Abstract

We develop a theory of political competition in legislative elections based upon the notion that majority party legislators collaboratively influence policy. Because of this team aspect, a candidate’s party label matters for voters, in addition to his own policy positions. In equilibrium, candidates may be unable to escape the burden of their party association, and primary voters in both parties can often nominate extremist candidates and still win. We also show that gerrymandering affects the equilibrium platforms not only in those districts that become more extreme, but also in those that ideologically do not change.

Keywords: Differentiated candidates, primaries, polarization.
1 Introduction

In the basic model of representative democracy, voters elect legislative representatives whose positions reflect the preferences of their respective districts’ median voters. These representatives convene in an amorphous assembly — one in which parties do not play an important role —, and national policy is set to correspond to the preferences of the median representative in this assembly. Thus, the legislature is composed of representatives who are more moderate than the voters who elect them, and actual policy and legislation reflects the most moderate position in this assembly of moderates (a prediction that appears somewhat counterfactual).

There is, of course, a large body of literature modeling interactions of representatives in a legislature and the effects of legislative institutions such as the power of specialized committees, but this literature takes the set of legislators as given. There is also a large literature on political competition, but that literature generally assumes that candidates either unilaterally choose or are exogenously endowed with policy positions that enter voters’ utility functions directly and exclusively. That is, voters care only about the positions of those candidates that they personally can decide between. In this paper, we build a model of electoral competition that combines these two strands of literature: When voting for their local representative, the voters in our model explicitly take into account that they are not in a position to determine the unilaterally decisive policy maker in the nation, but rather just one of many representatives who interacts with other representatives in the determination of policy.

Our model is based on two realistic ingredients: First, the majority party in a legislature is an important power center influencing the crafting of policy. Coordination of decision-making and voting according to the majority preferences in the majority party increases the influence of each majority party legislator on the policy outcome (Eguia, 2011a,b). Also, as in any large organization, specialization is an ubiquitous feature of modern legislatures because no single legislator can be an expert in all policy areas. Thus, individual legislators have considerably more influence on policy in their area of specialization than in other areas where they primarily rely on the expertise of their fellow party members (Shepsle and Weingast, 1987; Gilligan and Krehbiel, 1989).
This importance of the majority party for law-making creates important spillover effects between the candidates of the same party who run in different districts. Second, legislative candidates are elected by voters who understand the legislative stage, and are nominated by policy-motivated primary voters who take both the general election and the legislation process into account when deciding whom to nominate.

The importance of parties is uncontroversial among scholars of legislatures. However, there is surprisingly little analysis of how the fact that each candidate is connected to a party and thus, implicitly, to the positions of candidates of that party from other districts influences nomination decisions, as well as election outcomes in different legislative districts.

Applying the simplest Downsian model naively to Congressional elections – which much of the empirical literature implicitly does – generates counterfactual predictions: Since all candidates adopt the preferred position of their district’s median voter, all voters should be policy-wise indifferent between the Democratic candidate and his Republican opponent. Thus, Republicans in New England or Democrats in rural Western districts should have a substantial chance to be elected to Congress if only they match their opponent’s policy platform. Furthermore, in this framework, gerrymandering districts affects only candidate positions in the gerrymandered districts, but does not help a party to increase their expected representation in Congress. These predictions are certainly counterfactual, but understanding why that is so is challenging.

In our model, all voters care about both their local representative’s position, and the position of the majority party, which is determined endogenously as the median of elected majority party legislators. In the general election, voters vote for their preferred candidate, taking into account the two ways in which their local representatives may change the policy outcome:

1. The district result may change which party is the majority party in Congress;

2. If the district elects a candidate from the majority party, then his position may affect the majority party’s position.

\footnote{1}{See Table 1 in Winer et al. (2014) for evidence that a significant share of U.S. Senate elections are non-competitive. In 29.4 percent of U.S. Senate elections between 1922 and 2004 without an incumbent running, the winner received a vote share that was at least 20 percentage points larger than the loser’s vote share.}
If only the first effect is present, the general election voter generically strictly prefers one of the two party positions. The favored party's primary voter can exploit this situation by nominating a more extreme candidate than the general election voter would prefer. In particular, if voters care sufficiently strongly about national positions relative to local candidate positions, then the favored party's primary voter can simply nominate his own preferred candidate and still win, generally with a strict supermajority of votes.

The local general election loses its disciplining force vis-a-vis the local party because the determining factor is the national preference. The electoral prospects of candidates in a given district are influenced by the expected ideological position of their parties' winning candidates elsewhere. The association with a party that is not attuned with a district's ideological leanings may be poisonous for a candidate even if his own policy positions are tailor-made for his district.

Consider, for example, Lincoln Chafee, the former Republican U.S. senator from Rhode Island, who had taken a number of moderate and liberal positions that brought him in line with voters in his state. In the 2006 election, “exit polls gave Senator Lincoln Chafee a 62 percent approval rating. But before they exited the polls, most voters rejected him, many feeling it was more important to give the Democrats a chance at controlling the Senate.” His Democratic challenger Whitehouse “succeeded by attacking the instances in which Chafee supported his party’s conservative congressional leadership (whose personalities and policies were very unpopular, state-wide).”

In a review of 2006 campaign ads, factcheck.org summarized: “President Bush was far and away the most frequent supporting actor in Democratic ads […] The strategy is clear: whether they’re referring to a Republican candidate as a ‘supporter’ of the ‘Bush agenda’ or as a ‘rubber-stamp,’ Democrats believe the President’s low approval ratings are a stone they can use to sink their opponents […] Democratic Sen. Hillary Clinton of New York got the most mentions in Republican ads holding forth the supposed horrors of a Democratic-controlled Senate […] The runner-up is ‘San Francisco Liberal Nancy Pelosi,’ who is mentioned in at least 6 GOP ads as a

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2For example, Chafee was pro-choice, anti-death-penalty, supported gay marriage and voted against the Iraq war (see http://en.wikipedia.org/wiki/Lincoln_Chafee).
reason not to vote for a Democrat who would in turn vote to make her Speaker of the House.”

We now turn to the second effect mentioned above. For this, assume that the district election does not influence which party wins the majority in the legislature. In this case, the voters’ strategic calculations change because the only possible effect on national policy is that a vote for the majority party candidate may influence that party’s national position. Consider what happens when voters care primarily about national positions and the general election district median voter would prefer a more moderate position of the majority party. In this case, the median voter is willing to vote for a majority party candidate who would moderate the national party position (even if that local candidate is still far away from the median voter’s preferred position), but the local majority party primary voter might prefer not to nominate such a candidate because he cares about the national policy, too, and does not want such a diluting influence.

Again, Lincoln Chafee provides an instructive illustration of this principle. Before the 2006 general elections, when Republicans had a clear majority in the Senate, conservative Republicans in Rhode Island mounted a primary challenge. Chafee defeated his challenger who had attacked him for not being sufficiently conservative only by a margin of 53 percent to 47 percent, and there is reason to believe that a majority of “real” Republicans would have preferred to replace a popular incumbent Republican senator with an extremist whose policy positions would have implied a very low likelihood of prevailing against the Democrat in the general election in Rhode Island. Our model explains why this behavior may be perfectly rational for policy-motivated Republican primary voters: From their point of view, having Chafee as a member of the Republican Senate caucus caused more harm than good. In general, the effect just discussed homogenizes the majority party, i.e. diminishes the intra-party differences between majority party legislators.

In contrast to the classical one-district spatial model, the ideological composition of districts in our model does not only influence the ideological position of elected candidates, but also the chances of parties to win, thus increasing partisan incentives for gerrymandering. We also show

\[\text{See } \text{http://www.factcheck.org/elections-2006/our_2006_awards.html}\]

\[\text{Rhode Island’s open primary system allows registered Democrats and Independents to vote in the Republican primary. The New York Times article “To hold Senate, GOP bolsters its most liberal” (September 10, 2006) quotes a Republican consultant as saying that “There is no doubt that if the primary was held only among Republicans, Chafee would lose. He would be repudiated by the Republicans who he has constantly repudiated.”}\]
that gerrymandering or, more generally, the intensification of the median ideological preferences in some districts, also affects the political equilibrium in those districts where the median voter preferences remain moderate. Our results imply that testing for the causal effect of gerrymandering on polarization in Congress is more complicated than the existing literature has recognized.

Our paper proceeds as follows. Section 2 reviews the related literature. In Section 3, we provide some stylized facts about statewide executive and legislative elections, and explain why they are hard to explain within the standard model that looks at legislative elections in different districts in isolation. In Section 4, we set up the general model, and the main analysis follows in Sections 5, 6 and 7. We conclude in Section 8.

2 Related Literature

Ever since Downs’ (1957) seminal work, candidates’ position choice is a central topic in political economy. While the classical median voter framework identifies reasons for platform convergence, many subsequent electoral competition models develop different reasons for policy divergence, such as policy motivation (e.g., Wittman 1983; Calvert 1985); entry deterrence (e.g., Palfrey 1984; Callander 2005); agency problems (Van Weelden, 2013) and incomplete information among voters or candidates, in both one-period (e.g. Martinelli 2001; Callander 2008; Gul and Pesendorfer 2009) and multiperiod settings (e.g. Castanheira 2003; Bernhardt et al. 2004).

Most of the literature looks at isolated one-district elections. Exceptions are Austen-Smith (1984), Snyder (1994) and Ansolabehere et al. (2012).

In Austen-Smith (1984), the party that wins the majority of \( n \) districts implements an aggregate of its candidates’ positions. Each district candidate chooses his position to maximize his chance of winning. If an equilibrium exists, then both party positions fully converge to the median median, while individual candidates’ positions differ. In contrast, in our model, positions are chosen by policy-motivated primary voters, and voters care about both the local candidates’ positions and the national party positions. In our equilibrium, national party positions diverge, and we can analyze the effects of gerrymandering and of more or less radical primary voters.
Snyder (1994) considers a dynamic setting in which voters care only about national party positions that are chosen by the party’s representatives in the pre-election legislature to maximize their individual reelection chances. In Ansolabehere et al. (2012), a special version of this model, the left and the right party locate at the 25th and 75th percentile of the district median distribution.

In the probabilistic voting model (e.g., Lindbeck and Weibull 1987; Dixit and Londregan 1995), voters also receive “ideological” payoffs that are independent of the candidates’ positions. One could interpret these ideological payoffs as capturing the effects of the candidate being affiliated with a party, and therefore, implicitly, the party’s other legislators’ positions. However, the “ideology shock” in these models is exogenous, so that one cannot analyze the main point of interest in our model – how does the fact that a party’s national position matters for voters and is determined as an aggregate of all its representatives’ positions, affect both the voters’ choice between local candidates and the candidates’ equilibrium positions?

In the influential models of Erikson and Romero (1990) and Adams and Merrill (2003), voters receive, in addition to the payoff from the elected candidate’s position, a “partisan” payoff from his party affiliation, which, however, is exogenous and orthogonal to his policy position. Our model provides a microfoundation for these partisan payoffs, and shows how they depend on the equilibrium polarization between the parties’ candidates in other districts, and how they, in turn, affect the candidates’ equilibrium positions.

Our model belongs to the class of differentiated candidates models (Aragones and Palfrey 2002; Soubeyran 2009; Krasa and Polborn 2010a,b, 2012, 2014; Camara 2012). In these models, candidates have some fixed “characteristics” and choose “positions” to maximize their probability of winning. Voters care about both characteristics and positions. In contrast to existing differentiated candidates models, voters’ preferences over characteristics (i.e., the candidates’ party affiliations) are endogenously derived here from the positions of candidates in other districts.

Our model assumes that national party positions matter for voters,7 and a significant number of models explains why this is so. Conditional party government theory (Rohde, 2010; Aldrich, 2011; Halberstam and Montagnes (2015) provide empirical evidence of spillovers from national presidential campaigns on Senate elections and the positions of candidates in those elections.

7Halberstam and Montagnes (2015) provide empirical evidence of spillovers from national presidential campaigns on Senate elections and the positions of candidates in those elections.
1995) and endogenous party government theory (Volden and Bergman, 2006; Patty, 2008) argue that party leaders can use incentives and resources to ensure cohesiveness of their party. Procedural cartel theory (Cox and McCubbins, 2005) argues that party leadership can at least enforce voting discipline over procedural issues. Castanheira and Crutzen (2010), Eguia (2011a,b) and Diermeier and Vlaicu (2011) provide theories of endogenous institution choice leading to powerful parties. All these models of the importance of parties in Congress take the distribution of legislator preferences as exogenously given, while our model provides for an electoral model and thus endogenizes the types of elected legislators.

Our results are relevant for the large empirical literature that analyzes how primaries, the ideological composition of districts and especially the partisan gerrymandering of districts affects the ideological positions of representatives in Congress (e.g., Lee et al. 2004; McCarty et al. 2009a; Hirano et al. 2010). The empirical literature does not derive predictions about the expected correlations from an explicit model of legislative elections, but rather takes the intuition from the isolated election model and simply transfers it. For example, most of the empirical literature argues informally that the positions of representatives, measured by their DW-Nominate score, “should” reflect the conservativeness of their districts. Our model shows that this transfer is not always appropriate, and that the candidates’ equilibrium positions may correspond to the preferences of the parties’ respective primary electorates rather than those of the district median voters.

Since we assume that the nomination decision is made by a policy-motivated party median voter, our model is related to the literature on policy-motivated candidates pioneered by Wittman (1983) and Calvert (1985), who assume that candidates are the ones who are policy-motivated and get to choose the platform that they run on. In our model, the effective choice of platform is made by the primary election median voter, but this change does not substantively affect the analysis. This approach is also taken by Coleman (1972) and Owen and Grofman (2006). To our knowledge, no paper in this literature analyzes policy-motivated policy selectors in the type of “linked” elections in different districts that we focus on.

Implicitly, we assume that either candidates can commit to an ideological position in the primary, or that candidates are citizen-candidates with an ideal position that is common knowledge.
3 Partisanship in Legislative and Executive Elections

We now show that the electorate’s preference distribution influences the parties’ performance substantially stronger in legislative elections than in executive ones. Within the standard framework, this difference is puzzling, and one can interpret our model as a resolution of this puzzle.

The simple Downsian model predicts that both candidates locate at the median voter’s ideal point, so that all voters are indifferent between both candidates. A liberal or conservative district should not provide an advantage, in terms of the probability of winning, to Democrats or Republicans. In practice, though, a district’s ideological preferences do affect the electoral chances of the two parties – we talk of “deep red” or blue states, implying that the candidate of the ideologically favored party has a much clearer path to victory than his opponent.

However, we now show that voters’ ideological preferences have a much larger effect in legislative elections than in executive ones.\(^9\) We consider Gubernatorial and U.S. Senate elections from 1974 to 2012. While both of these types of contests are high-profile, state-wide races, Gubernatorial elections are for executive positions while Senate elections are for legislative ones. Consistent with the empirical literature, we measure the median state ideology by its Partisan Voting Index (PVI), that is, the difference of the state’s average Republican and Democratic Party’s vote share in the past U.S. Presidential election, relative to the nation’s average share of the same.\(^10\)

The dependent variable is the difference between the Democrat’s and the Republican’s share of the two party vote in a particular election. In addition to the main independent variables of interest (PVI and PVI×Senate election), we use incumbency and election type (i.e., Senate or Governor election) dummies and year fixed effects in order to control for the electoral advantage of incumbents, and for election-cycle national shocks in favor of one party.

Table 1 summarizes the results, with the first column (all years since 1974, all states) as the baseline case. For Gubernatorial elections, the omitted category, the PVI coefficient indicates that a

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\(^9\)While this feature is apparently a sort of folk wisdom among political scientists, to the best of our knowledge, there is no published account of this fact.

\(^10\)For example, if, in a particular state, the Republican wins by 7 percent, while nationally, he wins by 3 percent, then the state has a PVI of 7 − 3 = 4. Also note that vote shares are calculated relative to the two-party vote, i.e., votes for minor parties are eliminated before the vote share percentages are calculated.
Table 1: Senate and Gubernatorial elections

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<tr>
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<tbody>
<tr>
<td>PVI</td>
<td>0.591*** (0.104)</td>
<td>0.563*** (0.118)</td>
<td>0.511*** (0.111)</td>
<td>0.542*** (0.126)</td>
</tr>
<tr>
<td>PVI × Senate</td>
<td>0.457*** (0.141)</td>
<td>0.491*** (0.159)</td>
<td>0.517*** (0.149)</td>
<td>0.470*** (0.170)</td>
</tr>
<tr>
<td>N</td>
<td>1061</td>
<td>703</td>
<td>835</td>
<td>554</td>
</tr>
<tr>
<td>$r^2$</td>
<td>0.528</td>
<td>0.559</td>
<td>0.547</td>
<td>0.584</td>
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*** indicates significance at the 1% level.


one point increase in the Democratic vote share in Presidential elections increases the Democratic gubernatorial candidate’s vote share only by about 0.591 points. In contrast, in Senate elections, the same ideological shift increases the Democratic Senate candidate’s vote share by 0.591 + 0.457 = 1.048 points. Evidently, the difference between executive and legislative elections is substantial and highly significant. The remaining three columns confirm the qualitative robustness of this difference if we restrict to elections after 1990 and if we exclude the political South.\textsuperscript{11}

A coefficient of about 1 for Senate elections is remarkable — if Senate candidates were forced to locate at their party’s Presidential candidate’s position, irrespective of whether such a position is competitive in their respective state, then this should result in a coefficient of about 1. Whenever the disadvantaged candidate is able and willing to adjust his position to better fit the state’s voter preferences, this would reduce the advantage of the opponent, and thus the estimated coefficient. Somehow, only gubernatorial candidates appear, at least to some extent, capable of such a position adjustment, while Senate candidates are not.

Our model’s explanation for this phenomenon, detailed in the following sections, is that Governor candidates are perceived as more independent of their party, while Senate candidates, if elected, would become members of the Democratic and Republican Senate caucuses, and so the

\textsuperscript{11}At least until the 1990s, there were many conservative Southern Democrats in state politics in the South, so it is useful to check that our results are not driven by this region of the country.
party positions are more important for voters in these legislative elections.

4 Model

We consider a polity divided into a set of districts $I$, where the number of districts is $|I| = 2n + 1$. Each district $i$ contains three strategic agents: A median Democratic primary voter, a median Republican primary voter and a general election median voter.

For each district $i$ and each party $P$, the local primary voter chooses the position $x_{i,P} \in \mathbb{R}$ of party $P$'s candidate in district $i$. The general election median voter observes $(x_{i,D}, x_{i,R})$ (but not the candidates who are nominated in other districts), and chooses $(l_i, x_i) \in \{(D, x_{i,D}), (R, x_{i,R})\}$, where $l_i$ stands for the winning candidate’s party label, and $x_i$ for his position.

We refer to $(l_i, x_i) \equiv (l_i, x_i)_{i \in I}$ as the legislature. We say that party $P$ is the majority party in legislature $(l, x)$ if the party label of at least $n + 1$ representatives in the assembly is $P$. Let $x_M(l, x)$ denote the median policy position among the majority party legislators.\(^\text{12}\)

Each agent has an ideal position $\theta$, and the utility of a voter with ideal position $\theta$ from district $i$ is

$$u(\theta, x_M, x) = -(1 - \gamma)(x_M - \theta)^2 - \gamma(x_i - \theta)^2,$$

where $\gamma \in (0, 1)$, and $x_M = x_M(l, x)$. Here, the first term is the utility from the majority party’s position, and the second term is the utility from the policy position of district $i$’s representative. Note that $\gamma \to 1$ corresponds to the standard case where voters only care about the election outcome in their own district, and $\gamma \to 0$ means that voters care only about the majority party’s position and not their own representative’s position per se.

We assume that the position of the majority party that is relevant for voters is given by the position of the median elected legislator of the majority party. One way to think about this is that the majority party caucus elects a speaker, and a lot of power is then concentrated in the speaker. By \(^\text{12}\)If the number of majority party legislators is even, we let $x_M(l, x)$ be the average of the two most central positions in the majority party caucus.
standard arguments, the median majority party member is the Condorcet winner in this election.\textsuperscript{13}

Ex-ante there is uncertainty about the ideal position of district \(i\)'s median voter, which is chosen independently across districts according to cdfs \(\Phi_i(\cdot)\).\textsuperscript{14}

The game proceeds as follows:

**Stage 1** In each district, the local members of each party simultaneously select their candidates, who are then committed to their policies \(x_{i,D}, x_{i,R} \in \mathbb{R}\). We assume that the nomination process can be summarized by the preference parameter of a “decisive voter,” whom we take to be the median party member in the district, and whose ideal position is denoted \(m_{i,R}\) for Republicans and \(m_{i,D}\) for Democrats.

**Stage 2** In each district \(i\), the median voter \(M_i\) is realized, observes the candidate positions \(x_{i,D}\) and \(x_{i,R}\) in his own district, and chooses whom to vote for. For the other districts, the median voter of district \(i\) does not observe the candidates’ positions.\textsuperscript{15}

**Stage 3** The elected candidates from all districts form the legislature, which determines the majority party and its positions, and payoffs are realized; no strategic decisions take place in this stage.

We consider pure strategy sequential equilibria of this game.\textsuperscript{16}

## 5 Partial Equilibrium Analysis

There are two ways in which the election outcome in a particular district can affect national policy:

First, the district election outcome can decide which party has a majority in the legislature. Second,

\textsuperscript{13}Of course, in reality, the positions of several legislators might matter for the implemented policy, and one can generalize our model along these lines without changing the fundamental results.

\textsuperscript{14}We also discuss the deterministic case where \(\Phi_i\) is degenerate.

\textsuperscript{15}Of course, players have correct expectations about what happens in other districts along the equilibrium path.

\textsuperscript{16}Note that incomplete information matters in stage 3, since the median voter in district \(i\) cannot observe a deviation in stage 1 by primary voters in a different district \(j \neq i\). Thus, the question arises how these voter beliefs about other districts are affected, if one of the median primary voters in district \(i\) deviates in stage 1 (recall that only nature moves in stage 2). Consistency of beliefs, as defined in a sequential equilibrium, immediately implies that median voter \(i\) assumes that no deviation has also occurred in the other districts, i.e., positions \(m_{j,R}, m_{j,D}\) remained the same for \(j \neq i\).
it could be the case that one party will be the majority party no matter who is elected in the particular district we consider. Then, a district may still influence national policy if it elects the candidate of the majority party, and that candidate affects the position of the majority party.

Of course, in general, both of these effects will be present in the actual equilibrium of the game. However, to gain some intuition before we analyze the general case, it is useful to analyze the two effects separately.

5.1 No Influence on Party Positions

We first consider the case of an election in a particular district $i$ in which the election outcome has no influence on median party policies. In this section, this is by assumption, but in the general analysis below, we will show that this case can indeed arise in an equilibrium of the general model.

Suppose that the party positions are $\bar{x}_D < \bar{x}_R$, and that $\gamma < 1$. Furthermore, assume that the median Democratic and Republican primary voters in district $i$ are located at $-\bar{m}$ and $\bar{m}$, respectively, and the general election median voter is at $M_i$ (there is no uncertainty about this position).

What are the equilibrium candidate positions in district $i$? The election here either determines which party wins a majority in the legislature, or the district is not pivotal for the majority in the legislature.

Consider first the case that party $P$ wins a majority irrespective of the election outcome in district $i$. Then, the median voter’s utility from the Democratic candidate is $-(1-\gamma)(\bar{x}_P - M_i)^2 - \gamma(x_{iD} - M_i)^2$. The payoff from the Republican is $-(1-\gamma)(\bar{x}_P - M_i)^2 - \gamma(x_{iR} - M_i)^2$. Since the first term is the same in both cases, the median voter will simply elect the candidate whose position is closer to his own. As a consequence, in equilibrium both candidates must locate at the median voter’s ideal point $M_i$ as in the standard case.

The situation changes dramatically in the second case, namely that the district is pivotal. Now the median voter’s payoff from the Democrat is $-(1-\gamma)(\bar{x}_D - M_i)^2 - \gamma(x_{iD} - M_i)^2$, and $-(1-\gamma)(\bar{x}_R - M_i)^2$. If utility does not depend on party position at all ($\gamma = 1$), then we are in a standard model where both candidates locate at the median voter’s ideal point $M_i$ in the unique equilibrium, i.e., the centripetal forces of candidate competition lead to moderation.
\(M_i^2 - \gamma(x_{i,R} - M_i)^2\) from the Republican. If \((\bar{x}_D - M_i)^2 \neq (\bar{x}_R - M_i)^2\), then one of the candidates has an advantage in the race. For example, suppose that \(\epsilon_D = -(\bar{x}_D - M_i)^2 > -(\bar{x}_R - M_i)^2 = \epsilon_R\), then the median voter \(M_i\) receives a utility advantage of \(\epsilon_D - \epsilon_R\) from the Democratic candidate. This means that there exists \(x_{i,D} < M_i\) such that a Democratic candidate with position \(x_{i,D}\) can be nominated and win.

The equilibrium positions themselves depend on the median Democratic primary voter’s location \(-\bar{m}\) as well as on \(\gamma\). Specifically, if \(x_{i,D} > -\bar{m}\), then \(M_i\) must be indifferent between the two candidates.\(^{18}\) Furthermore, the Republican must be located at \(M_i\), else he could win by moving close to \(M_i\). Thus,

\[-(1 - \gamma)(\bar{x}_D - M_i)^2 - \gamma(x_{i,D} - M_i)^2 = -(1 - \gamma)(\bar{x}_R - M_i)^2,\]

which implies

\[x_{i,D} = M_i - \sqrt{\frac{1 - \gamma}{\gamma} (\epsilon_D - \epsilon_R)}.\]

It follows immediately that \(x_{i,D} \to -\infty\) as \(\gamma \downarrow 0\). Clearly, the median primary voter has no benefit from selecting a candidate \(x_{i,D} < -\bar{m}\). As a consequence, if the party policy matters sufficiently much, and \(\gamma\) is therefore sufficiently small, Democratic primary voters select a candidate located at their ideal point \(-\bar{m}\).

In summary, in the case where a district is pivotal, candidate competition no longer leads to moderation. The example used a “partial equilibrium” approach by assuming that party positions are fixed and most importantly are different. One might expect that candidate competition in the other districts would moderate positions. However, we will see that this is not always the case.

\(^{18}\)If \(M_i\) strictly prefers the Democrat, then primary voters would be better off nominating an even more liberal candidate, who would win. If \(M_i\) strictly prefers the Republican, then the Democratic primary voter can win by nominating a candidate that is to the left of the Republican and the median voter, given the utility advantage \(\epsilon_D - \epsilon_R\) of the Democratic candidate.
5.2 A District that Influences Party Positions

We now turn to the case that one party is the majority party no matter who is elected in district $i$. In that case, district $i$ may still influence national policy if it elects the candidate of the majority party, and that candidate affects the position of the majority party.

This effect is clearest in a special election that is held in only one district, say, because the previously elected representative resigned. Suppose that the Republicans already have a majority of seats in the legislature without the representative from district $i$, that $\gamma$ is small (i.e. party positions matter most for voters), and that the Republican position without the representative from district $i$ (i.e., if a Democrat wins the by-election) is to the right of district $i$'s median voter’s ideal point.

If the Republican candidate in district $i$ is more moderate than $\bar{x}_R$, then his election would result in a more moderate national platform and therefore be attractive to the median voter in district $i$. However, a candidate who moves the national Republican position to a significantly more moderate position is not attractive for the Republican primary voter in district $i$, even considering his benefit from a Republican local representative. At most, the Republican primary voter might offer a candidate who is very close to the national party platform and does not moderate the national Republican position by much, while being able to win in district $i$ against a Democratic candidate located at median voter $i$’s ideal point.

To analyze this setting more formally, suppose that candidate $x_{i,R}$ is located so close to $\bar{x}_R$ that he would be the median Republican in the legislature, if elected. To make the median voter of district $i$ indifferent between this Republican candidate and a Democratic candidate located at $M_i$, it has to be true that

$$-(1-\gamma)(x_{i,R}-M_i)^2 = -(1-\gamma)(\bar{x}_R-M_i)^2$$

which we can solve for $x_{i,R} = \sqrt{1-\gamma} \bar{x}_R + (1-\sqrt{1-\gamma}) M_i$, i.e., the local Republican’s position must be some weighted average between the median voter’s ideal point and the position of the Republican party without the Republican from district $i$, and close to the latter if $\gamma$ is small.
Substituting this position into the utility function of the Republican primary voter \( m_i;R \), we get

\[
-\left( \sqrt{1 - \gamma(m_i;R - \bar{x}_R)} + (1 - \sqrt{1 - \gamma})(m_i;R - M_i) \right)^2. 
\] (4)

Alternatively, the Republican primary voter can always nominate a candidate who is unacceptable for the median voter and get

\[
-(1 - \gamma)(m_i;R - \bar{x}_R)^2 - \gamma(m_i;R - M_i)^2.
\] (5)

Subtracting (5) from (4) and simplifying yields

\[
2\left( \sqrt{1 - \gamma - (1 - \gamma)}(m_i;R - M_i)\left((m_i;R - M_i) - (m_i;R - \bar{x}_R)\right) \right), 
\] (6)

which is positive because \( \bar{x}_R > M_i \) and \( 0 < \gamma < 1 \). Hence, the Republican primary voter in district \( i \) indeed wants to offer an acceptable candidate to the general election median voter. However, note that (3) implies that, as \( \gamma \to 0 \), \( x_{i,R} \to \bar{x}_R \). Thus, if voters care mostly about the majority party’s national position, the ability to influence party policy through local elections will result in a very homogeneous majority party caucus. However, as we will now see, this does not mean that the party overall will be moderate.

6 A Deterministic Multidistrict Model

We now analyze a simple setting in which the two types of partial equilibrium effects identified in the previous section interact with each other. Specifically, consider three districts that are ordered with respect to the position of the median voters.\(^{19}\) Let \( M_1 < 0 \), \( M_2 = 0 \), and \( M_3 > 0 \). Suppose that the position of median primary voters are at \(-\bar{m}\) for Democrats, and \( \bar{m} \) for Republicans, respectively, and that \( M_1 + M_3 \geq 0 \).

We now show that there exists an equilibrium in which the Republican candidates win in all

\(^{19}\)The argument and the equilibrium derived below can be generalized to any odd number of districts. We refrain from doing so to simplify the exposition.
districts, despite not being located at the median voters’ respective ideal points, against Democratic candidates who are located at \( x_{i,D} = M_i \). Intuitively, each district median voter in this equilibrium votes for the Republican candidate because, given that the voters in the other two districts vote for their Republican candidate, it is (at least weakly) more important to influence the Republican position than to vote for the ideal local candidate. In turn, Republican primary voters exploit this effect by nominating more extreme candidates, to the point where median voters in at least two of the districts are indifferent.

Specifically, we claim that the Republican positions in districts 1, 2 and 3 are given by the following equations that specify that median voters in districts 1 and 2 are indifferent between the candidates, while the median voter in district 3 may be either indifferent or may prefer the Republican candidate.

For example, equation (7) below states that district 1’s median voter, \( M_1 \), is indifferent between the Democratic and the Republican candidate. The left-hand side is the payoff from electing the Republican. In this case, Republicans win all districts, and, as we will show, the median Republican is located in district 2. Thus, if the Republican is elected, the voter derives utility from the local position, with weight \( \gamma \), and from the national party position, \( x_{2,R} \), with weight \( 1 - \gamma \). If, instead, the Democrat is elected in district 1, then his position, \( x_{1,D} \) counts for the local component of the voter’s utility, but the national position is changed to the median of the remaining Republican representatives, which is the average of \( x_{2,R} \) and \( x_{3,R} \).

Similarly, if district 2 elects a Democrat, then the Republican party position is changed to 0.5(\( x_{1,R} + x_{3,R} \)), and to 0.5(\( x_{2,R} + x_{3,R} \)) if district 3 elects a Republican.

\[
-(1-\gamma)(x_{2,R} - M_1)^2 - \gamma(x_{1,R} - M_1)^2 = -(1-\gamma)\left(\frac{x_{2,R} + x_{3,R}}{2} - M_1\right)^2; \tag{7}
\]
\[
-(x_{2,R} - M_2)^2 = -(1-\gamma)\left(\frac{x_{1,R} + x_{3,R}}{2} - M_2\right)^2; \tag{8}
\]
\[
-(1-\gamma)(x_{2,R} - M_3)^2 - \gamma(x_{3,R} - M_3)^2 \geq -(1-\gamma)\left(\frac{x_{1,R} + x_{2,R}}{2} - M_3\right)^2, \tag{9}
\]
where \( x_{3,R} = \bar{m} \) if the inequality in (9) is strict. Note that, as long as the median voter in district 3 strictly prefers the Republican candidate, the primary voter can nominate a more extreme candidate and still win, so if (9) holds as strict inequality, it must be that nominating a more extreme candidate is not any more attractive because the Republican candidate is already at the primary voter’s ideal point \( \bar{m} \). In addition, we have

\[
M_i \leq x_{i,R}, \text{ for } i = 1, 2, 3, \text{ and } x_{1,R} < x_{2,R} < x_{3,R},
\]

i.e., Republican candidates take at least weakly more conservative positions than the median voter in each district, and the positions of the Republican candidates have the same ordering as the ideal positions of the median voters in these districts.

We now show that, if there exist Republican positions \( (x_{1,R}, x_{2,R}, x_{3,R}) \) that satisfy (7) to (10) then there is an equilibrium in which Republicans take these positions, Democrats take the district median voter’s ideal position in each district (i.e., \( x_{i,D} = M_i \)), and all districts elect Republicans. (In the Appendix, in the proof of Proposition 1, we then show that a solution to the equation system (7) to (10), and thus an equilibrium, indeed exists).

First note that (7) and (8) ensure that the median voter in districts 1 and 2 is indifferent between the candidates, while the median voter is either indifferent or strictly prefers the Republican in district 3. Hence, median voters cannot profitably deviate.

Next, note that all types \( \theta > M_i \) strictly prefer the Republican in district \( i \). To see this, first consider district \( i = 1 \). The net benefit of electing a Republican in district 1 for voter type \( \theta \) is given by

\[
-(1 - \gamma)(x_{2,R} - \theta)^2 - \gamma(x_{1,R} - \theta)^2 + (1 - \gamma)\left(\frac{x_{2,R} + x_{3,R}}{2} - \theta\right)^2 + \gamma(M_1 - \theta)^2,
\]

where the first two summands are \( \theta \)'s payoff to type if the Republican is elected, and the last two summands are negative of \( \theta \)'s payoff if the Democrat is elected — in this case the Republican party position is changed to the median of the candidates in the remaining two districts given by \( 0.5(x_{2,R} + x_{3,R}) \).
This net benefit is a linear function of $\theta$ (because the $\theta^2$ terms drop out against each other). By construction, the net benefit is zero for $\theta = M_1$. Now consider voter $\theta$ in district 1, where $x_{1,R} < \theta < x_{2,R}$. Then $M_1 \leq x_{1,R}$ and $x_{2,R} < 0.5(x_{2,R} + x_{3,R})$ imply that $\theta$ is at least as close to $x_{1,R}$ than to $M_1$, and closer to $x_{2,R}$ than to $0.5(x_{2,R} + x_{3,R})$. As a consequence, voter $\theta$ strictly prefers the Republican candidate, i.e., the net benefit is strictly greater than 0. Hence linearity in $\theta$ implies that the net benefit from the Republican is strictly greater than zero for all $\theta > M_1$ and strictly less than zero for all $\theta < M_1$. The argument for the other districts is analogous.

Now consider the first stage of the game. Democratic candidates are already located at $M_i$, so Democratic primary voters cannot change their strategy in a way that increases their winning probability. Since $x_{3,R} \leq m_{3,R}$, (10) implies $x_{i,R} < x_{3,R} \leq m_{3,R} = m_{i,R}$. Thus, a deviation in district $i = 1, 2$ is only profitable for the respective Republican primary voters if the new candidate position $x'_{i,R}$ satisfies $x_{i,R} < x'_{i,R} \leq m_{i,R}$. However, it follows that the left-hand sides of (7) and (8), respectively, are strictly less than the right-hand sides if we replace $x_{i,R}$ by $x'_{i,R}$. Hence the Republican loses. As shown above, voters $\theta > M_i$, and hence also the median Republican primary voters, are strictly worse off if the Democrat is elected. Thus, the deviation is not profitable. The argument for district 3 is similar.

In order to show that an equilibrium exists, we still need to show that there exist candidate positions that satisfy conditions (7) to (10). We prove this in the Appendix. Further, we can also show that any equilibrium must satisfy these equations, and that one party wins all districts. In particular, unless $\gamma = 1$ this also excludes the case that candidates located at the district medians, or at the same positions, as the numerical simulations in Figure 1 indicate.

**Proposition 1** Consider a three district model without uncertainty with $M_1 < M_2 = 0 < M_3$. Suppose that the median positions of Democratic and Republican primary voters are at $\bar{m}$ and $\bar{m}$ in all districts. Then

1. If $\gamma$ is close to zero or close to one (i.e., voters derive either most utility nor almost no utility from party positions) then, in any pure strategy equilibrium, one party wins in all districts.

2. If $M_1 + M_3 \geq 0$ then, for any $\gamma$, there exists a pure strategy equilibrium in which Republicans
win in all three districts. Democratic candidates are located at the median voters’ ideal points. Republicans at \( x_{1,R} < x_{2,R} < x_{3,R} \) with \( x_{i,R} > M_i \).

3. If \( M_1 + M_3 \leq 0 \) then, for any \( \gamma \), there exists a pure strategy equilibrium with similar properties in which Democrats win in all districts.

4. If \( \gamma \) is small and Republicans win in all districts, then candidate positions \( x_{i,R} \) are close to \( M_3 \) for \( i = 1, 2, 3 \). Similarly, if Democrats win in all districts then \( x_{i,D} \) is close to \( M_1 \), for \( i = 1, 2, 3 \).

In Section 5.2 we argued that an equilibrium in which a party gets a majority with probability 1, results in a homogenous party, if the voters primarily care about the party’s policy, i.e., if \( \gamma \) is close to zero. Proposition 1 shows that positions converge to the ideal point of the median of the most conservative district. A numerical simulation is shown in Figure 1. The positions of the Democratic candidates, who are always at the district median, are indicated by the dashed lines. The solid lines are the positions of the Republican candidates. Note that there is substantial divergence between parties even when \( 1 - \gamma \) is small and voters care primarily about the local candidates. As predicted by Proposition 1 candidate positions converge to \( M_3 \) when \( 1 - \gamma = 1 \). In this case there is policy divergence in districts 1 and 2, and substantial heterogeneity among the
losing party’s representatives, whereas the candidates of the winning party all support the same platform.

Of course, the result of Proposition 1 that one party wins all districts is somewhat unrealistic, and is due to the assumption that there is complete information about median voter preferences so that, in a deterministic world, all players know for sure which party wins. In practice, we may not know for sure which party will win an election, but one can nevertheless point to many instances where there is a very high likelihood that a particular party will control the legislature. Proposition 1 applies very well in these situations. In particular, while general election voters would be happy to elect a majority party candidate who is marginally more moderate than their representatives in the legislature from other districts, the majority party primary voters are, for the same reason, reluctant to nominate such moderate candidates. Proposition 1 predicts that these opposing forces lead to an internally homogenous, but rather extreme party.

7 A Stochastic Multidistrict Model

In pure strategy equilibria in a deterministic model, all districts are safe for one of the parties (i.e., all winning probabilities are either zero or one). We now address what happens with competitive districts where both candidates have a strictly positive probability of winning. In this case, parties have a non-trivial trade-off because nominating an ideologically too extreme candidate reduces the chance of winning the district election, and thus the probability of winning a majority in the legislature. In the previous section we saw that multidistrict elections give rise to externalities between districts which in turn lead to substantial political polarization that is not present in the single-district case. The objective of this section is to investigate how these externalities manifest themselves in competitive districts. Moreover, in order for the model to address the empirical literature dealing with issues of gerrymandering, it is useful to analyze a setting in which there are competitive districts that may elect Democrats or Republicans, each with strictly positive probabilities.

In order to model a competitive district, we assume that there is uncertainty about the position
of the median voter at the time when candidates announce their platforms. More formally, suppose that the median voter is realized according to a probability distribution described by a cdf $\Phi(\cdot)$ with mean $\mu$ and standard deviation $\sigma$. To perform comparative statics with respect to $\mu$ we will assume that $\Phi(x) = \Phi_0(x-\mu)$, where $\Phi_0$ is the cdf of a symmetric distribution with mean zero. The median primary voters in the competitive districts are located at $-m$ and $m$, respectively.\textsuperscript{20} For our results it will be irrelevant whether there is one or multiple identical competitive districts.

In addition to competitive districts there are districts that lean sufficiently far to the left and right, respectively, so that in equilibrium they are safe for one of the parties. For the moment suppose that, in the non-competitive districts, there is no uncertainty about the location of the median voter and, specifically, suppose that the median voter coincides with the median primary voter. Furthermore, suppose that there are $k_D$ districts $i$ in which $m_{D,i} = M_i = -m_G < 0$ and $k_R$ districts with $m_{R,i} = M_i = m_G > 0$. Corollary 1 below shows that the results are not affected if median primary voters and the general election median voters differ.

In an equilibrium that is symmetric across districts it is intuitive that extreme Democratic districts nominate candidates at position $-m_G$ and Republican districts at $m_G$, while the competitive districts determine which party wins the majority.

The nature of the equilibrium depends critically on the number of extreme districts. First, we consider the case where the number of extreme districts is small. In particular, we assume that $k_D, k_R < (n-1)/2$ so that, since the majority party has at least $n+1$ representatives (as there are $2n+1$ districts), a supermajority of the majority party’s representatives come from competitive districts. Thus, even if a legislator from a competitive districts deviates, the winning party’s position is still determined by legislators from competitive districts.

**Proposition 2** Suppose that the distribution of the median voter in each competitive district is i.i.d. with mean $\mu$, following a distribution $\Phi(x) = \Phi_0(x-\mu)$, where $\Phi_0$ is symmetric around zero, and has a strictly positive, continuous density $\phi_0$. Suppose that the median primary voters are at $m_{R,i} = -m_{i,D} = m$ in the competitive districts $k_D < i < 2n + 1 - k_R$, and at $m_{R,i} = -m_{i,D} = m_G$ in the

\textsuperscript{20}Note that this symmetry assumption is without loss of generality. Alternatively, we could choose $\mu = 0$ and the primary voters would be located at $m_D = -m + \mu$ and $m_R = m + \mu$. 21
remaining districts. Let $k_D, k_R < (n - 1)/2$.

Then for any $\mu$ in a neighborhood of zero and for small $\gamma$, there exists a unique equilibrium that is symmetric across competitive districts, i.e., $x_{D,i} = x_D(\mu)$, and $x_{R,i} = x_R(\mu)$ for all districts $i$ with $k_D < i < 2n + 1 - k_R$.

1. $x_D(\mu)$ and $x_R(\mu)$ are independent of the number of competitive districts and independent of $\gamma$. Furthermore, $x_D(\mu) < x_R(\mu)$ for all $\mu$.

2. $x_D(0) = -\frac{m}{1 + 2m_0(0)m}$, and $x_R(0) = \frac{m}{1 + 2m_0(0)m}$.

3. $x_R(\mu) - x_D(\mu) < x_R(0) - x_D(0)$ for $\mu \neq 0$ in a neighborhood of zero.

4. The probability that the Republican wins in a competitive district is strictly increasing in $\mu$ for $\mu$ in a neighborhood of zero.

5. Democrats win districts $i \leq k_D$ and Republicans in districts $i \geq 2n + 1 - k_R$.

Given the assumption that there are few extreme districts and the party position is therefore determined by legislators from competitive districts, it is intuitive that the positions of median voters in the extreme districts, $m_G$ and $-m_G$ are irrelevant for candidate positions in competitive districts. It is probably more surprising that the exact number of safe districts $k_D$ and $k_R$ does not affect candidate choice in the competitive districts, although it obviously affects the probability of winning a majority in the legislature.

For example, suppose there are seven districts (i.e., $n = 3$), one of which is an extreme Democratic district, while all remaining districts are moderate (i.e., $k_D = 1$ and $k_R = 0$). If $\mu = 0$ then the positions in the competitive districts, $x_D$ and $x_R$ must be symmetric around zero, and hence, since $\Phi$ is symmetric, each candidate wins with probability 0.5 in a competitive district. However, since Democrats already have a safe district, they only have to win three of the competitive seats to get a majority in the legislature. Hence, the probability of attaining a Democratic majority is about 66%. Nevertheless, this electoral advantage does not affect policy position and polarization, i.e., the difference between candidates, $x_R - x_D$, in the competitive district.
Finally, there is no externality between competitive districts. As indicated in the first statement of Proposition 2, both $\gamma$ and the number of competitive districts are irrelevant for the equilibrium positions in the competitive districts. In other words, the equilibrium positions, $x_D$, and $x_R$, are the same as in the single district model.

Now consider the opposite case, with many extreme districts. In this case, party positions are determined by candidates from extreme districts, and the main question of interest is how the externalities imposed by these extreme districts affect political competition in moderate districts.

**Proposition 3** Make the same assumptions as in Proposition 2, except that now $n + 1 > k_D, k_R > (2n + 1)/3$. Then the following is true for all $\mu$ in a neighborhood of zero.

1. For small $\gamma$, there exists a unique equilibrium that is symmetric across competitive districts, i.e., $x_{Di} = x_D(\mu)$, and $x_{Ri} = x_R(\mu)$ for all districts $i$ with $k_D < i < 2n + 1 - k_R$.

2. There exists $\bar{m} < \infty$ such that for all $\mu \neq 0$, and $m > \bar{m}$, polarization is larger than in the case with few extreme districts from Proposition 2. If $\mu = 0$, polarization is the same as in Proposition 2.

3. $x_D$ and $x_R$ become independent of the median voter position in extreme districts, $m_G$, as $\gamma \to 0$.

To illustrate the individual statements of Proposition 3 consider an example model with one competitive district, with $m = 1$, $m_G = 2$ and $\Phi$ following a normal distribution with mean $\mu$ and standard deviation $\sigma = 0.2$.

Figure 2 shows how the equilibrium candidate positions change as the expected position of the median voter changes. For comparison, when there are few extreme districts (i.e., the setting of Proposition 2), then both candidates move to the right by approximately equal amounts when the median voter moves to the right (the dashed lines).

$^{21}$Clearly, the functions $x_D(\cdot)$ and $x_R(\cdot)$ implicitly depend on all parameters of the game (and not just on $\mu$). Consequently, these functions are different from the corresponding ones defined in Proposition 2.
Figure 2: Republican (red) and Democratic (blue) positions in a competitive district for $\gamma = 0.1$ and for Proposition 2 (dashed lines)

In contrast, when extreme districts determine the positions of the parties (and when those are what matters most for voters), then who wins in a moderate district is determined primarily by the realized median voter position, and not so much by the positions of the local candidates. The Republicans, who have the advantage in terms of winning probability, move to more conservative positions as the expected median voter position becomes more conservative because they know that their candidate becomes more and more likely to be elected, independent of his own position and the Democrat’s position. In contrast, Democrats know that their candidate has a chance of winning only if the realized median voter is very close to zero and thus indifferent between national Democrats and Republicans – but in that case, they do not need to compromise too much with respect to their local candidate’s position.

Figure 3 summarizes the resulting degree of local polarization (i.e., the difference between the local Republican and the local Democratic candidate), as we vary the expected median voter position, keeping other parameter values as in Figure 2. The middle line and the upper line show local polarization for the cases of $\gamma = 0.5$ (the middle curve) and $\gamma = 0.001$ (the top curve). The dashed line in Figure 3 shows the benchmark case with few extreme districts from Proposition 2. Polarization in the competitive district is larger when positions are determined by extreme districts.
than it is in the benchmark case, which illustrates the second statement of Proposition 3.

This result therefore addresses an important argument about the impact of gerrymandering on polarization. McCarty et al. (2009a,b) claim that, while Congress has become more polarized in a time during which electoral districts became more heterogeneous due to gerrymandering, this is merely a temporal coincidence. “Political scientists have demonstrated that whenever a congressional seat switches parties, the voting record of the new member is very different from that of the departing member, increasing polarization. In other words, it is becoming more common to observe a very liberal Democrat replaced by a very conservative Republican (and vice versa).” They argue that, since these switches happen in competitive districts, this effect cannot be explained by gerrymandering.

Our result shows that this argument may be flawed because the candidates in districts that are not directly affected by gerrymandering (in the sense that their median voter’s ideal position remains constant) nevertheless choose more extreme positions because of the increased extremism in other districts. Propositions 3 shows that voter preference polarization in some districts — possibly brought about by gerrymandering — does not just affect these districts, but other districts as well. In particular, if the more extreme legislators from gerrymandered districts determine the
national policy, then we should observe increased polarization also in the remaining competitive districts, exactly the behavior noted by the empirical literature. In contrast to the argument above, this, of course, does not imply that gerrymandering has no effect.

The same arguments also apply to the Senate where there is obviously no “gerrymandering,” but where increased regional preference differences have created an increasing number of safe seats for the parties. More extreme candidates elected in these safe states impose an externality on the remaining competitive states, creating increased polarization in those states as well.

It is also important to point out that the equilibrium local polarization is very similar for the case that both local and national policy matter equally for voters ($\gamma = 0.5$), and for the case that essentially only national policy matters. Thus, while the formal statement of Proposition 3 focuses on the case that $\gamma$ is small, the qualitative results hold for a wide range of values of $\gamma$.

Figure 4 summarizes the equilibrium degree of local polarization, as we vary the position of primary voters in extreme districts, $m_G$, for $\gamma = 0.9$ (bottom curve), 0.5 (middle curve) and 0.1 (top curve). We choose $\mu = 0.2$ and keep the remaining parameter values from the previous figures. As expected, polarization is larger when national positions are more important (small $\gamma$). Further, as the third statement of Proposition 3 indicates, $m_G$ has a decreasing influence on polarization as
\( \gamma \) decreases. In other words, if voters care primarily about party positions, then the existence of sufficiently many safe districts is sufficient to generate polarization in competitive districts; it is not necessary for these safe districts to be very extreme.

In Proposition 3, we assume that the median voters in extreme districts are deterministic and located at the same position as the Democratic and Republican primary voters. This is analytically convenient because it removes any doubt about what is happening in these districts, both in terms of positioning and in terms of the election outcome. However, if we relax the assumption, the equilibrium characterized in Proposition 3 remains an equilibrium for \( \gamma \to 0 \): The proof of Corollary 1 shows that candidates in “extreme” districts will locate at the primary voter’s ideal point.

**Corollary 1** Use the same assumptions as in Proposition 3 except that there is uncertainty about the distribution of median voters in extreme districts. Specifically, assume that there exists \( h > 0 \) such that cdf of median voter positions in extreme districts, \( \Phi_{\text{ext,D}}(-h) = 1 \) in the \( k_D \) liberal-leaning districts, and \( \Phi_{\text{ext,R}}(h) = 0 \) in the \( k_R \) conservative-leaning districts. Then, for small \( \gamma \), the equilibrium characterized in Proposition 3, in which the winning candidates in extreme districts locate at the primary voter’s ideal point, remains an equilibrium of this generalized game.

It is important to emphasize that the “extreme” districts in Corollary 1 can actually be quite moderate, as long as it is certain that the general election median voters in these districts strictly prefer one of the two party positions.

In spite of the fact that, in this equilibrium, the winning candidate in an extreme district locates away from the district median voter’s ideal point, the general election in the extreme districts is not competitive. Whether or not the opposition party nominates a candidate, and if yes, at which position that candidate locates, has no effect on the equilibrium position of the winning candidate.

The main competitive election when party positions are determined by candidates from extreme districts, is the favored party’s primary. For example, former House Speaker Dennis Hastert said in an October 7, 2013 interview with NPR, referring to members of Congress: “It used to be they’re looking over their shoulders to see who their general election opponent is. Now they’re looking over their shoulders to see who their primary opponent is.”
Consider, for example, the 2013 U.S. government shutdown. An October 7, 2013 Washington Post opinion poll shows that registered voters disapproved of the Republican party shutting down the government by 71 to 26. Thus, it is very likely that the median voters in most districts – even most of those held by Republican House members – opposed shutting down the government, but among voters who identify as Republicans, there was a 52-45 majority in favor of the shutdown, and it is likely that, among those voters who actually vote in Republican primaries, there was an even larger majority in favor of the shutdown.

There are media reports that many Republican representatives “would have liked” to end the government shutdown much sooner, but were afraid that taking this position publicly would put them at risk in their district primary. Our model shows that this fear is justified: Primary voters who refuse to renominate a “moderate” and replace him with a more extremist candidate are not irrational – because even the extremist can win. This makes the primary threat so credible.22

8 Conclusion

Much of the existing literature on electoral competition in legislative elections implicitly assumes that voters evaluate their local candidates based on their positions, but not on the party label under which they run. Such a model implies that both parties nominate candidates who are very close to the preferences of the respective district median voters. Therefore, even in districts with rather extreme preferences, both parties’ candidates should be competitive, and the position of Democratic and Republican Congressmen elected from similar districts should be very similar. It is safe to say that these predictions are not borne out in reality, and to understand why this is the case is of first-order importance for our understanding of the American democratic system.

22 Alternative explanations for non-median policy outcomes include lobbying and differential preference intensity. Arguably, neither of these alternative explanations is plausible for the government shutdown.

The lobbying explanation (a strong lobby in favor of the minority position is able to “buy” the support of legislators) requires that benefits on the minority side are highly concentrated, while the issue is a relatively minor issue for most voters. The intensity explanation requires that minority voters care more about the issue, but according to the same Washington Post poll cited above, 12 percent of registered voters “strongly approve” of the shutdown, 14 percent “approved somewhat,” while 53 percent “strongly disapprove” and 18 percent “disapprove somewhat.” Thus, intensity about the issue appears higher among those who disapprove.
In this paper, we have developed a theory of electoral competition in a world where majority party legislators collaboratively influence policy and voters therefore rationally care about candidates’ party labels. This model yields results that are fundamentally different from the standard model.

In our model, a candidate’s association with candidates of the same party that run in other districts generates an incentive for voters to focus less on the candidates’ own position when deciding whom to vote for — local candidates are “contaminated” by their party association. This leads to less competitive local elections, providing the ideologically favored party with the leeway to nominate more extreme candidates who are nevertheless elected. As a consequence, the equilibrium of our model can explain how electoral competition can beget a very polarized legislature.

Our analysis has two additional important empirical implications. First, it can explain why a district’s ideological preferences have a smaller partisan effect in elections in which a candidate has a more autonomous policy influence, such as elections for executive leadership positions than in legislative elections. Of course, in reality, even executive leader positions are not entirely autonomous, so there will be some contamination in executive elections as well, but we would expect this effect to be smaller than in legislative elections, and this expectation is borne out in our empirical analysis of Senate and Gubernatorial elections in Section 3.

Second, much of the existing empirical analysis of the effects of gerrymandering on polarization in Congress is implicitly based on applying a naive model in which voters care only about the local candidates’ positions. Such a model may lead to incorrect inferences about the importance of gerrymandering. For example, one cannot infer that gerrymandering does not matter for polarization in Congress from showing that there is no marginal effect of changes in district medians on ideological positions of legislators, and that the difference in voting records of Republicans and Democrats representing the same or very similar districts has increased. In general, an implication of our model for empirical work is that legislator behavior in different districts is intricately connected rather than independent, and this implies that one needs to be very careful with claims that difference-in-difference approaches can identify causation.
9 Appendix

Proof of Proposition 1. We first prove existence, i.e., statements 2 and 3.

First, note that solving (8) for \( x_{2,R} \), and using \( M_2 = 0 \) implies

\[
 x_{2,R} = \sqrt{1 - \gamma \frac{x_{1,R} + x_{3,R}}{2}}. \tag{12}
\]

If \( x_{1,R} \geq M_1 \) and \( x_{3,R} \geq M_3 \) then the assumption that \( M_1 + M_3 \geq 0 \) imply that \( x_{2,R} \geq 0 = M_2 \).

We can now use (7) and (12) to solve for \( x_{1,R} \) and \( x_{2,R} \) as functions of \( x_{3,R} \). In the process of doing this, we need to make sure that the conditions of (10) are satisfied, i.e., \( M_i \leq x_{i,R} \) for \( i = 1, 2 \) and \( x_{1,R} < x_{2,R} < x_{3,R} \) over the relevant range of \( x_{3,R} \).

Claim 1. For \( i = 1, 2 \), there exist differentiable functions \( g_i : [M_3, \infty) \to \mathbb{R} \) such that, if \( x_{i,R} = g_i(x_{3,R}) \), then \( x_{1,R}, x_{2,R}, \) and \( x_{3,R} \) satisfy (7), (8) and (10).

Let \( h(x_{1,R}, x_{3,R}) \) be defined as the difference between the right-hand side of (7) and the left-hand side, substituting (12) for \( x_{2,R} \), i.e.,

\[
 h(x_{1,R}, x_{3,R}) = (1 - \gamma)(x_{2,R} - M_1)^2 + \gamma(x_{1,R} - M_1)^2 - (1 - \gamma)\left(\frac{x_{2,R} + x_{3,R}}{2} - M_1\right)^2,
\]

where \( x_{2,R} \) is given by (12). We show that \( h(M_1, x_{3,R}) < 0 \).

Substituting \( x_{1,R} = M_1 \) into (12) yields \( x_{2,R} = \frac{0.5 \sqrt{1 - \gamma(M_1 + x_{3,R})}}{x_{3,R}} \). Further, as shown above \( x_{2,R} \geq 0 \). Thus, \( \frac{x_{2,R} + x_{3,R}}{2} - M_1 > x_{2,R} - M_1 > 0 \), which implies

\[
 h(M_1, x_{3,R}) = (1 - \gamma)(x_{2,R} - M_1)^2 - (1 - \gamma)\left(\frac{x_{2,R} + x_{3,R}}{2} - M_1\right)^2 < 0. \tag{13}
\]

Let

\[
 \bar{x}_{1,R} = \frac{\sqrt{1 - \gamma}}{2 - \sqrt{1 - \gamma}} x_{3,R}.
\]

Then (12) implies \( x_{1,R} < x_{2,R} \) for all \( x_{1,R} < \bar{x}_{1,R} \) and \( x_{1,R} = x_{2,R} \) for \( x_{1,R} = \bar{x}_{1,R} \). Further, at \( x_{1,R} = \bar{x}_{1,R} \) (12) implies \( (2 - \sqrt{1 - \gamma})x_{2,R} = x_{3,R} \) and hence \( x_{2,R} < x_{3,R} \). Since \( x_{2,R} \), as defined by (12), is strictly
monotone in $x_{1,R}$ it follows that $x_{2,R} < x_{3,R}$ for all $x_{1,R} \leq \bar{x}_{1,R}$.

Note that

\[
 h(\bar{x}_{1,R}, x_{3,R}) = (x_{2,R} - M_1)^2 - (1 - \gamma)\left(\frac{x_{2,R} + x_{3,R}}{2} - M_1\right)^2 =
\]

\[
 \left(\sqrt{1 - \gamma\frac{x_{1,R} + x_{3,R}}{2} - M_1}\right)^2 - (1 - \gamma)\left(\frac{x_{2,R} + x_{3,R}}{2} - M_1\right)^2 =
\]

\[
 = (1 - \gamma)\left(\frac{x_{2,R} + x_{3,R}}{2} - \frac{M_1}{\sqrt{1 - \gamma}}\right)^2 - (1 - \gamma)\left(\frac{x_{2,R} + x_{3,R}}{2} - M_1\right)^2 > 0,
\]

where the inequality follows since $x_{2,R} \geq M_2 = 0$, $x_{3,R} \geq M_3 > 0$, $M_1 < 0$, and $0 < \gamma < 1$. Hence, (13) and (14) imply, by the intermediate value theorem, that there exists $x_{1,R}$ with $M_1 < x_{1,R} < \bar{x}_{2,R}$ and $h(x_{1,R}, x_{3,R}) = 0$.

Next, note that for any fixed $x_{3,R} \geq M_3$ the exists a unique solution. Then $h(x, x_{3,R})$ is a quadratic function of $x$, and therefore there cannot exist a root of $g$ on $[M_1, \bar{x}_{1,R}]$ at which $\frac{\partial h}{\partial x} > 0$. In particular, since (as shown above) $h(M_1, x_{3,R}) > 0$ and $h(\bar{x}_{1,R}, x_{3,R}) < 0$ there would have to exist two additional roots, which is not possible since $h$ is quadratic in $x$ and not constant equal to zero.

Next, suppose that there exists a root $x^*$ in $[M_1, \bar{x}_{1,R}]$ at which $\frac{\partial h(x^*, x_{3,R})}{\partial x} = 0$. Since $h(M_1, x_{3,R}) > 0$ and $h(\bar{x}_{1,R}, x_{3,R}) < 0$ this implies that there must exists $x < x^* < \bar{x}$ with $\frac{\partial h(x, x_{3,R})}{\partial x}, \frac{\partial h(x, x_{3,R})}{\partial x} < 0$. This is a contradiction because, as a derivative of a quadratic function, $\frac{\partial h(x, x_{3,R})}{\partial x}$ is linear in $x$. As a consequence, $\frac{\partial h(x^*, x_{3,R})}{\partial x} < 0$ at any root in $[M_1, \bar{x}_1]$, which also implies that the solution is unique.

Let $g_1(x_{3,R})$ be the unique solution of the equation $h(g_1(x_{3,R}), x_{3,R}) = 0$. Then $g_1(\cdot)$ is a well-defined function. Further, we have shown that $\frac{\partial h(x^*, x_{3,R})}{\partial x} < 0$. The implicit function theorem therefore implies that $g_1(x_{3,R})$ is continuously differentiable for all $x_{3,R} \geq M_3$.\(^{23}\) Finally, define $g_2$ by

\[
 g_2(x_{3,R}) = \sqrt{1 - \gamma}\frac{g_1(x_{3,R}) + x_{3,R}}{2}.
\]

Then, by construction, $x_{1,R} = g_1(x_{3,R})$, $x_{2,R} = g_2(x_{3,R})$ and $x_{3,R}$ satisfy (7) and (8). Furthermore, we have shown that $x_{1,R} < x_{2,R} < x_{3,R}$ if $x_{3,R} \geq M_3$, and that $x_{i,R} \geq M_i$ for $i = 1, 2$. This concludes the

\(^{23}\)Note that the implicit function theorem implies that there exists a continuously differentiable function $f$ in a neighborhood of $x_{3,R}$ that solves the equation. However, since the solution to the equation is unique, $f$ coincides with $g_1$ and hence $g_1$ is continuously differentiable at $x_{3,R}$.
proof of Claim 1.

Claim 2: There exist $x_{i,R}$, $i = 1, 2, 3$ that satisfy (7) to (10).

Now let $x_{3,R} = M_3$, $x_{1,R} = g_1(x_{3,R})$ and $x_{2,R} = g_2(x_{3,R})$. (10) holds since $x_{3,R} \geq M_3$. Thus,

$$(1 - \gamma)(x_{2,R} - M_3)^2 + \gamma(x_{3,R} - M_3)^2 = (1 - \gamma)(x_{2,R} - M_3)^2 < (1 - \gamma) \left( \frac{x_{1,R} + x_{2,R}}{2} - M_3 \right)^2, \quad (15)$$
i.e., the median voter $M_3$ in district 3 is strictly better off voting for the Republican. Thus, the set $D$ of all $x_{3,R}$ that satisfy (7), (8), (10) and

$$(1 - \gamma)(x_{2,R} - M_3)^2 + \gamma(x_{3,R} - M_3)^2 \leq (1 - \gamma) \left( \frac{x_{1,R} + x_{2,R}}{2} - M_3 \right)^2, \quad (16)$$
is non-empty.

Now choose $x_{3,R} = m_{3,R}$. If (9) holds then we have a solution for (7) to (10). Else, the intermediate value theorem implies that there exists $x_{3,R}$ that solves (9) with equality. This proves Claim 2.

Claims 1 and 2 imply existence. We next show the final statement of the Proposition, that the winning parties candidate position converge to those of the median of the most extreme district.

Consider a sequence $\gamma_n$, $n \in \mathbb{N}$ with $\lim_{n \to \infty} \gamma_n = 0$. Let $x_{i,R}^n$, $n \in \mathbb{N}$ be the equilibrium Republican candidate positions. Since these positions are bounded from below by $M_1$ and from above by $\bar{m}$, there exist convergent subsequences such that $\lim_{k \to \infty} x_{i,R}^{n_k} \to x_{i,R}$ for all $i$. Taking the limit in equations (7) to (9) implies that $x_{1,R} = x_{2,R} = x_{3,R}$. Suppose that $x_{3,R} > M_3$. Suppose the Democrat selects $x_{3,D} = M_3$. Then the median voter is strictly better off with the Democrat for sufficiently large $k$. In particular, there exists $\varepsilon > 0$ such that $x_{i,R}^{n_k} > M_3 + \varepsilon$ for all sufficiently large $k$. Thus, if district 3 elects a Democrat, the new Republican party position is more moderate, and since $\bar{x}_R \geq M_3$ it is closer to $M_3$, a contradiction. Hence $x_{i,R} = M_3$. Since all subsequences converge to the same limit, it follows that the candidate policies converge to $M_3$ as $\gamma$ goes to zero.

We finally, prove the first statement of the Proposition, i.e., that if $\gamma$ is close to 0 or close 1, one party wins in all districts. We start with the case where $\gamma$ is close to 1.
Claim 3: For any $\gamma$ let $x_{i,P}^\gamma$, $i = 1, 2, 3$, $P = D, R$ be a collection of equilibrium policies. Then $\lim_{\gamma \to 1} x_{i,P} = M_i$.

Suppose by way of contradiction that there exists a sequence $\gamma_n$ such that $x_{i,D}^\gamma$ is bounded away from $M_i$. The median voter $M_i$ must be indifferent between the candidates, else the winner could improve by moving closer to their primary voter’s ideal point — for large $n$ it is not possible for a candidate to win at the primary voters’ ideal point, since the opposing candidate could win by nominating a candidate close to $M_i$. Then $\lim_{n \to \infty} (\gamma_n - M_i)^2 = \lim_{n \to \infty} (\gamma_n - M_i)^2 > 0$, i.e., both candidates are equally far away from $M_i$ in the limit.

Thus, if candidate $D$ wins and $x_{i,D}^\gamma < M_i$, then for $\gamma_n$ close to 1, Republicans could win with a candidate $x_{i,R} = M_i$, which would make them better off. If $x_{i,D}^\gamma > M_i$, then the Democrats could improve by nominating a marginally more liberal candidate, and still win. The argument if candidate $R$ wins is symmetric. This proves Claim 3.

Now assume by way of contradiction that Democrats win in district 1, Republicans in the remaining districts, and suppose that $\gamma$ is close to 1. Then the median voter must be indifferent between the candidates: Because of Claim 3, $x_{1,D}$ is close to $M_1$ and hence $x_{1,D} > -\bar{m}$. As a consequence, a more liberal Democrat could also win in district 1, which contradicts that $x_{1,D}$ is an equilibrium policy.

Next, $x_{1,R} \neq M_1$. In particular, if $x_{1,R} = M_1$, then the median voter would be better off electing the Republican since this would move the party position from $0.5(x_{2,R} + x_{3,R})$ to $x_{2,R}$. This latter position is further to the left than the former since, as Claim 3 proves, $x_{i,R}$ is close to $M_i$ when $\gamma$ is close to 1. Thus, in order to have an equilibrium in which the Democrat wins in district 1, the Republican primary voters must be worse off if the Republican candidate won with $x_{1,R} = M_1$. Thus, $x_{1,R} > M_1$, and the Republican primary voters in district 1 must be better off (or indifferent) if their candidate loses, else, it would be optimal for them to win by nominating a marginally more liberal candidate.

We have established that the net benefit from voting for the Republican is linear in a voter’s type $\theta$. Since type $\theta = M_1$ is indifferent, and because the median Republican primary voter type $\theta = \bar{m} > M_1$ is either indifferent or worse off if the Republican is elected, it follows that the net
benefit from the Republican is zero or strictly less than zero for all \( \theta \geq M_1 \).

Now consider type \( \theta = 0.5(x_{1,R} + x_{2,R}) \). Then \( \theta \) is closer to the local Republican’s policy \( x_{1,R} \) than to the Democrat’s at \( x_{1,D} < M_1 < x_{1,R} \). Similarly, \( \theta \) is closer to the Republican party position \( x_{2,R} \) that arises if district 1 elects the Republican, than to party position \( 0.5(x_{2,R} + x_{3,R}) \) when only districts 2 and 3 elect Republicans. As a consequence, there exists a type \( \theta \geq M_1 \) who strictly prefers the Republican, a contradiction. Hence, no equilibrium exists in which the Democrat wins district 1. The proof for cases where the Democrats win one of the other districts is analogous.

It remains to consider the case where \( \gamma \) is close to zero. Suppose the Democrat wins district 1, and the Republicans in the remaining districts. Then \( \bar{m}_D = -\bar{m} \leq x_{1,D} \leq M_i \) because, if \( x_{1,D} > M_i \), then Democrats would be better off replacing their candidate with one at \( M_1 \), who would still win. The same is true for candidates \( x_{1,R} < \bar{m}_D \). Since the Democrats do not win a majority, the party position is irrelevant, which would provide the only possible reason for nominating a candidate to the left of \( \bar{m}_D \).

Let \( \bar{x}_R^\gamma \) be the equilibrium position of the Republican party. Note that the Democrats can always win district 2 for large \( \gamma \) by selecting a candidate \( x_{2,D} = -x_{1,D}^* \) so that the Democratic party position would be \( M_2 \)’s ideal position of 0. An equilibrium therefore requires that Democrats do not want to win with such a candidate, i.e., it must be the case that \( \limsup_{\gamma} \bar{x}_R^\gamma \leq 0 \).

Now consider district 3. If \( \gamma \) is small, then the Republican would get elected with a position \( x_{3,R} \) with \( 0.5(x_{2,R} + x_{3,R}) = M_3 \). Hence in the limit, the Republican party position should be \( \bar{x}_R \geq M_3 \), i.e., \( \liminf_{\gamma} \bar{x}_R^\gamma \geq M_3 \), a contradiction. Hence, no equilibrium exists in which the Democrat wins solely in the first district. The other cases are similar. □

**Proof of Proposition 2.** Consider a particular competitive district, and let \( p_k \) be the probability that \( k \) of the remaining \( 2n \) districts vote Republican. Suppose that the Republican in district \( i \) deviates to policy \( y \). Since \( k_R < (n - 1)/2 \), the median party policy remains at \( x_R \) if the Republican
wins. The payoff of a type \( \theta \) voter from the Democrat is

\[
-(1 - \gamma) \left( \sum_{k=0}^{n} p_k(\theta - x_D)^2 + \sum_{k=n+1}^{2n} p_k(\theta - x_R)^2 \right) - \gamma(\theta - x_D)^2,
\]

while the payoff from the Republican is

\[
-(1 - \gamma) \left( \sum_{k=0}^{n-1} p_k(\theta - x_D)^2 + \sum_{k=n}^{2n} p_k(\theta - x_R)^2 \right) - \gamma(\theta - y)^2.
\]

If \( x_D = x_R = x \) then the voter \( \theta \) who is indifferent between the candidates, i.e., the cutoff voter is given by \( \theta = (x + y)/2 \). Either \( x > -m \) or \( x < m \). Suppose that \( x < m \). It follows immediately that the Republican primary voters can increase their payoff by nominating a candidate \( y \) with \( x < y \leq m \): the Republican gets elected with probability \( 1 - \Phi(y) > 0 \) since the density, \( \phi \), is strictly positive; if the Republican candidate loses, the policy remains at \( x \). Thus, \( x_D = x_R \) is not an equilibrium.

The cutoff voter is given by

\[
\theta_R(x_D, x_R, y) = \frac{1}{2} \left( 1 - \gamma \right) p_n(x_R^2 - x_D^2) + \gamma(y^2 - x_D^2).
\]

It follows immediately that

\[
\theta_R(x_D, x_R, x_R) = \frac{x_D + x_R}{2}, \quad \frac{\partial \theta_R(x_D, x_R, y)}{\partial y} \bigg|_{y=x_R} = \frac{1}{2} \frac{\gamma}{(1 - \gamma)p_n + \gamma}.
\]

Next we show that \( x_R(\mu) > x_D(\mu) \).

Suppose, by way of contradiction, that \( x_R(\mu) \leq x_D(\mu) \). Since \( x_D \neq x_R \) it follows that \( x_R(\mu) < x_D(\mu) \), and hence liberals vote for the Republican and conservatives for the Democrat. The cutoff voter is located at \( (x_D(\mu) + x_R(\mu))/2 \). Now suppose the Republican’s position is changed to \( x_R = x_D(\mu) \). Then (17) implies that the cutoff is unchanged, and hence winning probabilities in the district are unaffected. However, the Republican primary voter in district \( i \) is strictly better off, since \( x_R = x_D(\mu) \) is strictly more conservative than the original policy, a contradiction. Hence \( x_R(\mu) > x_D(\mu) \).
Since $\Phi(\theta_R(x_D, x_R))$ is the probability that the Democrat gets elected, the Republican primary voter solves

$$\max_y -\Phi(\theta_R(x_D, x_R, y)) \left( (1 - \gamma) \left( \sum_{k=0}^{n-1} p_k (m - x_D)^2 + \sum_{k=n+1}^{2n} p_k (m - x_R)^2 \right) + \gamma (m - x_D)^2 \right)$$

$$- \left[ 1 - \Phi(\theta_R(x_D, x_R, y)) \right] \left( (1 - \gamma) \left( \sum_{k=0}^{n-1} p_k (m - x_D)^2 + \sum_{k=n+1}^{2n} p_k (m - x_R)^2 \right) + \gamma (m - y)^2 \right).$$

(19)

The first derivative with respect to $y$ is given by

$$-\phi(\theta_R) \frac{\partial \theta_R}{\partial y} \left( (1 - \gamma) p_n \left( (m - x_D)^2 - (m - x_R)^2 \right) + \gamma \left( (m - x_D)^2 - (m - y)^2 \right) \right)$$

$$+ (1 - \Phi(\theta_R)) 2\gamma (m - y),$$

(20)

and the second derivative is

$$-\left( \phi(\theta_R) \frac{\partial^2 \theta_R}{\partial y^2} + \phi'(\theta_R) \left( \frac{\partial \theta_R}{\partial y} \right)^2 \right) \left( (1 - \gamma) p_n \left( (m - x_D)^2 - (m - x_R)^2 \right) + \gamma \left( (m - x_D)^2 - (m - y)^2 \right) \right)$$

$$- 4\gamma \phi(\theta_R) \frac{\partial \theta_R}{\partial y} (m - y) - 2\gamma (1 - \Phi(\theta_R)).$$

(21)

Equation (17) implies

$$\lim_{\gamma \to 0} \frac{1}{\gamma} \frac{\partial \theta_R(x_D, x_R, y)}{\partial y} = -\frac{x_D + x_R - 2y}{p_n(x_R - x_D)}, \quad \lim_{n \to \infty} \frac{1}{\gamma} \left( \frac{\partial \theta_R(x_D, x_R, y)}{\partial y} \right)^2 = 0,$$

(22)

and

$$\lim_{n \to \infty} \frac{1}{\gamma} \frac{\partial^2 \theta_R(x_D, x_R, y)}{\partial y^2} = \frac{1}{p_n(x_R - x_D)}.$$  

(23)

Dividing (21) by $\gamma$, taking the limit for $n \to \infty$ and using (22) and (23) yields

$$-\frac{\phi(\theta_R)}{p_n(x_R - x_D)} p_n \left( (m - x_D)^2 - (m - x_R)^2 \right) - (1 - \Phi(\theta_R)) 2\gamma < 0,$$

(24)

since $x_R > x_D$, and $0 < \Phi(\cdot) < 1$ given that the density of $\Phi$ is always strictly positive. Thus, the
second order condition is satisfied for all $y$ when $\gamma$ is small.

Evaluating the first order condition (20) at $y = x_R$ yields (after canceling the common terms and multiplying by 2)

$$-\phi\left(\frac{X_D + X_R}{2}\right)\left((m - X_D)^2 - (m - x_R)^2\right) + \left(1 - \Phi\left(\frac{X_D + X_R}{2}\right)\right)4(m - x_R) = 0. \quad (25)$$

Similarly, the first order condition for the choice of the Democratic candidate is (again after canceling and multiplying)

$$-\phi\left(\frac{X_D + X_R}{2}\right)\left((m + X_D)^2 - (m + x_R)^2\right) - \Phi\left(\frac{X_D + X_R}{2}\right)4(m + X_D) = 0. \quad (26)$$

We now show that the first order conditions have a unique solution at $\mu = 0$.

Rearranging (25) and (26), we get

$$\phi\left(\frac{X_D + X_R}{2}\right) = \frac{(1 - \Phi\left(\frac{X_D + x_R}{2}\right))4(m - x_R)}{(m - X_D)^2 - (m - x_R)^2} = \frac{\Phi\left(\frac{X_D + x_R}{2}\right)4(m + X_D)}{-(m + X_D)^2 - (m + x_R)^2}. \quad (27)$$

Note that the denominator of the term in the middle of (27) is equal to $(x_R - x_D)(2m - x_R - x_D)$, and the denominator of the right term of (27) is equal to $(x_R - x_D)(2m + x_R + x_D)$. Furthermore, since $\phi(\cdot) > 0$, it follows that $x_R < m$ and $x_D > -m$, i.e., none of the candidates is located at the ideal point of the median primary voter.

Substituting this and canceling common terms, we get

$$\left(1 - \Phi\left(\frac{X_D + x_R}{2}\right)\right)(m - x_R)(2m + x_R + x_D) = \Phi\left(\frac{X_D + x_R}{2}\right)(m + x_D)(2m - x_R - x_D). \quad (28)$$

Suppose that $x_R + x_D > 0$. Since, for $\mu = 0$, $\Phi$ is symmetric, this implies that $\Phi\left(\frac{x_R + x_D}{2}\right) > \frac{1}{2} > 1 - \Phi\left(\frac{x_R + x_D}{2}\right)$. Since $-m < x_D < x_R < m$ it follows that $(m - x_R)(2m + x_R + x_D) \neq 0$, and $(m + x_D)(2m - x_R - x_D) \neq 0$. For (28) to hold, we must therefore have

$$(m - x_R)(2m + x_R + x_D) > (m + x_D)(2m - x_R - x_D), \quad (29)$$
which simplifies to \( x_R^2 < x_D^2 \), and hence \( |x_R| < |x_D| \). This and \( x_D < x_R \) imply that \( x_D + x_R < 0 \), a contradiction.

Similarly we get a contradiction if we assume that \( x_R + x_D < 0 \). Hence \( x_D = -x_R \).

The first order conditions together with the fact that \( x_D = -x_R \) imply

\[
2\phi(0)x_R m = 2(1 - \Phi(0))(m - x_R).
\]

Since \( \Phi(0) = 1/2 \), we get

\[
x_R = \frac{m}{1 + 2\phi(0)m}.
\]  

(30)

Now recall that \( \Phi(x) = \Phi_0(x - \mu) \) and \( \phi(x) = \phi_0(x - \mu) \). At \( \mu = 0 \) strategies are symmetric around zero and hence \( x_D + x_R = 0 \). We now take the derivatives of (25) and (26) with respect to \( \mu \), evaluated at \( \mu = 0 \). To shorten the notation we write \( x'_R \) and \( x'_D \) for \( x'_R(0) \) and \( x'_D(0) \).

\[
- \phi'_0(0) \left( \frac{x'_D + x'_R}{2} - 1 \right) \left( (m - x_D)^2 - (m - x_R)^2 \right) - \phi_0(0) \left( 2(m - x_D)x'_D + 2(m - x_R)x'_R \right)
\]

\[
- \phi'_0(0) \left( \frac{x'_D + x'_R}{2} - 1 \right) 4(m - x_R) - (1 - \Phi_0(0)) 4x'_R = 0,
\]

(31)

and

\[
- \phi'_0(0) \left( \frac{x'_D + x'_R}{2} - 1 \right) \left( (m + x_D)^2 - (m + x_R)^2 \right) - \phi_0(0) \left( 2(m + x_D)x'_D - 2(m + x_R)x'_R \right)
\]

\[
- \phi'_0(0) \left( \frac{x'_D + x'_R}{2} - 1 \right) 4(m + x_D) - \Phi_0(0) 4x'_D = 0.
\]

(32)

If \( \mu = 0 \) we have the symmetric equilibrium characterized above where \( x_R \) is given by (30). Thus, (31) and (32) imply

\[
x'_D(0) = x'_R(0) = \frac{4\phi_0(0)^2 m^2}{4\phi_0(0)^2 m^2 + 1}.
\]

(33)
The second derivatives of (25) and (26) with respect to $\mu$, evaluated at $\mu = 0$, are

$$-4\phi_0''(0)mx_R \left( \frac{x_D' + x_R'}{2} - 1 \right)^2 - 4\phi_0(0) \left( (m - x_R)x_R'' - x_R x_D'' - 2 \left( \frac{x_D' + x_R'}{2} - 1 \right) x_R' \right) - 2x_R'' = 0; \quad (34)$$

$$4\phi_0''(0)mx_R \left( \frac{x_D' + x_R'}{2} - 1 \right)^2 - 4\phi_0(0) \left( (m - x_R)x_D'' - x_R x_D'' + 2 \left( \frac{x_D' + x_R'}{2} - 1 \right) x_D' \right) - 2x_D'' = 0. \quad (35)$$

(34) and (35) imply $x_D''(0) = -x_R''(0)$. Let $S = 0.5(x_D'(0) + x_R'(0)) - 1$. Then (33) implies $S < 0$. We can solve (34) for $x_R''$ to get

$$x_R''(0) = \frac{2\phi_0''(0)mx_RS^2 + 4\phi_0(0)x_R'(0)S}{1 + 2\phi_0(0)m}. \quad (36)$$

Thus, $x_R''(0) < 0$. Since $x_D''(0) = -x_R''(0)$, it follows that $x_R''(0) - x_D''(0) < 0$. As a consequence, $x_R(\mu) - x_D(\mu)$ assumes a local maximum at $\mu = 0$. Therefore $x_R(\mu) - x_D(\mu) < x_R(0) - x_D(0)$ for $\mu \neq 0$ in a neighborhood of 0.

Finally, note that $x_D < 0 < x_R$ near $\mu = 0$, which implies that the median voters in districts 1 to $k_D$ strictly prefer that the Democrats win, while Republicans in districts $2n + 1 - k_R$ to $2n + 1$ strictly prefer that the Republicans win. As a consequence, districts 1 to $k_D$ are safe for the Democratic candidates, who get elected with policy $-m_G$, while districts $2n + 1 - k_R$ to $2n + 1$ are safe for the Republican candidates, who get elected with policy $m_G$, if $\gamma$ is not too large.

In particular, suppose that the median voter in district $i < k_D$ deviates and elects the Republican. Then the probability that policy $x_R$ is implemented increases, while the probability of policy $x_D$ decreases, which makes the median voter worse off as long as $\gamma$ is small. Since the median voter is strictly better off with the Democrat, the primary voter will propose a candidate with policy $x_{D,i} = -m_G$. The argument that districts $2n + 1 - k_R$ to $2n + 1$ are safe for the Republicans is analogous.

**Proof of Proposition 3.** Since the median voters in the gerrymandered districts are at $-m_G$ and $m_G$, the winning positions in these districts are $-m_G$ and $m_G$, respectively. By assumption, the gerrymandered districts account for at least 2/3 of all districts. Consequently, the national policy
is at $-m_G$ if the Democrats win a majority in the legislature, and at $m_G$ if the Republicans win.

Consider a particular competitive district, and let $p_k$ be the probability that $k$ of the remaining $2n$ districts vote Republican. Denote the Democrat’s and the Republican’s policies by $x_D$ and $x_R$. Then the payoff of a voter at $\theta$ from the Democrat is

$$-(1 - \gamma) \left( \sum_{k=0}^{n} p_k (\theta + m_G)^2 + \sum_{k=n+1}^{2n} p_k (\theta - m_G)^2 \right) - \gamma (\theta - x_D)^2,$$

while the payoff from the Republican is

$$-(1 - \gamma) \left( \sum_{k=0}^{n-1} p_k (\theta + m_G)^2 + \sum_{k=n}^{2n} p_k (\theta - m_G)^2 \right) - \gamma (\theta - x_R)^2.$$

The cutoff voter, who is indifferent between the candidates, is therefore given by

$$\theta(x_D, x_R) = \frac{1}{2} \frac{\gamma (x_R^2 - x_D^2)}{\gamma (x_R - x_D) + 2(1 - \gamma) p_n m_G}. \quad (37)$$

Note that

$$\lim_{\gamma \to 0} \frac{1}{\gamma} \frac{\partial \theta(x_D, x_R)}{\partial x_D} = -\frac{x_D}{2p_n m_G}, \quad \text{and} \quad \lim_{\gamma \to 0} \theta(x_D, x_R) = 0. \quad (38)$$

The Democratic primary voter therefore solves

$$\max_{x_D} -\Phi(\theta(x_D, x_R)) \left( (1 - \gamma) \sum_{k=0}^{n} p_k (m + m_G)^2 + (1 - \gamma) \sum_{k=n+1}^{2n} p_k (-m - m_G)^2 + \gamma (-m - x_D)^2 \right)$$

$$- (1 - \Phi(\theta(x_D, x_R))) \left( (1 - \gamma) \sum_{k=0}^{n-1} p_k (m + m_G)^2 + (1 - \gamma) \sum_{k=n}^{2n} p_k (-m - m_G)^2 + \gamma (-m - x_R)^2 \right).$$

The first order condition is

$$-\phi(\theta) \frac{\partial \theta(x_D, x_R)}{\partial x_D} \left( \gamma ((m + x_D)^2 - (m + x_R)^2) + (1 - \gamma) p_n ((-m + m_G)^2 - (m + m_G)^2) \right)$$

$$- 2\Phi(\theta) \gamma (m + x_D) = 0. \quad (39)$$
Dividing both sides of (39) by \( \gamma \), then taking the limit for \( \gamma \to 0 \), and using (38) yields

\[
\phi(0)(-x_D)m = \Phi(0)(m + x_D).
\] (40)

The Republican primary solves

\[
\max_{x_R} \Phi(\theta(x_D, x_R)) \left( (1 - \gamma) \sum_{k=0}^{n} p_k(2m)^2 + \gamma(m - x_D)^2 \right) \\
- (1 - \Phi(\theta(x_D, x_R))) \left( (1 - \gamma) \sum_{k=0}^{n-1} p_k(2m)^2 + \gamma(m - x_R)^2 \right).
\]

The first order condition is

\[
-\phi(\theta) \frac{\partial \theta(x_D, x_R)}{\partial x_R} \left( (1 - \gamma)4p_n m^2 + \gamma((m - x_D)^2 - (m - x_R)^2) \right) + 2\gamma \left( 1 - \Phi(\theta) \right) (m - x_R) = 0.
\]

It follows that

\[
\frac{\partial}{\partial x_R} \theta(x_D, x_R) \bigg|_{\gamma=0} = \frac{x_R}{2p_n m}.
\]

Again, dividing by \( \gamma \), setting \( \gamma = 0 \) and using the fact that \( \theta = 0 \) when \( \gamma = 0 \), yields

\[
\phi(0)x_R m = (1 - \Phi(0)) (m - x_R).
\] (41)

This implies

\[
x_D = -\frac{\Phi_0(-\mu)m}{\Phi_0(-\mu) + \phi_0(-\mu)m}, \quad x_R = \frac{(1 - \Phi_0(-\mu)m)}{(1 - \Phi_0(-\mu)) + \phi_0(-\mu)m}.
\] (42)

We now show that the objective of the Democrats’ maximization problems is strictly concave. The derivative of (39) is

\[
-\left( \phi(\theta) \frac{\partial^2 \theta(x_D, x_R)}{\partial x_D^2} + \phi'(\theta) \left( \frac{\partial \theta(x_D, x_R)}{\partial x_D} \right)^2 \right) \left( 2\gamma(2m)^2 - (m - x_R)^2 \right) - (1 - \gamma)4p_n m^2 \\
- 4\gamma\phi(\theta) \frac{\partial \theta(x_D, x_R)}{\partial x_D} (m + x_D) - 2\gamma\phi(\theta).
\] (43)
Note that
\[
\lim_{\gamma \to 0} \frac{\partial^2 \theta(x_D, x_R)}{\partial x_D^2} \frac{1}{\gamma} = -\frac{1}{2 p_n m}, \quad \text{and} \quad \lim_{\gamma \to 0} \left( \frac{\partial^2 \theta(x_D, x_R)}{\partial x_D^2} \right)^2 \frac{1}{\gamma} = 0. 
\]
(44)
Dividing both sides of (43) by \( \gamma \), taking the limit for \( \gamma \to 0 \), and using (38), and (44) yields \(-2\phi(\theta)(m + 1) < 0 \). Thus, for small \( \gamma \) the objective is concave for every \( x_D \). Concavity of the Republican’s objective follows similarly.

If \( \mu = 0 \) then \( \Phi(0) = 0.5 \). Hence (42) implies that the distance between the policies is the same as in Proposition 2, i.e., as in the case where all districts are symmetric.

Using the fact that \( \Phi \) is symmetric and hence \( \phi'(0) = 0 \) and \( \Phi(0) = 0.5 \), it is easy to verify that
\[
\frac{\partial (x_R - x_D)}{\partial \mu} \bigg|_{\mu = 0} = 0. 
\]
Hence, if \( \frac{\partial^2 (x_R - x_D)}{\partial \mu^2} \bigg|_{\mu = 0} < 0 \), then \( \mu = 0 \) is a local maximum, and polarization, i.e., the distance between the policies is smaller in a neighborhood of \( \mu = 0 \). The reverse is true if \( \frac{\partial^2 (x_R - x_D)}{\partial \mu^2} \bigg|_{\mu = 0} > 0 \).

Again, using (42), the fact that \( \phi'(0) = 0 \) and \( \Phi(0) = 0.5 \) it follows that
\[
\frac{\partial^2 (x_R - x_D)}{\partial \mu^2} \bigg|_{\mu = 0} = -\frac{4m^2 \left( 8\phi(0)^3 + \phi''(0) + 2m\phi''(0)\phi(0) \right)}{(1 + 2m\phi(0))^3}. 
\]
Thus, the second derivative is positive if and only if \( 8\phi(0)^3 + \phi''(0) + 2m\phi''(0)\phi(0) < 0 \), i.e., if \( m > \bar{m} \), where
\[
\bar{m} = -\left( \frac{4\phi(0)^2}{\phi''(0)} + \frac{1}{2\phi(0)} \right). 
\]
Let \( x_R^G(\mu) \), and \( x_D^G(\mu) \) the policies in district \( n + 1 \) in the gerrymandered model, and \( x_R(\mu), x_D(\mu) \) those in the symmetric model characterized in Proposition 2. Then for \( \mu \) in a neighborhood of zero, \( \mu \neq 0 \) we get \( x_R^G(\mu) - x_D^G(\mu) > x_R^G(0) - x_D^G(0) \), while item 3 in Proposition 2 implies \( x_R(\mu) - x_D(\mu) < x_R(0) - x_D(0) \). Since \( x_R^G(0) = x_R(0) \) and \( x_D^G(0) = x_D(0) \) it follows that the equilibrium policies in district \( n + 1 \) are more polarized in the gerrymandered model for \( m \geq \bar{m} \).

\textbf{Proof of Corollary 1.} Consider a Democratic district. We need to show that voting for a Democrat
located at $m_G$ remains optimal. Suppose that the median voter changes to voting for the Republican. Since the number of districts is sufficiently large, the party platforms remain unaffected. If the median voter elects a Republican, the probability of a Republican majority is strictly increased, thus increasing the probability that $m_G$ instead of $-m_G$ is implemented. Since the median voter is located to the left of $-h$, there exists $\bar{\gamma} > 0$, such that whenever $\gamma < \bar{\gamma}$, the median voter is strictly worse off. Hence, it is not optimal to deviate. Even a Republican located at the median voter’s ideal point would not win. ■
References


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