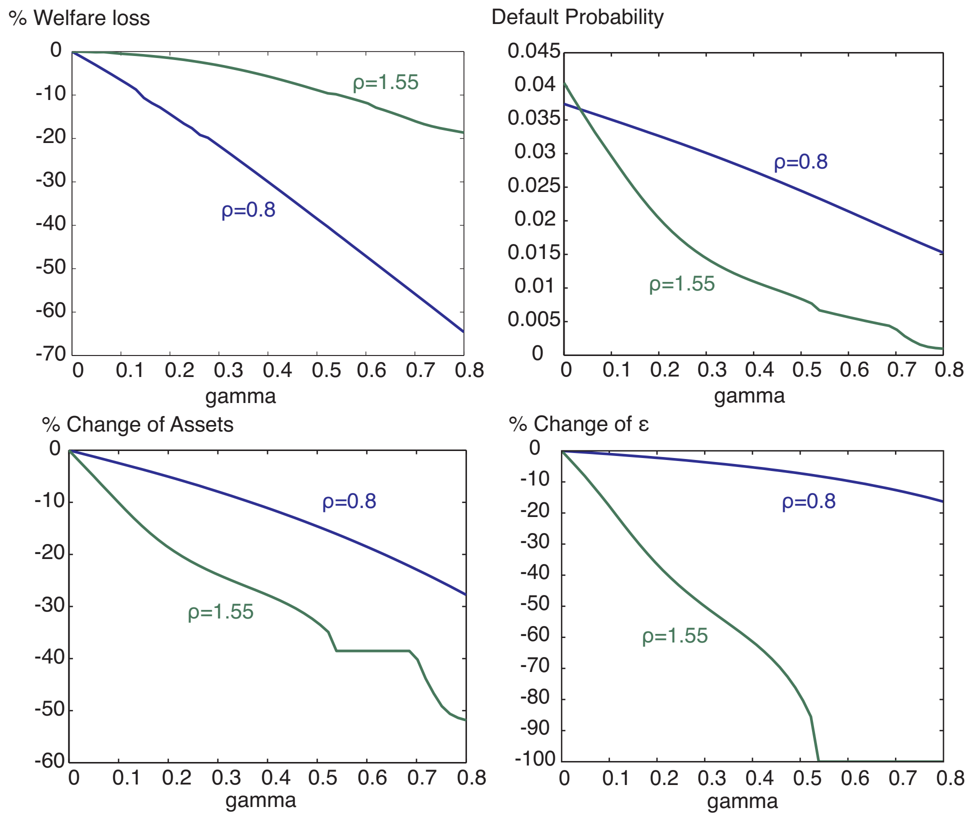

Figure 2: Impact of changes in γ on entrepreneurs with lower risk aversion levels

agents would wish to incorporate to protect personal assets and raising γ is detrimental them: While raising γ lowers default (see the top-right panel), this benefit is outweighed by the fact that raising γ discourages risk taking, resulting in a a substantial decrease in firm size (see the bottom-left panel). The size reduction is bigger for entrepreneurs with $\rho = 1.55$, but since less of their net worth is tied up in the firm, their welfare loss is lower. The reduction in the default probability with γ implies that interest rates and thus borrowing costs decline, which implies that ϵ decreases as the firm will use more outside funds. In summary, the net effect of increasing γ is to reduce the insurance provided by bankruptcy, which in turn discourages socially beneficial risk-taking.

In contrast, figure 3 shows that more risk averse agents could increase welfare by forgoing limited liability for some values of γ. This occurs because higher ρ agents run smaller firms, hence the loss from exclusion is smaller. This implies that ratio v_B/v_S is increasing in ρ, which in turn implies that x^* is larger.[37] If entrepreneurs could commit ex-ante to a default cutoff x_c^* (i.e., if constraint (8)

V_S^{γ}, $V_S^{\gamma'}(w)$ are value functions in solvency given exemption parameters γ and γ'. Proposition 1 implies that $V_S^{\gamma}(\lambda w) = \lambda^{1-\rho} V_S^{\gamma'}(w)$. Further, $V_S^{\gamma}(w) = w^{1-\rho} V_S^{\gamma}(1) = v_S^{\gamma}$ and $V_S^{\gamma'}(w) = w^{1-\rho} V_S^{\gamma'}(1) = v_S^{'\gamma}$. The welfare change is $\lambda = (v_S^{\gamma}/v_S^{\gamma'})^{\frac{1}{1-\rho}}$.

[37]Continuation value v_S is increasing in A, i.e., bigger firms have greater losses from exclusion. Thus $v_B/v_S < 1$,

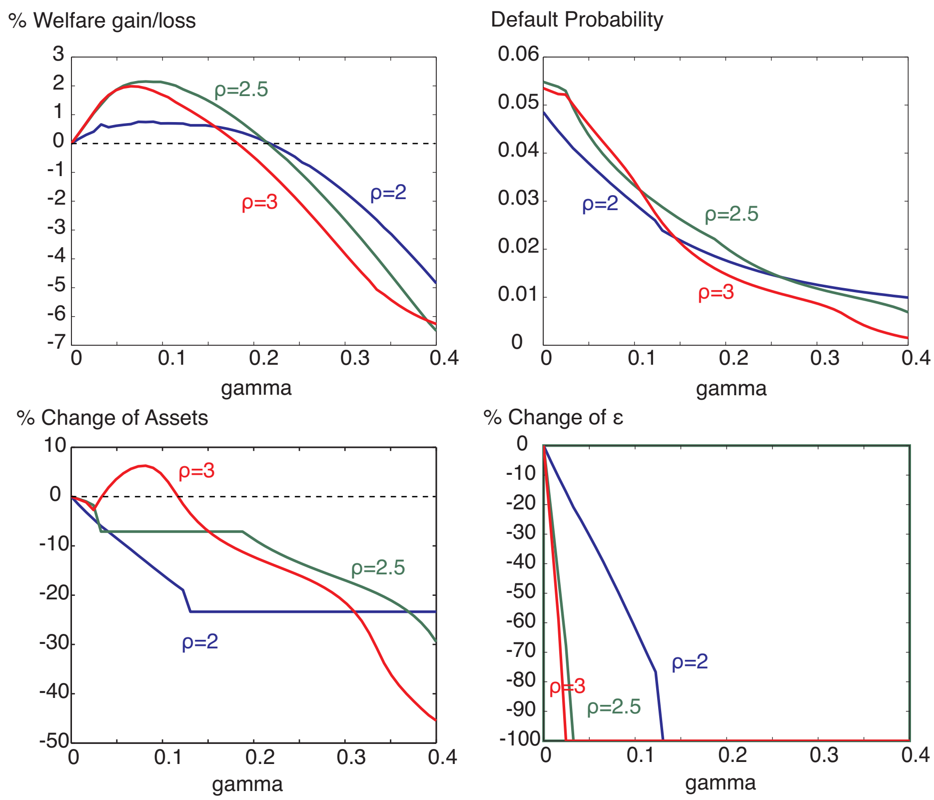

Figure 3: Impact of changes in γ on entrepreneurs with higher risk aversion levels

was eliminated), then $x_c^* < x^*$. Thus inability to commit leads to higher default, which is costly. Raising γ lowers x^*, moving the default cutoff closer to the efficient (full commitment) level, x_c^*. As long as γ is not raised too much this benefit outweighs the cost of discouraging risk taking. Figure 3 also shows that increasing γ increases firm (asset) size for entrepreneurs with $\rho = 3$, but not for those with $\rho = 2$ and $\rho = 2.5$. Borrowing constraint (10) binds when $\epsilon \to 0$, and when this occurs $A = b$, i.e., the ex-ante choice of A is constant. In contrast, for $\rho = 3$ the borrowing constraint is slack for all values of γ. Raising γ lowers borrowing costs and the entrepreneur first responds by lowering ϵ, i.e., by using more outside funds. Once it cannot be reduced further because $\epsilon = 0$, the lower borrowing costs induce the firm to increase A. If γ becomes too large, however, the loss of insurance from bankruptcy starts to dominate and A is reduced.

If the more risk-averse tend to remain unincorporated, there are two observable implications. First, table 5 shows in the baseline model ($\gamma = 0$), less risk averse agents run larger firms (higher A), use more personal funds (higher ϵ), and risk aversion has little effect on consumption (c). (15) implies for given $x > 0$, the more risk averse will have a smaller posterior level of A. Second, one

but it converges to 1 as $A \to 0$.

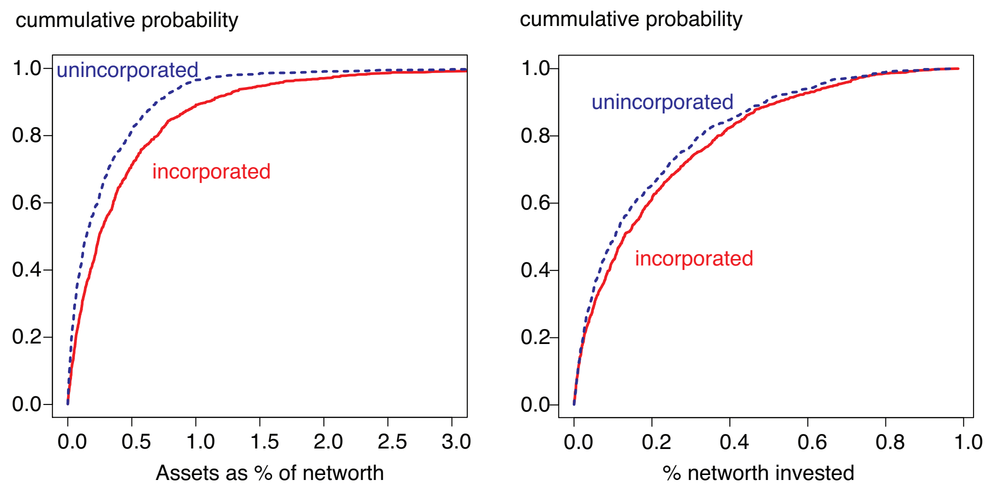

Figure 4: Assets as a percentage of net worth and net worth invested in the firm for 1998

can check that (12) implies for given x, more risk-averse agents will invest less personal wealth in the firm. Figure 4 shows that both implications are true in the SSBF data.[38]

Why do risk-averse entrepreneurs with a commitment problem not simply incorporate and pledge collateral? For example, the owner of a small unincorporated firm with retirement assets has two options: (i) Withdraw funds from the retirement account and post them as a bond with the lender. This is costly due to early withdrawal penalties and because long-term assets earn higher returns than more liquid investments. (ii) Leave the funds in the retirement account but promise to use them to cover business debts. The agent might renege on the promise or it may not be enforceable by a court. Remaining unincorporated effectively provides collateral when γ is known to all parties and enforced by bankruptcy courts at low cost.

In practice, remaining unincorporated and pledging collateral may be substitutes. The desirability of each alternative will depend on opportunity and enforcement costs. Furthermore, the effective amount of γ will differ significantly among entrepreneurs. For example, if most of an entrepreneur's net-worth is in home equity and the entrepreneur resides in a state that exempts all home equity γ will be very low, while if the state permits home equity to be seized γ will be higher. Second, tax advantages and disclosure requirements are likely to have differential impacts on entrepreneurs. Thus, the model suggests that more risk averse entrepreneurs are more likely to be unincorporated, but it does not imply a strict cutoff level of ρ.

[38]The distributions in figure 4 report firms with positive equity and owners with positive net worth.

Table 6: Comparative statics for T: Fix $r_L = 1.2\%$, $r = 4.5\%$, $\beta = 0.97$, $\delta = 0.10$

T	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**20**
med A%	56.2	54.4	52.7	51.1	49.6	48.1	46.7	45.4	44.3	43.3	42.4	38.9
default%	6.1	5.6	5.3	5.0	4.7	4.4	4.2	4.0	3.8	3.6	3.5	2.9
cons.%	3.7	3.7	3.7	3.7	3.7	3.6	3.6	3.6	3.6	3.6	3.6	3.6
neg eq%	8.4	8.8	9.3	9.7	10.0	10.6	11.1	11.9	13.4	15.1	17.0	21.0

7 Policy Experiments and Comparative Statics

Overall, the model is able to account for key properties of the data. In light of this success, we now undertake a series of policy experiments to better understand the effect of bankruptcy rules, liquidity constraints, risk aversion and optimism in explaining the data. We also perform a counterfactual exercise to show the importance of the return distribution. We now wish to evaluate the effect of policies versus innate characteristics, thus we conduct comparative static exercises and use equivalent variation to assess welfare (utilities of heterogeneous agents cannot be compared).

7.1 Bankruptcy Policy: T, δ

Bankruptcy Exclusion Period T:

Consider the effect of changes in T on welfare, where longer exclusion raises the penalty of bankruptcy. Table 6 fixes μ, σ, b, and evaluates the effect of altering the exclusion period from the benchmark $T = 11$. As T decreases default increases rapidly. Firm size increases, measured by median asset level A. Because b is fixed, the decrease in total investment results in a decrease in equity and an increase in debt, which raises negative equity. One of the main economic arguments in support of recent U.S. bankruptcy reform was that more stringent bankruptcy rules lower interest rates, and therefore help borrowers. Table 13 in Appendix A shows that the loan rate indeed decreases as T increases. However, stricter bankruptcy provides less insurance against bad realizations, and this effect dominates. In particular, table 7 shows that lowering the exclusion period increases welfare, and the model implies that it is optimal to set T as low as possible. Decreasing T is beneficial in the baseline model because it allows a firm to restart and be productive, in accordance with the historical rationale for bankruptcy. Mandating $T = 0$ may not be possible or desirable in practice.[39]

[39] For example, information frictions would make a very low T undesirable. Suppose entrepreneurs could choose between the blueprint return distribution and an alternative with more risk that is socially undesirable. In an institutional environment in which strong ex-ante and interim screening mechanisms exist and penalties are credible, a small T can be sufficient to avoid moral hazard or adverse selection. In contrast, a country with poor institutions would require a

Table 7: Welfare Effect as T Varies: % change in net-worth compared to benchmark

risk aversion ρ	**0.9**	**1.2**	**1.5**	**1.8**	**2.1**	**2.5**	**3.0**	**3.5**	**4.0**
$T = 6$	36.9	11.2	7.7	6.1	5.0	3.9	3.1	2.6	2.2
$T = 8$	19.8	5.5	3.9	3.0	2.4	1.9	1.5	1.3	1.1
$T = 10$	6.3	1.3	1.1	0.8	0.7	0.5	0.4	0.3	0.3
$T = 11$	—	—	—	—	—	—	—	—	—
$T = 12$	-3.6	-0.7	-0.9	-0.7	-0.5	-0.4	-0.3	-0.3	-0.2
$T = 14$	-7.6	-4.4	-2.1	-1.8	-1.4	-1.0	-0.8	-0.6	-0.5
$T = 16$	-12.7	-6.5	-3.8	-2.4	-2.0	-1.5	-1.1	-0.9	-0.7

The tradeoff between insurance provided by firm bankruptcy and higher interest rates induced by increased default has been analyzed for consumer bankruptcy by Chatterjee, Corbae, Nakajima, and Rios-Rull (2007) and Livshits, MacGee, and Tertilt (2007). In both models consumers trade off insurance against health, divorce or family shocks versus consumption smoothing; the signs of the tradeoffs differ but the welfare effects are modest.[40] The first paper finds that when the length of punishment is reduced from 10 to 5 years welfare drops by 0.05%, thus the negative effect from a higher interest rate and tighter borrowing constraint slightly dominates the insurance benefit of a shorter punishment period. The second paper shows that the insurance effect is sometimes weakly dominant, but again the effect is modest. Meh and Terajima (2008) add unincorporated entrepreneurs to the model, and find a larger welfare effect of 1.78%. In contrast, table 7 reports strong welfare effects from reducing the exclusion penalty in our model, particularly for agents with low levels of risk aversion. The main reason for the difference between our model of firm bankruptcy and the consumer bankruptcy models is that reducing the punishment period encourages entrepreneurs to invest more in their firms (operate at a larger scale), and increased output raises welfare. In this sense, even though we do not find extreme variations in ρ, risk interacts with the dynamic decision problem, return distribution and bankruptcy rules to have an important effect on some (heterogeneous) agents, namely those that invest most heavily in their firms.

Bankruptcy Cost δ: Appendix A analyzes bankruptcy cost δ (efficiency in liquidating firm assets) and table 16 shows the welfare effect is minor. However, if δ is very large and there are large fixed cost to creditors to recover payments in default, agents will try to avoid costly bankruptcy, through debt forgiveness or renegotiation. The static model of Krasa, Sharma, and Villamil (2008) shows that when courts are sufficiently inefficient substantial deadweight losses are possible.

larger T to deter entrepreneurs from choosing the alternative distribution, thus generating additional inefficiencies.

[40]In our model credit is secured, for example by a house, and "bad luck" is a poor return x rather than the health, job, divorce or family shocks in the consumer models.

Table 8: Comparative Statics for b: r_L = 1.2%, r = 4.5%, β = 0.97, δ = 0.10

b	**0.10**	**0.15**	**0.20**	**0.21**	**0.25**	**0.30**	**0.35**	**0.40**	**0.50**
median A %	46.9	47.1	47.8	48.1	49.0	50.5	51.8	52.4	52.5
default %	3.0	3.6	4.3	4.4	4.8	5.2	5.4	5.6	5.9
cons. %	3.6	3.6	3.6	3.6	3.7	3.7	3.7	3.6	3.6
neg Eq. %	5.7	7.7	10.0	10.6	11.7	12.9	13.6	14.2	15.0

7.2 Liquidity Constraints

Policy can also affect credit constraint parameter b. Table 8 shows that increasing b allows firms to borrow more, and hence operate at a larger scale A. The higher levels of firm debt, however, increase the percentage of firms who default or have negative equity. Table 9 shows substantial welfare effects from raising b for the least risk averse agents, but not for the more risk averse because for sufficiently high b the credit constraint does not bind. Comparing the welfare effects of T and b shows that an entrepreneur with median ρ benefits more from reducing T than from relaxing the borrowing constraint, in the baseline model. In practice, relaxing the borrowing constraint could be achieved by providing subsidized loans targeted to small business, such as SBA loans.

Table 9: Welfare Effect as b Varies: % change in net-worth compared to benchmark

risk aversion ρ	**0.9**	**1.2**	**1.5**	**1.8**	**2.1**	**2.5**	**3.0**	**3.5**	**4.0**
b = 0.100	-13.1	-8.5	-6.2	-4.9	-3.7	-2.2	-0.9	-0.2	-0.1
b = 0.150	-6.1	-4.8	-3.2	-2.0	-1.4	-0.4	0.0	0.0	0.0
b = 0.200	-1.8	-0.2	-0.6	-0.4	-0.1	0.0	0.0	0.0	0.0
b = 0.215	—	—	—	—	—	—	—	—	—
b = 0.250	8.2	1.7	1.3	0.6	0.0	0.0	0.0	0.0	0.0
b = 0.300	14.8	4.2	2.4	0.7	0.0	0.0	0.0	0.0	0.0
b = 0.400	26.6	7.2	2.7	0.7	0.0	0.0	0.0	0.0	0.0
b = 0.500	35.0	7.5	2.7	0.7	0.0	0.0	0.0	0.0	0.0

7.3 Risk Aversion

Now consider the effect of changes in risk aversion. Clearly policy cannot modify μ, but comparative statics show how owner risk aversion affects the firm. In table 10, as μ increases, owners run smaller firms. Because b is fixed, these smaller firms have higher debt, which explains why negative equity and default rise with μ. Mazzocco (2006) finds that women are more risk averse than men (ρ of 5 versus 1.7). In our model this parameter change would imply that (i) less women own

Table 10: Comparative Statics for μ: $r_L = 1.2\%$, $r = 4.5\%$, $\beta = 0.97$, $\delta = 0.10$

μ	**1.15**	**1.25**	**1.35**	**1.45**	**1.55**	**1.65**	**1.75**	**1.85**
median A %	74.3	65.4	58.3	52.7	48.1	44.4	41.2	38.6
default %	4.2	4.2	4.3	4.4	4.4	4.5	4.6	4.7
cons. %	2.8	3.0	3.2	3.5	3.6	3.8	4.0	4.1
neg Eq. %	8.4	8.9	9.5	10.0	10.6	11.1	11.7	12.3

businesses, (ii) they run smaller firms,[41] and (iii) they have higher negative equity. The SSBF data indicate that all three model implications are consistent with the data. In 1993 and 1998 women owned 16% and 24% of businesses, respectively. In 1998 median assets, normalized by net-worth outside the firm, were 39% for firms owned by women and 53% for men (the only year net-worth is reported). Finally, negative equity for women was 19.5% versus 14.8% for men, and 26.1% versus 19.4% in 1993 and 1998, respectively. Absent the model, the observation that firms run by more risk averse owners have more negative equity might seem counterintuitive.

7.4 Entrepreneur Optimism

How does optimism by entrepreneurs affect our results? Intuition suggests that less risk averse, less optimistic agents will behave similarly to more risk averse, more optimistic agents. This leads to an identification problem: optimistic agents may be observationally equivalent to less risk averse, non-optimistic agents. We now investigate whether the model has observable implications that are uniquely induced by optimism. Assume that an optimistic entrepreneur believes the firm's return exceeds the true return by some fixed percentage $\bar{o}$. Formally, this implies the entrepreneur assumes that firm returns are $X + \bar{o}$, which yields cdf $H(x - \bar{o})$ in the objective of problem 3. Assume the lender uses the correct distribution to determine payoff (7) in problem 3.

Tables 17 and 18 in Appendix A vary $\bar{o}$ by 5% and 10% respectively, and fix all other parameters.[42] The tables show that slight optimism improves the fit in the baseline model with $T = 11$ while keeping μ, σ and the default rate in acceptable ranges. Liquidity constraint parameter b increases slightly, as does A. Negative equity increases to a level consistent with SSBF data because optimistic entrepreneurs run larger firms: they expect higher future returns relative to the baseline

[41] In a very interesting study of nascent entrepreneurs, **?**) table 6 documents that women plan to run smaller firms than men, suggesting an innate difference.

[42] Differences in manager ability could be modeled by considering a distribution $H(x, a)$, where a denotes ability. We focus on heterogeneity in risk aversion because it is central to theories of entrepreneurship. Furthermore, firms with very high and low ability will exit the SSBF sample – those with low ability will tend to close down and those with high ability will become too large to be included in the survey. See, for example, Antunes, Cavalcanti, and Villamil (2008), Cagetti and DeNardi (2006) or Meh and Terajima (2008) for models with ability heterogeneity.

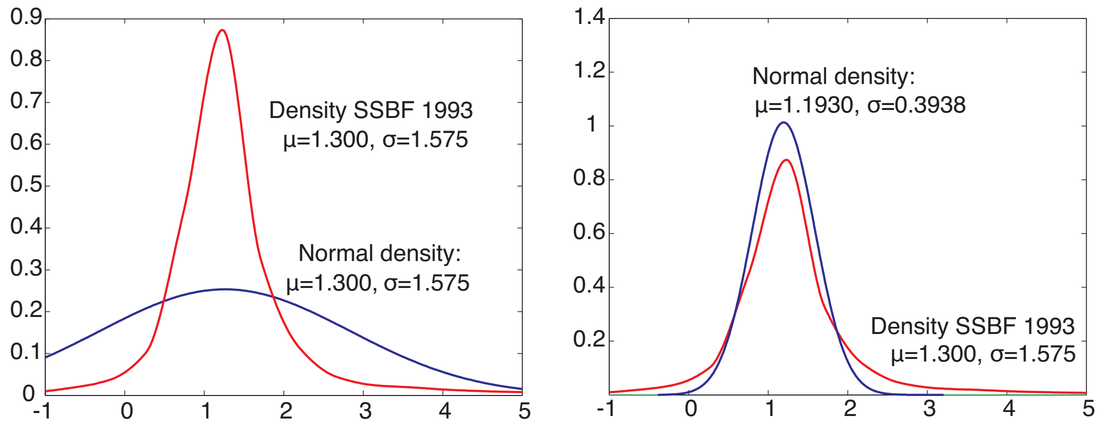

Figure 5: Empirical firm return pdf versus normal pdfs, SSBF 1993

and increase the total amount of debt $\bar{v}$. Equity is negative if $x < \bar{v}$. When $\bar{v}$ is higher, $x < \bar{v}$ is more likely and this increases the percentage of projects with negative equity. Mild entrepreneur optimism can thus account for the level of negative equity observed in the 1993 SSBF (15.7%) and still accommodate the relatively low level of default observed in the data.

7.5 Counterfactual Exercise: Empirical vs. Normal Returns

The features of return distribution $f(x)$ are important for understanding entrepreneur behavior. Figure 5 compares the empirical distribution of return on assets for incorporated firms in the 1993 SSBF to two normal distributions with different means and variances.[43] Clearly, small firms have risky, non-normal returns. The standard deviation is high, with the higher risk somewhat compensated by a higher mean, and the distribution is skewed right with high kurtosis (i.e., a long upper tail), see table 2. About 12% of firms lost more than 20% of assets invested (debt plus equity), 7.4% lost more than 40%, and 3.8% lost more than 100%. However, positive returns are even more substantial: 20.7% exceeded 50%, 10.4% exceeded 100%, and 3.8% exceeded 200%.

We conduct two counterfactual experiments to show that the return distribution is important. The experiments replace the empirical ROA distribution computed from SSBF data, keeping all other benchmark settings the same, with two different normal distributions. In figure 5 the right panel shows the "best fit" normal distribution that minimizes the maximum distance between the normal and empirical cdfs and the left panel shows the normal distribution with the same mean and variance as the empirical distribution.

[43] We use 1993 data because it is the only SSBF data set with interest expenses, which are required to compute ROA. We consider only firms with at least $50,000 in assets that are incorporated.

Best Fit Normal Distribution. Let $g_{\mu,\sigma}$ be the density of a normal distribution with mean μ and standard deviation σ and f be the density of the SSBF distribution. We solve $\min_{\mu,\sigma} \sup_x |g_{\mu,\sigma}(x) - f(x)|$ to find a normal distribution that best approximates the empirical density function. The resulting values are $\mu = 1.193$ and $\sigma = 0.394$, shown in the right panel of figure 5. In order to fit the "middle" this normal distribution has less mass in the tails and, as a consequence, is less risky. Thus, when re-calibrating the model, median risk aversion increases from 1.55 to 2.33 but at the same time, for given ρ, the lower project risk in this normal distribution encourages entrepreneurs to run larger firms. Default is lower, again because this normal distribution has a thinner lower tail. Finally, the thinner upper tail implies that less firms will be "lucky" and have a very good realization. In order to match the distribution of net-worth invested, firms must be more leveraged: Given two solvent firms with the same realization, a more leveraged firm earns a higher return because the owner receives a higher residual after making the fixed debt payment.[44] The somewhat higher level of debt also implies that more low realizations will result in negative equity, and the predicted percentage of firms with negative equity increases from 10.6% to 13.7%.

Table 11: Counterfactual Experiment: Normal Distributions

Parameter	**Data**	**Empirical** $f(x)$ SSBF 1993	**Best Fit Normal** $g(x)$ μ= 1.193, σ=0.3938	μ, σ **Normal** $g(x)$ μ=1.300, σ=1.193
μ	1-3	1.55	2.33	$4.4 * 10^8$
σ	NA	.83	1.11	$7.9 * 10^8$
b%	NA	21.5	30.0	23.4
fit	NA	0.042	0.040	.045
median A%	[43.1,51.9]	48.1	54.7	38.6
default %	3.5	4.4	1.5	61.0
cons. %	3-5	3.6	4.9	3.1
neg. Eq %	15.7	10.6	13.7	64.4

Normal Distribution with SSBF μ, σ. The left panel of figure 5 compares the SSBF pdf with a normal distribution with the same mean and standard deviation. Table 11 shows the results for this distribution are significantly at odds with the data, highlighting the importance of the return distribution. First, the fat tails lead to μ and σ with all point mass at $\underline{\rho}$ and $\bar{\rho}$, where $\bar{\rho}$ is the highest risk aversion for which we compute a solution. Generally, we can choose $\bar{\rho}$ sufficiently high that the mass above $\bar{\rho}$ is negligible; this cannot be done for this normal distribution with fat tails and $\bar{\rho}$ affects the results.[45] Second, the model predictions in the last column of table 11 are implausible.

[44] This also explains the higher value of b.

[45] Upper bound $\bar{\rho}$ is needed for computation; it is impossible to compute solutions for a fine grid $[\underline{\rho}, \infty]$.

8 Concluding Remarks

This paper assesses the quantitative effects of changes in bankruptcy rules, credit constraints and optimism on firms when agents differ in willingness to bear risk. Corporate bankruptcy insures owners against extreme personal loss, but preserves the possibility of very high future firm returns. Figure 5 shows the empirical return distribution for small U.S. firms has most mass centered around the middle, which is attractive to individuals with standard degrees of risk aversion, and a long upper tail. Entrepreneurs trade off the value of absorbing a current loss against the option value of maintaining the firm. We find that modest differences in risk aversion interact with policies to generate significant effects on output and welfare for some agents. The model also permits us to link firm legal status with owner risk aversion. Less risk-averse owners incorporate to protect personal assets because higher firm option value leads to lower default rates, while if more risk-averse owners run firms they tend to remain unincorporated. This seemingly paradoxical behavior occurs because placing some personal assets at risk of seizure allows more risk-averse owners to solve their "excess default" problem. In other words, remaining unincorporated permits them to post a bond a postiori. Of course, if the legal system is too costly, slow, corrupt, or otherwise not credible, bankruptcy will not improve outcomes.

Default is beneficial in risky lending relationships because it introduces a contingency into a non-contingent contract, debt. Bankruptcy allows risk-averse agents to protect themselves against extremely bad outcomes. Indeed, we have shown that even when current realizations are poor owners may use personal assets to "bail out" their firms to avoid bankruptcy due to the firm's option value. One point of the paper is that firms use many strategies to manage risk – including altering their size, capital structure (including injections of equity from personal net-worth), and sometimes choosing to default. Thus, lowering the default rate to zero is not the desideratum. For example, the default rate is zero when no lending occurs, but this is not a desirable outcome.

Finally, a number of extensions are possible. First, the large welfare effects we find are likely to be an upper bound. In future work it would be useful to examine the effect of adverse selection and moral hazard. These information frictions would raise the exclusion period in order to penalize socially inefficient default more heavily, and lower the welfare gains from bankruptcy. Second, we model the many sources from which firms obtain loans, including, banks, trade credit associations, leasing companies, and credit cards, as a composite lender. In future work, it would be useful to model the problems of these different lenders. For example, it would be instructive to consider the problem of a bank that must attract deposits and make loans, subject to default risk and regulation. Similarly, trade credit and leasing are important when lenders face information and enforcement problems, as is the case for small firms. Also, general equilibrium effects are important in credit markets. Increased loan demand will raise the cost of external finance, which will offset some of the welfare gains. Third, we focus on idiosyncratic firm risk, which is particularly interesting in this setting, because firms are not tradable, and hence the owner cannot diversify this risk. Nonetheless, aggregate risk and correlated shocks would be interesting extensions to further explore macroeconomic implications of the model.

9 Appendix A: Experiments

Table 12 shows that the model is roughly stable when T changes. As T increases, μ remains between 1.5 and 1.6 and σ between 0.74 and 0.9. Liquidity constraint b decreases somewhat because the penalty increases with T; entrepreneurs become more cautious and run smaller firms (lower A) and to achieve the best model fit, the optimization procedure lowers b to ensure they use enough personal funds. Default decreases with T because it is more costly to the entrepreneur. Consumption and negative equity are stable. Table 13 shows that the loan rate decreases as T increases given ρ.

Table 12 Benchmark Exogenous Variables: $r_L = 1.2\%$, $r = 4.5\%$, $\beta = 0.97$, $\delta = 0.10$

T	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**20**
μ	1.62	1.55	1.49	1.51	1.52	1.52	1.51	1.50
σ	0.90	0.83	0.75	0.74	0.76	0.76	0.76	0.78
b %	20.6	21.5	22.0	19.8	18.4	17.7	17.3	15.4
fit	0.046	0.042	0.037	0.034	0.034	0.034	0.035	0.036
median A %	46.9	48.1	49.2	47.0	45.3	44.3	43.8	41.3
default %	4.7	4.4	4.2	3.8	3.5	3.3	3.1	2.5
cons. %	3.7	3.6	3.6	3.6	3.6	3.6	3.6	3.5
neg Eq. %	10.2	10.6	10.8	10.5	10.8	11.1	11.6	11.1

Table 13 Loan Interest Rate as T Varies

risk aversion ρ	**0.9**	**1.2**	**1.5**	**1.8**	**2.1**	**2.5**	**3.0**	**3.5**	**4.0**
$T = 0$	19.6	17.7	17.4	18.1	19.4	21.6	24.4	27.2	29.9
$T = 6$	18.0	15.3	14.2	14.0	14.3	14.4	14.3	14.1	14.1
$T = 8$	17.3	14.5	13.3	13.0	13.3	13.3	13.2	13.1	13.0
$T = 10$	16.6	13.7	12.4	12.1	12.3	12.4	12.2	12.0	11.9
$T = 11$	16.3	13.3	12.0	11.6	11.9	11.9	11.6	11.5	11.4
$T = 12$	16.0	12.9	11.7	11.2	11.4	11.4	11.2	11.0	10.9
$T = 14$	15.3	12.3	10.9	10.5	10.6	10.5	10.3	10.2	10.1
$T = 16$	14.7	11.8	10.4	9.8	9.8	9.8	9.6	9.5	9.4
$T = 20$	13.6	10.7	9.3	8.7	8.5	8.8	8.6	8.5	8.4

Table 14, 15 and 16 show the results are unaffected by substantial changes in bankruptcy cost δ. Compared to the $\delta = 0.1$ benchmark in table 12, table 14 triples δ and re-estimates the model: μ, σ, b are virtually unaffected, thus the model is robust and detailed cost measurement is not essential in this range. Table 15 reports comparative static results in which δ varies between 0 and 100%, fixing b, μ, σ at the benchmark values (i.e., the model is not re-estimated). Again, δ has almost no impact on endogenous variables – in contrast to the comparative statics with respect to T. Table 16 shows that δ has a minor effect on welfare (at the median level of risk aversion the gains/losses are less than 0.1%) because (a) bankruptcy occurs with only a small probability, and (b) assets Ax in bankruptcy states tend to be small so deadweight loss δAx is small. Clearly, the expected costs, i.e., the product of (a) and (b) is second order.

Table 14 Higher Cost δ: r_L = 1.2%, r = 4.5%, β = 0.97, δ = 0.30

T	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**20**
μ	1.79	1.67	1.55	1.50	1.52	1.52	1.51	1.50
σ	1.08	0.95	0.81	0.74	0.76	0.76	0.76	0.78
b %	14.9	16.9	19.8	20.1	18.4	17.6	17.2	15.4
fit	0.052	0.046	0.040	0.035	0.034	0.034	0.035	0.036
median A %	39.8	42.6	46.3	47.3	45.3	44.3	43.6	41.3
default %	4.0	4.0	4.0	3.8	3.5	3.2	3.1	2.5
cons. %	3.8	3.7	3.7	3.6	3.6	3.6	3.6	3.5
neg Eq. %	8.7	9.2	10.2	10.5	10.7	11.0	11.4	11.1

Table 15 Comparative Statics for δ: Fix r_L = 1.2%, r = 4.5%, β = 0.97, δ = 0.10

δ	**0.00**	**0.10**	**0.20**	**0.30**	**0.40**	**0.50**	**0.60**	**0.80**	**1.00**
fit	0.042	0.042	0.046	0.050	0.054	0.057	0.060	0.063	0.065
median A %	48.3	48.1	48.0	47.9	47.8	47.8	47.7	47.6	47.5
default %	4.5	4.4	4.4	4.4	4.3	4.3	4.3	4.2	4.2
cons. %	3.6	3.6	3.6	3.6	3.6	3.6	3.6	3.6	3.6
neg Eq. %	10.8	10.6	10.3	10.2	10.1	10.1	10.0	9.9	9.7

Table 16 Welfare Effect as δ Varies: % increase or decrease of net-worth compared to benchmark

risk aversion ρ	**0.9**	**1.2**	**1.5**	**1.8**	**2.1**	**2.5**	**3.0**	**3.5**	**4.0**
δ = 0.00	0.0	0.0	0.0	0.0	0.1	0.1	0.1	0.0	0.0
δ = 0.10	—	—	—	—	—	—	—	—	—
δ = 0.20	0.0	0.0	0.0	0.0	-0.1	-0.1	0.0	0.0	0.0
δ = 0.40	0.0	0.0	0.0	0.0	-0.2	-0.2	-0.1	-0.1	-0.1
δ = 0.60	0.0	0.0	0.0	-0.1	-0.4	-0.3	-0.2	-0.1	-0.1
δ = 0.80	0.0	0.0	0.0	-0.2	-0.5	-0.3	-0.2	-0.2	-0.1
δ = 1.00	0.0	0.0	0.0	-0.2	-0.5	-0.4	-0.3	-0.2	-0.1

The remaining tables show the effects of slight optimism.

Table 17 5% Optimism: $r_L = 1.2\%$, $r = 4.5\%$, $\beta = 0.97$, $\delta = 0.10$, optimism=5%

T	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**20**
μ	1.69	1.65	1.61	1.58	1.55	1.52	1.50	1.48
σ	0.75	0.71	0.68	0.66	0.64	0.63	0.62	0.62
b %	26.4	26.2	26.3	26.7	27.0	27.3	27.2	24.4
fit	0.032	0.030	0.029	0.028	0.028	0.028	0.028	0.029
median A %	55.1	54.9	54.8	54.7	54.7	54.7	54.5	51.5
default %	4.7	4.4	4.1	3.9	3.7	3.6	3.4	2.7
cons. %	4.4	4.4	4.3	4.3	4.2	4.2	4.1	4.0
neg Eq. %	12.6	13.4	14.5	15.9	17.1	17.7	17.8	16.2

Table 18 10% Optimism: $r_L = 1.2\%$, $r = 4.5\%$, $\beta = 0.97$, $\delta = 0.10$, optimism=10%

T	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**20**
μ	1.92	1.89	1.83	1.79	1.76	1.73	1.70	1.61
σ	0.83	0.81	0.77	0.74	0.72	0.70	0.69	0.63
b %	26.6	26.2	27.0	27.2	27.3	27.3	27.3	27.4
fit	0.030	0.030	0.029	0.029	0.029	0.029	0.029	0.028
median A %	54.9	54.1	54.8	54.8	54.8	54.8	54.8	54.7
default %	4.4	4.0	3.8	3.6	3.4	3.3	3.1	2.7
cons. %	5.2	5.1	5.1	5.0	5.0	4.9	4.9	4.7
neg Eq. %	15.8	16.7	17.5	17.8	17.8	17.8	17.7	17.6

Appendix B: Proofs

Proof of Proposition 1. First, substitute $V_S(w) = w^{1-\rho}v_S$ and $V_B(w) = w^{1-\rho}v_B$ into the right-hand side of the objective of problem 1 and in constraint 2. Thus, we get

$$
\begin{aligned}
V_S(w) = \max_{c,A,\epsilon,\overline{v}} u(c) + \beta\Big[\int_{\mathfrak{B}} ((1+r)(w-\epsilon A - c))^{1-\rho} v_B \, dF(x) \\
+ \int_{\mathfrak{B}^c} (A(x-\overline{v}) + (1+r)(w-\epsilon A - c))^{1-\rho} v_S \, dF(x)\Big];
\end{aligned}
$$

Subject to:

$$
\int_{\mathfrak{B}} (1-\delta)x \, dF(x) + \int_{\mathfrak{B}^c} \overline{v} \, dF(x) \geq (1-\epsilon)(1+r_L) \tag{18}
$$

$$
x \in \mathfrak{B} \Longleftrightarrow v_B \Big((1+r)\big(w-\epsilon A - c\big)\Big)^{1-\rho} > v_S \Big(A(x-\overline{v}) + (1+r)\big(w-\epsilon A - c\big)\Big)^{1-\rho} \tag{19}
$$

$$
(1-\epsilon)A \leq bw \tag{20}
$$

$$
c, A \geq 0, \;\; 0 \leq \epsilon \leq 1. \tag{21}
$$

Let $\lambda > 0$ and let current wealth be w. We must prove that $V_S(\lambda w) = \lambda^{1-\rho} w$.

Suppose that the entrepreneur's wealth is λw and consumption is changed to λc, the firm's assets to λA, while ϵ remains unchanged. Then

$$
\begin{aligned}
&\lambda^{1-\rho} v_B \Big((1+r)\big(w-\epsilon A - c\big)\Big)^{1-\rho} = v_B \Big((1+r)\big(\lambda w - \epsilon\lambda A - \lambda c\big)\Big)^{1-\rho}, \text{ and} \\
&\lambda^{1-\rho} v_S \Big(A(x-\overline{v}) + (1+r)\big(w-\epsilon A - c\big)\Big)^{1-\rho} = v_S \Big(\lambda A(x-\overline{v}) + (1+r)\big(\lambda w - \epsilon\lambda A - \lambda c\big)\Big)^{1-\rho}.
\end{aligned}
$$

This and (19) imply that bankruptcy set $\mathfrak{B}$ remains unchanged. Thus, (18), (20) and (21) are satisfied. Next, note that the right-hand side of the objective changes by the factor $\lambda^{1-\rho}$. Because $V_S(\lambda w)$ is the maximum utility of the entrepreneur given wealth λw, it follows that

$$
V_S(\lambda w) \geq \lambda^{1-\rho} V_S(w), \tag{22}
$$

for all $\lambda > 0$. Thus,

$$
V_S(w) = V_S\left(\tfrac{1}{\lambda}\lambda w\right) \geq \tfrac{1}{\lambda^{1-\rho}} V_S(\lambda w),
$$

which implies that (22) holds with equality. Substituting $w = 1$ and $\lambda = w$ in (22) immediately implies that $V_S(w) = w^{1-\rho} v_S$. The proof that $V_B(w) = w^{1-\rho} v_B$ is similar. ■

Lemma 1 *Constraint 1 of Problem 1 binds.*

Proof of Lemma 1. Immediate: Suppose by way of contradiction that constraint (1) is slack. Then $\overline{v}$ can be lowered thereby increasing $w_s'(x)$, which increases the objective of problem 1.[46] ■

Lemma 2 *Suppose that $\mathfrak{B}$ is non-empty. Let*

$$x^* = \overline{v} - \left[1 - \left(\frac{v_B}{v_S}\right)^{\frac{1}{1-\rho}}\right] \frac{(1+r)(1-\epsilon A - c)}{A} \tag{23}$$

Then $\mathfrak{B} = \{x|\underline{x} \le x < x^*\}$. Conversely, if $x^* > \underline{x}$, then bankruptcy set $\mathfrak{B}$ is non-empty.[47]

Proof of Lemma 2. If the entrepreneur chooses to default, the entrepreneur's utility is

$$u^B(x) = \left[\eta A x + (1+r)\big(1 - \epsilon A - c\big)\right]^{1-\rho} v_B. \tag{24}$$

Otherwise, if the entrepreneur does not default, then the utility is

$$u^S(x) = \left[A(x - \overline{v}) + (1+r)\big(1 - \epsilon A - c\big)\right]^{1-\rho} v_S. \tag{25}$$

Note that $x \in \mathfrak{B}$ if $u^B(x) > u^S(x)$ and $x \notin \mathfrak{B}$ if $u^S(x) \ge u^B(x)$.

Suppose that $u^S(x) \ge u^B(x)$. We show that $u^S(x') > u^B(x')$ for all $x' > x$. Note that

$$\frac{d(u^S(x) - u^B(x))}{dx} = \frac{(1-\rho)(1-\eta)A v_S}{\left[\eta A x + (1+r)\big(1 - \epsilon A - c\big)\right]^{\rho} v_B} > 0$$

Thus, $u^S(x) - u^B(x) \ge 0$ implies that $u^S(x') > u^B(x')$ for all $x' > x$. Similarly, $u^B(x) > u^S(x)$ implies $u^B(x') > u^S(x')$ for all $x' < x$. Let x^* solve $u^B(x^*) = u^S(x^*)$. Then the bankruptcy set is given by $\mathfrak{B} = \{x|\underline{x} \le x < x^*\}$. (24) and (25) imply

$$\left[\eta A x^* + (1+r)\big(1 - \epsilon A - c\big)\right]\left(\frac{v_B}{1-\rho}\right)^{1-\rho} = \left[A(x^* - \overline{v}) + (1+r)\big(1 - \epsilon A - c\big)\right]\left(\frac{v_S}{1-\rho}\right)^{1-\rho},$$

which implies (23).

Now suppose that x^* is given by (23) and $x^* > \underline{x}$. Then by construction, $u^S(x^*) = u^B(x^*)$. Further, the monotonicity result established above implies $u^B(x) > u^S(x)$ for all $x < x^*$ and $u^S(x) \le u^B(x)$ for all $x \ge x^*$. Thus, the bankruptcy set is given by $\mathfrak{B} = \{x|\underline{x} \le x < x^*\}$. ■

[46] The direct effect is to increase the entrepreneur's payoff by decreasing required payments to the lender and the indirect effect is to lower the bankruptcy probability.

[47] At realization x^*, the entrepreneur is indifferent between default and continuing to operate the firm. Thus, (2) must hold with equality. Solving (2) for x^* implies (23).

Proof of Proposition 2. Let $\Gamma(v_S)$ be the maximum entrepreneur utility in Problem 3. We must prove there exists v_S^* such that $\Gamma(v_S^*) = v_S^*$. First let $\rho > 1$. Suppose that $v_S = 0$. Then $v_B < 0$. As a consequence, $\Gamma(0) < 0$. Now let $\hat{v}_S$ be the entrepreneur's expected utility from autarky.

$$\hat{v}_S = \max_{c_0, c_1, \ldots} \sum_{t=0}^{\infty} \beta^t u(c_t)$$

Subject to:

$$\sum_{t=0}^{\infty} \frac{c_t}{(1+r)^t} \leq w \text{ and } c_0, c_1, \ldots \geq 0,$$

Note that if $v_S = \hat{v}_S$ and we choose $A = 0$ in problem 3 then we get the autarky utility $\hat{v}_S$. Thus, optimization implies that $\Gamma(\hat{v}_S) \geq \hat{v}_S$. Since Γ is continuous, the intermediate value theorem implies that there exists a fixed point v_S^*.

For $\rho \leq 1$ we re-normalize $u_\rho(x) = (x^{1-\rho} - 1)/(1-\rho)$. Then $\lim_{\rho \to 1} u_\rho(x) = \ln(x)$. Suppose that $v_S = 0$ and that $u(x) = \ln(x)$. We show that $\Gamma(v_S) < 0$.

Let $w_0 = 1 - \epsilon A$ be the amount of net-worth not invested in the firm. Because the continuation payoff from non-default is zero we get

$$\Gamma(0) = \max_{c_0, c_1, \ldots, c_T} \sum_{t=0}^{T} \beta^t \ln(c_t) \tag{26}$$

Subject to:

$$\sum_{t=0}^{\infty} \frac{c_t}{(1+r)^t} \leq w_0$$

Furthermore, it is sufficient to prove that the objective of (26) is negative for $w_0 = 1$, because the objective is increasing in w_0.

The first order conditions immediately reveal that

$$c_t = (1+r)^t \beta^t c_0, \quad c_0 = \frac{1-\beta}{1-\beta^{T+1}}. \tag{27}$$

Substituting (27) into the objective of (26) yields

$$\sum_{t=0}^{T} \beta^t \ln((1+r)^t \beta^t) + \sum_{t=0}^{T} \beta^t \ln(c_0). \tag{28}$$

If $\beta(1+r) \leq 1$ then (28) is strictly less than 0. Thus, there exists $\overline{r}(\beta)$ with $(1+\overline{r}(\beta))\beta > 1$ such that $\Gamma(0) < 0$ for all $r \leq \overline{r}(\beta)$. By continuity there exists $\underline{\rho} < 1$ such that $\Gamma(0) < 0$ for $\rho \geq \underline{\rho}$. Finally, $\Gamma(\hat{v}_S) \geq \hat{v}_S$ for the autarky level of utility $\hat{v}_S$. Thus, continuity of Γ implies the existence of a fixed point v_S^*. ■

Appendix C: Match Criterion

We compare criterion (17) to the alternative square distance criterion.

Table 19 Supremum Norm: r_L = 1.2%, r = 4.5%, β = 0.97, δ = 0.10, optimism=0.0%

T	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**20**
μ	1.62	1.55	1.49	1.51	1.52	1.52	1.51	1.50
σ	0.90	0.83	0.75	0.74	0.76	0.76	0.76	0.78
b %	20.6	21.5	22.0	19.8	18.4	17.7	17.3	15.4
fit	0.046	0.042	0.037	0.034	0.034	0.034	0.035	0.036
median A %	46.9	48.1	49.2	47.0	45.3	44.3	43.8	41.3
default %	4.7	4.4	4.2	3.8	3.5	3.3	3.1	2.5
cons. %	3.7	3.6	3.6	3.6	3.6	3.6	3.6	3.5
neg Eq. %	10.2	10.6	10.8	10.5	10.8	11.1	11.6	11.1

Table 20 Square Norm: r_L = 1.2%, r = 4.5%, β = 0.97, δ = 0.10, optimism=0.0%

T	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**20**
μ	1.53	1.49	1.47	1.46	1.44	1.41	1.42	1.41
σ	0.74	0.70	0.70	0.69	0.67	0.65	0.67	0.69
b %	21.4	21.8	20.9	20.3	20.3	20.7	19.3	17.3
fit	0.020	0.019	0.019	0.019	0.019	0.019	0.019	0.020
median A %	50.2	50.8	49.6	49.0	48.9	49.3	47.4	44.7
default %	4.7	4.4	4.0	3.8	3.6	3.4	3.2	2.6
cons. %	3.7	3.7	3.6	3.6	3.6	3.5	3.5	3.5
neg Eq. %	9.8	10.2	10.1	10.3	10.9	11.7	11.8	11.5

10 Appendix D

10.1 Construction of the Distribution of Firm Returns

Herranz, Krasa, and Villamil (2008) use the 1993 SSBF to compute the return on assets (ROA) because it includes interest payments. They exclude unincorporated firms because the SSBF data do not account for the entrepreneur's wage from running the firm. The firm's nominal after-tax ROA is:

$$x = \frac{\text{Profit after taxes} + \text{Interest Paid}}{\text{Assets}} + 1. \tag{29}$$

Interest paid is added to after tax profit because the ROA must include payments to both debt and equity holders.[48] The nominal rate is adjusted by 3% for inflation (BLS CPI 1993). ROA is computed instead of return on equity because many firms had negative equity (about 16% in the 1993 SSBF and 21% in 1998). Many of these firms stay in business because owners use personal funds to "bail out the firm." Computing a ROA and modeling owners' allocations of equity and debt accounts for this.[49]

10.2 Numerical Procedure

Given model parameters, compute solutions to problem 3 as follows. For fixed v_S, use the first order conditions to solve for the optimum. (9) is always slack, since $c + \epsilon A = 1$ would imply zero future consumption. We need only verify if (10) and (or) (11) bind by checking for positive Lagrange multipliers in the first order conditions. Inserting the solution of the first order conditions into the objective yields $\Gamma(v_S)$. To find a fixed point, compute slope $\Gamma'(v_S)$ by the Envelope Theorem or compute the difference of Γ between v_S and a point v_S', giving solution ϵ, A, c, $\overline{v}$. Section 3.2 explains how to go from these point estimates to cdfs. Compute $\underline{\rho}$ from the first order condition using the fact that $v_S \to \infty$ as $\rho \downarrow \underline{\rho}$.[50]

Appendix E: Limited Liability

Suppose that entrepreneur can be forced to pay a percentage γ of private assets in the case of default. This yields the following optimization problem for an individual entrepreneur.

Problem 4 $V_S(w) = \max_{c,A,\epsilon,\overline{v}} u(c) + \beta\Big[\int_{\mathfrak{B}} V_{B,1}((1-\gamma)(1+r)(w-\epsilon A - c))\, dF(x)$

$$+ \int_{\mathfrak{B}^c} V_S(A(x-\overline{v}) + (1+r)(w-\epsilon A - c))\, dF(x)\Big]$$

Subject to:

$$\begin{aligned}&\int_{\mathfrak{B}\cap\mathbb{R}_-} x\, dF(x) + \int_{\mathfrak{B}\cap\mathbb{R}_+} (1-\delta)x\, dF(x)\\ &+ \int_{\mathfrak{B}} \gamma(1-\delta)\left(\frac{w}{A} - \epsilon - \frac{c}{A}\right) dF(x) + \int_{\mathfrak{B}^c} \overline{v}\, dF(x) \geq (1-\epsilon)(1+r_L)\end{aligned} \tag{30}$$

$$x \in \mathfrak{B} \textit{ if and only if } V_{B,1}((1-\gamma)(1+r)(w-\epsilon A - c)) > V_S(A(x-\overline{v}) + (1+r)(w-\epsilon A - c)) \tag{31}$$

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[48] We use after tax returns as this is relevant for an entrepreneur to decide how much net-equity to invest.

[49] Computing ROE is misleading for firms near distress. For firms with low but positive equity, small profit gives a high percentage return. Also, many loans are collateralized; book value of equity understates owner contribution (the "correct" value of equity).

[50] Choose a large value for v_S, solve for the remaining parameters including ρ, which approximates $\underline{\rho}$. In other words, rather than solving the fixed point problem for v_S, solve it for ρ.

$$(1-\epsilon)A \leq bw \tag{32}$$

$$c \geq 0,\ \ A \geq 0,\ \ 0 \leq \epsilon \leq 1. \tag{33}$$

Note that the investor's constraint is normalized by assets. Thus, the payment in bankruptcy states made out of the entrepreneur's personal assets must be divided by A.

Again, suppose that the entrepreneur's wealth is λw and consumption is changed to λc, the firm's assets to λA, while ϵ remains unchanged. Then as in the proof of Lemma 1 we can show that the constraints of Problem 4 are satisfied and that $V_S(\lambda w) = \lambda^{1-\rho} V_S(w)$. Similarly, it follows again that $V_B(\lambda w) = \lambda^{1-\rho} V_B(w)$. Thus, we get an optimization problem that is analogous to Problem 3.

Problem 5 $v_S = \max_{c,A,\epsilon,\overline{v}} u(c) + \beta v_B \int_{\underline{x}}^{x^*} \left[(1+r)\Big((1-\gamma)(1-\epsilon A - c)\Big)\right]^{1-\rho} dF(x)$

$$+\beta v_S \int_{x^*}^{\overline{x}} \left[A(x-\overline{v}) + (1+r)\Big(1-\epsilon A - c\Big)\right]^{1-\rho} dF(x)\Big]$$

Subject to:

$$\int_{\underline{x}}^{x^*} \min\{x, (1-\delta)x\}\, dF(x) + \int_{\underline{x}}^{x^*} \gamma(1-\delta)\left(\frac{1}{A} - \epsilon - \frac{c}{A}\right) dF(x) + \int_{x^*}^{\overline{x}} \overline{v}\, dF(x) \geq (1-\epsilon)(1+r_L) \tag{34}$$

$$x^* = \max\left\{\overline{v} - \left[1 - (1-\gamma)\left(\frac{v_B}{v_S}\right)^{\frac{1}{1-\rho}}\right] \frac{(1+r)(1-\epsilon A - c)}{A}, \underline{x}\right\} \tag{35}$$

$$c + \epsilon A \leq 1 \tag{36}$$

$$(1-\epsilon)A \leq b \tag{37}$$

$$c \geq 0, A \geq 0, 0 \leq \epsilon \leq 1. \tag{38}$$

Note that for $\gamma = 0$ this problem is equivalent to Problem 3.

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