Political Competition in Legislative Elections*

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Abstract

We develop a theory of candidate nomination processes predicated upon the notion that members of the majority party in a legislature collaboratively influence policy. Because of this team aspect, a candidate’s party label matters for voters, in addition to his own policy positions: For example, in a liberal district, electing even a liberal Republican may be unattractive for voters because it increases the chance that Republicans obtain the majority in Congress, thereby increasing the power of more conservative Republicans. We show that candidates may be unable to escape the burden of their party association, and that primary voters in both parties are likely to nominate extremist candidates. We also show that gerrymandering affects the equilibrium platforms not only in those districts that become more extreme, but also in those that ideologically do not change.

Keywords: Differentiated candidates, primaries, polarization.

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1 Introduction

In the most basic model of representative democracy, voters elect legislative representatives whose positions reflect the preferences of their respective districts’ median voters. These representatives convene in an amorphous assembly (one in which there are no parties, or parties at least do not play an important role), and national policy is set, in equilibrium, to correspond to the preferences of the median representative in this assembly. Thus, in this basic model, the legislature is composed of representatives who are very moderate relative to the voters who elect them, and actual policy and legislation reflects the most moderate position in this assembly of moderates. Suffice it to say that few observers of Congress believe that reality corresponds closely to these predictions; the central question is why this is the case.

In this paper, we build a model of electoral competition that can account for a much higher degree of polarization in the legislature, and which is based on two realistic ingredients: First, the majority party in a legislature is an important power center influencing the crafting of policy. Coordination of decision-making and voting according to the majority preferences in the majority party increases the influence of each majority party legislator on the policy outcome (Eguia, 2011a,b). Second, legislative candidates are nominated by policy-motivated primary voters who take both the general election and the legislation process into account when deciding whom to nominate.

The importance of parties is uncontroversial among scholars of legislatures. However, there is surprisingly little analysis of how the fact that each candidate is connected to a party and thus, implicitly, to the positions of candidates of that party from other districts influences the types of candidates who are nominated by their party to run for legislative office and the outcomes of elections in different legislative districts.

If one were to apply the simplest Downsian model naively to Congressional elections – which much of the empirical literature implicitly does – then it generates counterfactual predictions: In each district, both candidates should adopt the preferred position of the district median voter, and so, policy-wise, all voters should be indifferent between the Democratic candidate and his Republican opponent. Republicans in New England or Democrats in rural Southern districts should
have a substantial chance to be elected to Congress if only they match their opponent’s policy platform.\textsuperscript{1} Furthermore, in this model framework, gerrymandering districts would not help parties, at least not in the sense that it would increase the party’s expected representation in Congress. It is safe to say that both of these predictions are counterfactual, but understanding why that is so is challenging.

In our model, the implemented policy is determined by a function that maps the ideal positions of majority party legislators into a policy, and satisfies some basic intuitive requirements such as efficiency and monotonicity. In the general election for the legislature, voters vote for their preferred candidate, taking into account the two ways in which their local representatives may change the policy outcome: First, the district result may change which party is the majority party in Congress, and second, if they elect a candidate who will be in the majority party, they may affect the ideological composition of the majority party.

In this framework, there are spillovers between different districts: The electoral prospects of candidates in a given district are influenced by the expected ideological position of their parties’ winning candidates elsewhere. The association with a party that is not attuned with a district’s ideological leanings may be poisonous for a candidate even if his own policy positions are tailor-made for his district.

Consider, for example, Lincoln Chafee, the U.S. senator from Rhode Island from 1999 to 2006. In spite of being a Republican, Chafee had taken a number of moderate and liberal positions that brought him in line with voters in his state.\textsuperscript{2} In the 2006 election, “exit polls gave Senator Lincoln Chafee, a popular moderate Republican from a long-admired political family, a 62 percent approval rating. But before they exited the polls, most voters rejected him, many feeling it was more important to give the Democrats a chance at controlling the Senate. […] ‘I’m caught between the state party, which I’m very comfortable in, and the national party, which I’m not,’ said Mr. Chafee.”\textsuperscript{3}

\textsuperscript{1}See Table 1 in Winer et al. (2014) for evidence that a significant share of U.S. Senate elections are non-competitive. Even in elections without an incumbent running, the winner received a vote share that was at least 20 percentage larger than the loser’s vote share, in 29.4 percent of U.S. Senate elections between 1922 and 2004.

\textsuperscript{2}For example, Chafee was pro-choice, anti-death-penalty, supported gay marriage and voted against the Iraq war (see http://en.wikipedia.org/wiki/Lincoln_Chafee).

His Democratic challenger Whitehouse “succeeded by attacking the instances in which Chafee supported his party’s conservative congressional leadership (whose personalities and policies were very unpopular, state-wide).”

In a review of 2006 campaign ads, factcheck.org summarized: “President Bush was far and away the most frequent supporting actor in Democratic ads […] The strategy is clear: whether they’re referring to a Republican candidate as a ‘supporter’ of the ‘Bush agenda’ or as a ‘rubber-stamp,’ Democrats believe the President’s low approval ratings are a stone they can use to sink their opponents […] Democratic Sen. Hillary Clinton of New York got the most mentions in Republican ads holding forth the supposed horrors of a Democratic-controlled Senate […] The runner-up is ‘San Francisco Liberal Nancy Pelosi,’ who is mentioned in at least 6 GOP ads as a reason not to vote for a Democrat who would in turn vote to make her Speaker of the House.”

We show that “contamination” – as we call this spillover effect – makes most legislative elections uncompetitive and results in an equilibrium in which party members are able to nominate their ideal candidate, rather than the ideal candidate of the district median voter, and nevertheless win by a healthy margin. The other party either cannot effectively compete because, even if it nominates a candidate at the ideal position of the median district voter, that voter still prefers the more extreme competitor because he is associated with an average party position that is ideologically preferred by the district median voter; or the other party could, in principle, compete, but prefers to nominate a losing extremist. The latter case arises if a winning moderate might “taint” the party’s position in the legislature.

Again, Lincoln Chafee provides an instructive illustration of this principle. Before the 2006 general elections, when Republicans had a clear majority in the Senate, conservative Republicans in Rhode Island mounted a primary challenge. Chafee defeated his challenger who had attacked him for not being sufficiently conservative only by a margin of 53 percent to 47 percent, and there is reason to believe that a majority of “real” Republicans would have preferred to replace a popular incumbent Senator with an extremist whose policy positions would have implied a very

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6Rhode Island’s open primary system allows registered Democrats and Independents to vote in the Republican
low likelihood of prevailing against the Democrat in the general election in Rhode Island. Our model explains why this behavior may be perfectly rational for policy-motivated Republicans: From their point of view, having Chafee as a member of the Republican Senate caucus caused more harm than good.

In contrast to the classical one-district spatial model, the ideological composition of districts in our model does not only influence the ideological position of elected candidates, but also the chances of parties to win. Thus, partisan incentives for gerrymandering are much larger in our model. We also show that gerrymandering or, more generally, the intensification of the median ideological preferences in some districts, also affects the political equilibrium in those districts where the median voter preferences remain moderate. Thus, our results imply that testing for the causal effect of gerrymandering on polarization in Congress is more complicated than the existing literature has recognized.

Our paper proceeds as follows. Section 2 reviews the related literature. In Section 3, we provide some stylized facts about statewide executive and legislative elections, and explain why they are hard to explain within the standard model that looks at legislative elections in different districts in isolation. In Section 4, we set up the general model, and the main analysis follows in Sections 5 and 6. We conclude in Section 7.

2 Related literature

Ever since the seminal work of Downs (1957), the position choice of candidates and the determinants of policy convergence or divergence are arguably the central topics in political economy models of elections. While the classical median voter framework identifies reasons for equilibrium platform convergence, there is a large number of subsequent variations of the spatial model of electoral competition that develop different reasons for policy divergence, such as policy motivation (e.g., Wittman 1983; Calvert 1985; Martinelli 2001; Gul and Pesendorfer 2009); entry deterrence primary. The New York Times article “To hold Senate, GOP bolsters its most liberal” (September 10, 2006) quotes a Republican consultant as saying that “There’s no doubt that if the primary was held only among Republicans, Chafee would lose. He would be repudiated by the Republicans who he has constantly repudiated.”
(e.g., Palfrey 1984; Callander 2005); and incomplete information among voters or candidates (e.g. Castanheira 2003; Callander 2008; Bernhardt et al. 2009).

Overwhelmingly, the existing literature looks at isolated elections – usually, two candidates compete against each other, and voters care only about their positions. In the probabilistic voting model (e.g., Hinich 1978; Lindbeck and Weibull 1987; Dixit and Londregan 1995; Banks and Duggan 2005), voters also receive “ideological” payoffs that are independent of the candidates’ positions. While, to the best of our knowledge, these authors do not interpret the ideological payoff as capturing the effects of the candidate being affiliated with a party, and therefore implicitly the party’s other legislators’ policy positions, this is a possible interpretation. However, the “ideology shock” in these models is exogenous, so that the main point of interest in our model – How does the fact that policy is determined within the legislature, rather than unilaterally by the local candidate, affect both the candidates’ equilibrium positions and the voters’ choice between local candidates? – cannot be analyzed in these models.

Our model belongs to the class of differentiated candidates models (Soubeyran 2009; Krasa and Polborn 2010a,b, 2012, 2014; Camara 2012). In these models, candidates have some fixed “characteristics” and choose “positions” in order to maximize their probability of winning. Voters care about outcomes derived from a combination of characteristics and positions. In contrast to existing differentiated candidates models, voters’ preferences over characteristics (i.e., the candidates’ party affiliations) are endogenously derived from the positions of Democrats and Republicans in other districts.

Erikson and Romero (1990) and Adams and Merrill (2003) introduce an influential model framework in the political science literature in which voters receive, in addition to the payoff from the elected candidate’s position, a “partisan” payoff from the candidate’s party affiliation. However, this partisan payoff is not derived from any multidistrict model, and is in fact orthogonal to the policy positions chosen by the candidates. The contribution of our model to this literature is to show that one can interpret it as providing a microfoundation for these partisan payoffs: The association of the local candidates with the two parties matters because the parties in the legislature determine the implemented policy, and so the fact that voters care about the candidates’ party
labels is perfectly rational in our model, and the degree to which it matters for voters depends on the equilibrium polarization between the parties’ candidates in other districts.

The legislative part of our model assumes that parties in Congress have a strong influence on policy outcomes. A significant number of models explain why parties matter. Conditional party government theory (Rohde, 2010; Aldrich, 1995) and endogenous party government theory (Volden and Bergman, 2006; Patty, 2008) argue that party leaders can use incentives and resources to ensure cohesiveness of their party. Procedural cartel theory (Cox and McCubbins, 2005) argues that party leadership can at least enforce voting discipline over procedural issues, and Diermeier and Vlaicu (2011) provide a theory where legislators endogenously choose procedures and institutions that lead to powerful parties. All these models of the importance of parties in Congress take the preference distribution of legislators as exogenously given, while our model provides for an electoral model and thus endogenizes the types of elected legislators.

Since we assume that the nomination decision is made by a policy-motivated party median voter, our model is related to the literature on policy-motivated candidates pioneered by Wittman (1983) and Calvert (1985), who assume that candidates are the ones who are policy-motivated and get to choose the platform that they run on. In our model, the effective choice of platform is made by the primary election median voter, but this change does not substantively affect the analysis. This approach is also taken by Coleman (1972) and Owen and Grofman (2006). To our knowledge, no paper in this literature analyzes policy-motivated policy selectors in the type of “linked” elections in different districts that we focus on.

Our results are relevant for the large empirical literature that analyzes how primaries, the ideological composition of districts and especially the partisan gerrymandering of districts affects the ideological positions of representatives in Congress (e.g., Lee et al. 2004; McCarty et al. 2009; Hirano et al. 2010). Most empirical papers in this literature do not include a formal model from which they derive predictions about the “expected” correlations, but rather take the intuition from the isolated election model and simply transfer them to the setting of legislative elections. For

Implicitly, we assume that either candidates can commit to an ideological position in the primary, or that candidates are citizen-candidates with an ideal position that is common knowledge.
example, there is a general expectation in the empirical literature that the positions of district representatives, i.e. U.S. Senators or House members, measured by their DW-Nominate score should more or less reflect the conservativeness of their districts. Our model shows that this transfer of results derived in the isolated-election model to legislative elections is not always justified, and that the candidates’ equilibrium positions may correspond to the preferences of the parties’ respective primary electorates rather than those of the district median voter.

3 Consistent lopsided elections: A puzzle for the single-district model

In this section, we argue that the influence of the electorate’s preference distribution on the parties’ performance is substantially larger in legislative elections than in executive ones. This stylized fact is puzzling when viewed through the lens of the simplistic one-district spatial model which does not distinguish between executive and legislative elections. As we show, one can interpret our model as a resolution of this puzzle.

3.1 Some stylized facts

The simplest Downsian model predicts that both candidates in a plurality rule election choose their position at the median voter’s ideal point, so that all voters are indifferent between the candidates. A rather liberal or conservative district should not provide a particular advantage – in terms of the probability of winning the district – to Democrats or Republicans. In Section 3.2, we look at somewhat more sophisticated one-district models of candidate competition, but argue that this intuition is quite robust.

In practice, it is well known that the ideological preferences of voters do affect the electoral chances of the different parties’ candidates – we talk of “deep red” (or blue) states, implying that the candidates of the ideologically favored party have a much clearer path to victory than their opposition.
However, we now argue that the voters’ ideological preferences have a substantially larger effect in legislative elections than in executive ones. To demonstrate this phenomenon, we consider Gubernatorial and U.S. Senate elections from 1978 to 2012. Both of these types of contests are state-wide races, but evidently, Gubernatorial elections are for executive positions while Senate elections are for legislative ones. Consistent with the empirical literature, we measure the median state ideology by its Partisan Voting Index (PVI), which is calculated as the difference of the state’s average Democratic and Republican Party’s vote share in the past two U.S. Presidential elections, relative to the nation’s average share of the same.8

The dependent variable is the difference between the Democrat’s and the Republican’s vote share of the two party vote in a particular election. In addition to the main independent variables of interest (PVI and $PVI \times \text{Senate election}$), we use incumbency dummies and year fixed effects in order to control for the electoral advantage of incumbents, and for election-cycle national shocks in favor of one party.

Table 1 summarizes the results, with the first column as the baseline case (all years since 1978, all states). For Gubernatorial elections (the omitted category), the PVI coefficient indicates that a one point increase in the Democratic vote share in Presidential elections increases the Democratic gubernatorial candidate’s vote share only by about 0.519 points. In contrast, in Senate elections, the same ideological shift increases the Democratic Senate candidate’s vote share by $0.519 + 0.645 = 1.164$ points, more than twice the effect in Gubernatorial elections; evidently, the difference between executive and legislative elections is substantial and highly significant. The remaining three columns confirm the qualitative robustness of this difference if we restrict to elections after 1990 and if we exclude the political South.9

A coefficient of about 1 for Senate elections is quite remarkable — if Senate candidates were

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8For example, if, in a particular state, Democratic presidential candidates run ahead of Republicans by 7 percent (on average in the last two elections), while nationally, Democratic candidates win by 3 percent (in the same two elections), then the state has a PVI of $7\% - 3\% = 0.04$. Also note that vote shares are calculated relative to the two-party vote, i.e., votes for minor parties are eliminated before the vote share percentages are calculated. See http://cookpolitical.com/application/writable/uploads/2012_PVI_by_District.pdf for the PVI based on the 2004 and 2008 Presidential elections.

9The reason for excluding the South is that, at least until the 1990s, there were a lot of conservative Southern Democrats in state politics in the South, so it is useful to check that our results are not just driven by this region of the country.
Table 1: Senate and Gubernatorial elections

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<tr>
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<tbody>
<tr>
<td>PVI</td>
<td>0.519***</td>
<td>0.589***</td>
<td>0.529***</td>
<td>0.614***</td>
</tr>
<tr>
<td></td>
<td>(0.111)</td>
<td>(0.124)</td>
<td>(0.117)</td>
<td>(0.132)</td>
</tr>
<tr>
<td>PVI × Senate</td>
<td>0.645***</td>
<td>0.596***</td>
<td>0.597***</td>
<td>0.514***</td>
</tr>
<tr>
<td></td>
<td>(0.149)</td>
<td>(0.167)</td>
<td>(0.156)</td>
<td>(0.177)</td>
</tr>
<tr>
<td>N</td>
<td>1103</td>
<td>702</td>
<td>871</td>
<td>553</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.551</td>
<td>0.595</td>
<td>0.571</td>
<td>0.62</td>
</tr>
</tbody>
</table>

*** indicates significance at the 1% level.

Additional explanatory variables used: Election type (Senate or Governor), year dummies, and incumbency status.


hard-wired at their Presidential party position, irrespective of whether such a position is competitive in their respective state, then this should result in a coefficient of (about) 1. Any degree of willingness of the disadvantaged candidate to adjust his position to better fit the state’s voter preferences should reduce the advantage of the opponent, and thus the estimated coefficient. Somehow, only gubernatorial candidates appear (at least to some extent) capable of such a position adjustment, while Senate candidates are not.

3.2 Inconsistency with the simple single-district model

These stylized facts are difficult to reconcile with the standard model of political competition that implicitly assumes that the electoral competition between the two candidates in each district is not influenced by what happens outside the district. Specifically, it is very difficult to set up a one-district model in which a particular party wins almost certainly, and does so with a substantial winning margin.

Without loss of generality, let the median voter be located at zero and the party medians at $m_D$ and $m_R$. Even if party medians are far apart from each other, parties have to nominate relatively moderate candidates in order to remain competitive. This is obvious for the model without
uncertainty where both parties nominate candidates that maximize the median voter’s utility, i.e., \( x_D = x_R = 0 \), and both parties have equal vote shares, even if one party’s ideal point is substantially closer to the median voter’s ideal position than the opposition’s. Since this is true for arbitrary ideal positions of the parties, it implies that, even if party members become more extreme, i.e., \( m_D \) moves to the left and \( m_R \) to the right, the equilibrium policies remain moderate, and the margin of victory is close to zero (if the distribution of voter types is continuous). Statement 2 of Proposition 2 in Section 6 below shows that this insight also extends to the case with uncertainty about the median. In particular, even if the positions of the primary voters \(-m\) and \( m\) go to \(-\infty\) and \(+\infty\), the position of the candidates remain moderate (i.e., bounded).

Thus, if party members become more extreme then the Downsian model predicts at most a small effect on policy: Party members continue to nominate moderate candidates, and both parties receive approximately one-half of the votes. In contrast, it is a widespread view that the rise of activist and more ideological party members has resulted in more extreme candidates being nominated for office (e.g., Fiorina et al. (2006)). Further, many political commentators and scholars diagnosed a rise in polarization between the two parties. In order to generate such polarization in a standard model with policy-motivation, uncertainty, e.g., about the median voter’s location, must increase. In other words, we would need that the quality of political polls deteriorates over time, which is somewhat implausible.

The prediction that both candidates in executive elections (i.e., those where the elected candidate can set policy without being tied to their party) will be competitive is borne out in U.S. presidential elections. For example, between 1988 and 2012 the difference between the Republican and Democratic vote share in Presidential elections was between -5.6% and 7.7%, with a median of -0.5%.\(^{10}\) Furthermore, the results of Table 1 above indicate that Gubernatorial elections, even in ideologically skewed states, are at least considerably more competitive than the Presidential election in those same districts. In contrast, as shown above, many legislative elections result in one party receiving a substantially higher vote share than its opposition.

\(^{10}\) U.S. presidents are elected in many districts through the electoral college system rather than by a majority of the popular vote. However, to the extent that state ideological leanings are fixed and known, the objective for the parties’ primary electorates is essentially to nominate a candidate who can win in the decisive swing state.
Can we generate lopsided outcomes if one of the candidates has a “valence” advantage? Suppose that the net-valence of the Republican candidate, \( v_R - v_D \), is \( \varepsilon > 0 \). Then, in equilibrium, \( x_D = 0 \) and \( x_R = \sqrt{\varepsilon} \). Given these positions, the median voter at 0 is again indifferent: If he votes for \( D \) the utility is 0, if he votes for \( R \) the utility is \(-x_R^2 + \varepsilon = 0\), but in equilibrium he supports the Republican who wins the election. If voter types are continuously distributed, then the margin of victory is (almost) zero.

In order to generate a vote margin that is bounded away from 0, one would have to assume a valence advantage that is so large that the median voter prefers the favored candidate even if he is located at his party median’s ideal point, and the opposition candidate is located at the median voter’s ideal point. Usually, valence is interpreted as a small personal preference; for rational voters, the most of the utility-relevant payoff from a legislature should come from the laws the legislature enacts, rather than from legislators’ valences. Furthermore, it would be hard to understand why one party should be consistently much better than the other party in terms of the quality of candidates that they select, and why that party should necessarily be the one that is ideologically closer to the median voter.

It is easy to show that, if there is some uncertainty about the median, then the higher valence candidate wins with probability close to one, but the margin of victory is close to zero. However, if the uncertainty is not too large, then the winning margin is close to zero. The reason is that in any Downsian model without uncertainty, the median voter is indifferent between the candidates, while with some uncertainty he is close to indifferent, and hence the electorate splits close to 50-50. One of the key insights of this paper is that this is not longer true in a multi-district setting.

4 Model

The determination of policy in the legislature. We consider a polity divided into a set of districts \( I \), where the number of districts \( i \in I \) is either a continuum or an odd number, \( 2n + 1 \). Each district \( i \) elects one representative to the legislature, who is characterized by a position \( x_i \). All candidates are attached to one of two parties, called Democrats and Republicans.
Let $x = (x_{i,D}, x_{i,R})_{i \in I}$ be the positions of candidates in all districts, and let $X$ be the set of all such positions. Let $k_i \in \{D, R\}$ denote the party of the winning candidate in district $i$, and $K = (k_i)_{i \in I}$. Then the policy selection function $\xi$ maps the candidates’ ideal positions and the election outcomes in districts into an implemented policy, i.e. $\xi : X \times K \rightarrow \mathbb{R}$. For example, suppose that $\xi$ selects the preferred policy of the majority party’s median legislator. The intuition for our results can most easily be understood with this policy selection function.

**Voter utility.** The utility of a voter with ideal position $\theta$ from district $i$ is

$$u_\theta(x, K, v_i) = -(1 - \gamma)(\xi(x, K) - \theta)^2 - \gamma(x_i - \theta)^2,$$

where $\gamma \in [0, 1]$. Here, the first term is the utility from the legislature’s policy, and the second term is the utility from the policy position of district $i$’s representative. Note that $\gamma \rightarrow 1$ corresponds to the standard case where voters only care about the election outcome in their own district, and $\gamma \rightarrow 0$ means that voters care only about the implemented policy and not their own representative’s position per se.

Ex-ante there is uncertainty about the state of the world, captured by a collection of states $(\omega_i)_{i \in I}$, one for each district, where $\omega_i \in \Omega_i$ captures the uncertainty pertaining to the ideal position of district $i$’s median voter.

**Timeline.** The game proceeds as follows:

**Stage 1** In each district, the local members of each party simultaneously select their candidates, who are then committed to their policies $x_{i,D}, x_{i,R} \in \mathbb{R}$. We assume that the nomination process can be summarized by the preference parameter of a “decisive voter,” whom we take to be the median party member in the district, and whose ideal position is denoted $m$ for Republicans and $-m$ for Democrats.

**Stage 2** In each district $i$, the median voter $M_i(\omega_i)$ is realized, observes the candidate positions $x_{i,D}$ and $x_{i,R}$ in his own district, and chooses whom to vote for. Note that, for the other districts,
the median voter does not observe the candidates’ positions, but he has correct expectations in equilibrium.

Stage 3 The elected candidates from all districts form the legislature, which determines the implemented policy via function \( \xi \), and payoffs are realized; no strategic decisions take place in this stage.

**Equilibrium concept.** We consider Perfect Bayesian Nash equilibria of this game. For completeness we now describe in detail the strategy spaces for the players. In a slight abuse of notation, let 
\[ k_i : (M_i(\omega_i), \omega_i, x_{i,D}, x_{i,R}) \to \{D, R\} \]
denote the voting strategy of district \( i \)’s median voter \( M_i(\omega_i) \) when choosing between candidates \( x_{i,D} \) and \( x_{i,R} \) in state \( \omega_i \).

**Definition 1** A collection of policies \( x \in X \) and of voter strategies \( k(x, \omega) = (k_i)_{i \in I} \) is a pure strategy equilibrium if and only if

1. for every district \( i \) and for every state \( \omega_i \) in that district, the median voter \( M_i(\omega_i) \) chooses his optimal candidate: If \( w_i(M_i(\omega_i), \omega_i, x_{i,D}, x_{i,R}) = P_i \) and \( \bar{P}_i \) denotes the other candidate, then

\[
E_{\omega_i} u_{M_i(\omega_i)}(x, k_i(\cdot), k_{-i}(\cdot), v_{i,P_i}(\omega_i)) \geq E_{\omega_i} u_{M_i(\omega_i)}(x, k_i'(\cdot), k_{-i}(\cdot), v_{i,P_i}(\omega_i)) \text{ for all } k'_i \in \{D, R\}
\]

where the expectation is taken over the realization of uncertainty in the other districts;

2. the candidate choices of the decisive primary voters \( m_{i,P} \in \{-m, m\} \) are optimal for them, respectively:

\[
E_{\omega} u_{m_{i,P}}(x, k_i(\cdot), v_{i,k_i}) \geq E_{\omega} u_{m_{i,P}}(x_{-i,P}, \bar{x}_i,P, k_i(\cdot), v_{i,k_i}) \text{ for both parties } P \text{ and all alternative positions } \bar{x}_i,P
\]

where the expectation is taken with respect to the ex-ante distribution of \( \omega \).
5 Equilibrium analysis in the case without uncertainty

We start by considering the setting of a continuum of districts, modeling a large legislature in which the influence of each individual representative on their party’s position is negligible. The location of the district median voters are uniformly distributed (the argument also works for more general distribution at the cost of more cumbersome notation). We assume that primary voters have the same ideal positions of $m_D < 0$ (Democrats) or $m_R > 0$ (Republicans) in all districts. Since $m_D$ and $m_R$ are arbitrary, it is without loss of generality to assume that the uniform distribution of district median voters is centered at 0. In other words, the set of districts is represented by an interval $[-\mu, \mu]$. We also assume that $\mu < -m_D, m_R$ (i.e., in all districts, general election median voters are less extreme than primary voters) and that $\xi(\cdot)$ picks the median position of the majority party in the legislature.

Let $x_{i,D}$ and $x_{i,R}$ be the positions of the local candidates for Congress, and let $\xi_D$ and $\xi_R$ be the expected median Democratic and Republican positions in Congress (the voters’ expectations are correct in equilibrium). If $\xi_P, P \in \{D, R\}$ denotes the policy implemented by party that wins the majority in the legislature and if $x_{i,P_i}$ denotes the policy position of the winner of the district’s election, then (1) implies that a voter’s utility is given by $-(1 - \gamma)(\xi_P - \theta)^2 - \gamma(x_{i,P_i} - \theta)^2$.

Now consider the decision problem of the median voter in district $i$, located at position $M_i$. We will be looking for a symmetric equilibrium in which liberal districts, i.e. those with $M < 0$, vote for Democrats, and conservative districts ($M > 0$) vote for the republican candidates. In such an equilibrium, each voter’s choice is decisive for which party wins the majority of the legislature.

Consider a conservative district $i$ with a median voter $M_i > 0$ as show in Figure 1. If both local candidates locate at the district median $M_i$, then the median voter strictly prefers the Republican.
since $\xi_R$ is closer to $M_i$ than $\xi_D$. Thus, primary voters can nominate a more conservative candidate, indicated by the arrow pointing to the right. The maximally conservative candidate who still wins in this district makes the median voter indifferent to the Democratic candidate. If the median voter in the district selects the Republican, which he does in equilibrium, then the votes split perfectly and each party gets the majority with probability 0.5. If, instead, he selects the Democrat, the Democrats win the legislature. In order to be indifferent between these two choices, the following equation must hold.

$$-\gamma(M_i - x_{i,R})^2 - \frac{1 - \gamma}{2}(M_i - \xi_R)^2 - \frac{1 - \gamma}{2}(M_i - \xi_D)^2 = -(1 - \gamma)(M_i - \xi_D)^2 - \gamma(M_i - M_i)^2. \quad (2)$$

Using symmetry (i.e., $\xi_D = -\xi_R$) and solving for $x_{i,R}$ gives

$$x_{i,R} = M_i + \frac{2(1 - \gamma)}{\gamma} \sqrt{M_i \xi_R}. \quad (3)$$

Similarly, it follows that, for a district with $M < 0$,

$$x_{i,D} = M_i - \frac{2(1 - \gamma)}{\gamma} \sqrt{M_i \xi_D}. \quad (4)$$

The right-hand side of (3) and (4) consists of two terms: The first one is the median voter’s ideal position, and the second one is the leeway that the Republican primary voter has to choose a more conservative candidate because the median voter in a district $M_i > 0$ prefers the position of other Republicans, $\xi_R$ to that of the other Democrats, $\xi_D$. This leeway arises because, even though the Democratic candidate in district $i$ proposes $M_i$’s ideal position, he is contaminated by his association with the Democratic party whose representatives are on average too liberal for the taste of the conservative median voter in district $i$. The extent of this leeway depends on the relative importance of this effect as captured by $1 - \gamma$, and on the extent of the median voter’s preference for the generic Republican position, which depends on the median voter’s ideal position $M_i$.

Of course, if the right-hand side of (3) is larger than $m_R$, then the primary election median voter simply nominates his ideal candidate $x_{i,R} = m_R$, and the median voter has a strict preference for the
Republican candidate, so that this candidate receives a strict supermajority of votes.

In equilibrium the legislature’s policy \( \xi_R \) if Republicans win a majority must be equal to the median Republican legislator position in Congress. Since (3) is evidently increasing in \( M_i \), and since Republicans win exactly the districts in \([0, \mu]\), it is equal to the position of the Republican legislator elected from district \( \mu/2 \). Substituting, we get

\[
\xi_R = \min \left\{ \frac{\mu}{2} + \sqrt{\frac{2(1 - \gamma)}{\gamma} \sqrt{\frac{\mu}{2} \xi_R}}, m_R \right\} 
\]

Solving for an interior solution (i.e., in the first term in (5)), and using the fact that the distributions of medians is symmetric around zero we get

\[
\xi_R = -\xi_D = \frac{\mu}{2} + \frac{\sqrt{1 - \gamma^2}}{\gamma}
\]

For the case of \( \gamma = 1 \), i.e., if voters care exclusively about their local candidates and there is no spillover from party positions, we simply get that each elected candidate is located at the median voter’s ideal position, and so the median Republican in Congress is just the median voter of the median conservative district, \( \mu/2 \), and the median Democrat in Congress is the median voter of the median liberal district, \( -\mu/2 \).

However, the smaller is \( \gamma \), the larger is the factor in square brackets in (6). For example, if \( \gamma = 1/2 \) and that the district medians are between \(-1 \) and \( 1 \) then \( \xi_R = -\xi_D \approx 1.87 \), generating a lot more polarization in Congress compared to the case where \( \gamma = 1 \) and hence \( \xi_R = -\xi_D = 0.5 \).\(^{11}\)

The equilibrium winning position in a given district depends on the extent of preference heterogeneity between districts. We can think of an increase in \( \mu \) as a increased preference heterogeneity between districts, maybe brought about by gerrymandering that generates more “secure” Democratic and Republican districts. Holding constant the preferences of a particular moderate district (i.e., \( M_i \)), how does this change in other districts affect the equilibrium position of the winning

\(^{11}\) Of course, in order for this to be an interior equilibrium we must have \(-m_D, m_R > 1.87\).
candidate? If we have an interior value of $\xi_R$ and substitute in (3), we get

$$x_{i,R} = M_i + \sqrt{2\mu \left( 1 - \gamma + \sqrt{1 - \gamma} \right)}$$

Thus, an increase in the heterogeneity between district median voters leads to more extreme positions of winning candidates even in those districts where the voter preference distribution remains unchanged.

We now summarize these results.

**Proposition 1** Suppose there are a continuum of districts $I$, indexed by the position of the median voter. Median voters are uniformly distributed across districts between $-\mu/2$ and $\mu/2$. Suppose that the primary voters are located at $m_D < 0 < m_R$ in all districts.

1. Suppose that $|m_D|$ and $m_R$ are larger than the right-hand side of (6). Then there exists an equilibrium in which both parties receive the same number of seats, each median voter is decisive, and the legislature’s equilibrium policies $\xi_D$ and $\xi_R$ are given by (6).

2. If voters put more emphasis on the party’s policy, i.e., if $\gamma$ is decreased, then polarization (i.e., $\xi_R - \xi_D$) increases.

3. If $m_D \leq \xi_D$ but close to $\xi_D$ (as given (6)) then in all districts with sufficiently liberal median voters, the median voter strictly prefers the Democrat, and the Democrat wins the district with a supermajority. Similarly, if $m_R \geq \xi_R$ but close to $\xi_R$ then in all sufficiently conservative districts the median voter strictly prefers the Republican candidate, and Republicans win a supermajority.

4. Suppose that $\mu$ increases. Then the positions of winning candidates in each district that are not at the ideal position of the primary voter become more extreme.
5.1 Discussion

Extremism in the legislature. The standard intuition in the existing literature is based on a naive application of the single district model and suggests that two-party competition leads both parties to propose positions close to the ideal of the respective district median voters. Proposition 1 shows that this standard intuition does not carry over to our model. Rather than an assembly of district median voters from all over the country, Proposition 1 predicts a legislature in which representatives are more extreme than the median and in some district as extreme as the median primary voter in the majority party (statement 3). General elections lose their disciplining and moderating power because voters correctly anticipate that both parties are extreme.

Since one of these two extremist parties necessarily wins and therefore controls the legislature, we should not expect that this legislature necessarily adopts policies with broad popular support, as long as they are unpopular with their own party base. For example, an October 7, 2013 Washington Post opinion poll shows that registered voters disapproved of the Republican party shutting down the government by 71 to 26. Thus, it is very likely that the median voters in most districts – even most of those held by Republican House members – opposed shutting down the government, but among voters who identify as Republicans, there was a 52-45 majority in favor of the shutdown, and it is likely that, among those voters who actually vote in Republican primaries, there was an even larger majority in favor of the shutdown.

There are media reports that many Republican representatives “would have liked” to end the government shutdown much sooner, but were afraid that taking this position publicly would put them at risk in their district primary. For example, former House Speaker Dennis Hastert said in an October 7, 2013 interview with NPR: “It used to be they’re looking over their shoulders to see who their general [election] opponent is. Now they’re looking over their [shoulders] to see who their primary opponent is.”

Our model shows that this fear is justified: Primary voters who refuse to renominate a “moderate” and replace him with an extremist are not irrational – because even in a relatively moderate
district, an extremist is very likely to win. This makes the primary threat so credible.\footnote{Alternative explanations for non-median policy outcomes include lobbying and differential preference intensity. Arguably, neither of these alternative explanations is plausible for the government shutdown.}

**Safe and lopsided districts.** As mentioned in the introduction, the standard spatial model in which voters look at their district’s candidate positions in isolation cannot explain why there are “safe” districts that are essentially guaranteed to be won by one party’s candidate. Statement 3 of Proposition 1 shows that the median voters in less centrist districts have a strict preference for the election winner in their district, implying that winning margins are bounded away from zero, even though there is no uncertainty. Thus, without appealing to a large “party valence” or “partisanship” that is also unrelated to policy, our model generates districts that are safe and lopsided (i.e., the winner’s percentage is significantly larger than 50 percent).

Hence, our model can explain why, for example, a Democrat has a hard time to be elected in Wyoming, even if he adopted a platform that would actually be preferred by the median voter in his district to the platform of the Republican. The contamination effect produces exactly such a result because voters in a biased district are reluctant to vote for the representative of the party whose representatives from other districts are unpopular. For example, a vote for the Democrat in Wyoming would also strengthen the probability that Democrats from other parts of the country get the chance to enact legislation, and their positions are too liberal to be palatable for Wyoming voters.

**Changes in the Distribution of Voters:** The last statement of Proposition 1 shows that if the districts become more heterogeneous, then equilibrium policies become more extreme. This finding has important implications for the empirical analysis of the effects of gerrymandering. For example, McCarty et al. (2009) argue that, while Congress has become more polarized in a time during which electoral districts became more heterogeneous thanks to gerrymandering, this is merely a

\footnote{The lobbying explanation (a strong lobby in favor of the minority position is able to “buy” the support of legislators) requires that benefits on the minority side are highly concentrated, while the issue is a relatively minor issue for most voters. The intensity explanation requires that minority voters care more about the issue, but according to the same Washington Post poll cited above, 12 percent of registered voters “strongly approve” of the shutdown, 14 percent “approved somewhat,” while 53 percent “strongly disapprove” and 18 percent “disapprove somewhat.” Thus, intensity about the issue appears higher among those who disapprove.}
temporal coincidence. They draw this conclusion by arguing that also legislators from districts that were not gerrymandered (e.g., in the Senate, or in House districts in small states where there is less scope for gerrymandering) became more partisan in their voting behavior. Our result shows that this argument may be flawed because the candidates in districts that are not directly affected by gerrymandering (in the sense that their median voter’s ideal position remains constant) nevertheless choose more extreme positions because of the evolution of more extreme districts elsewhere. We will return to this argument in Section 6.

**Probability that Districts are Pivotal.** The equilibria considered in Proposition 1 have the feature that the election outcome in any single district determines with strictly positive probability which party wins the majority of seats. In particular, in equilibrium each party wins with 50% probability, and if, for example, a Democratic district elects a Republican, then Republicans win the majority with probability 1. If, a single district is never pivotal, then voters will only choose according to their preference for their individual candidate, and the candidate would therefore have to locate at the district median. In a model without uncertainty district medians can only be pivotal if one party has bare majority. In the following section we allow the position of the district median to be uncertain, which will automatically generate a situation in which none of the parties can be certain to win a majority. We will see that even if the probability of a district to be pivotal is very small, policies will not just be marginally more extreme. In other words, even in a model where the pivot probability goes to zero as we increase the number of districts, the equilibria of our model will not converge to the moderate equilibria of the standard one-district model.

6 The Effect of Contamination on Competitive Districts

In the model without uncertainty analyzed in the last section, all districts are safe for one of the parties, and parties nominate extreme candidates. We now address what happens with competitive districts where both candidates have a strictly positive probability of winning. In this case, parties have a non-trivial trade-off between nominating ideologically close candidates and nominating
moderates who have a better chance of winning the election, and thus giving the party a majority in the legislature.

This modification of the model is important in order to take the model closer to the data: In reality, there are some “swing districts” in which both Democrats and Republicans are competitive, and the behavior of representatives from these swing districts plays a key role in the empirical analysis of the effects of “gerrymandering.” How does the creation of districts that are internally ideologically homogeneous (either very conservative or very liberal) and in which representatives are conceivably much more afraid of a primary challenge from within their own party than of the opposition candidate in the general election affect equilibrium policy divergence between parties?

Many observers suspect that increased polarization in Congress is caused by gerrymandering. However, there is an empirical literature in political science that claims that gerrymandering has no significant effect on polarization between the two parties in Congress, arguing that polarization and gerrymandering increase at the same time, but that there is no causal relationship between the two. In this view, the principal driver of polarization between parties is “sorting” in the sense that conservative districts are increasingly represented by Republicans. McCarty et al. (2009) use the DW-Nominate score of representatives as their dependent variable, and analyze how it correlates with district characteristics (in particular, the district’s PVI as a proxy of its ideological bent) and the party affiliation of the district representative. They argue (p. 667) that “for a given set of constituency characteristics, a Republican representative compiles an increasingly more conservative record than a Democrat does. Gerrymandering cannot account for this form of polarization” because this change in behavior occurs even in swing districts.

Implicitly, this argument assumes that the only effect of gerrymandering is on the equilibrium positions of candidates in those districts that are directly affected: For example, if a district is gerrymandered to be more conservative, then positions of candidates in that district will be more conservative, but there are no spill-over effects on the positions of candidates in other districts that remain moderate. In order to analyze whether this is a logically justified argument, we extend the framework of the last section to allow for uncertainty about the district median voter positions.

Consider again $2n + 1$ districts, with primary voters located at $-m$ and $m$, respectively. In
Proposition 2 Suppose that \( k_D, k_R < (n - 1)/2 \). Let \( \Phi_0 \) be a symmetric distribution with mean zero. Suppose that the distribution of the median voter in each competitive district is i.i.d. with distribution \( \Phi(x) = \Phi_0(x - M) \), and that \( \xi(x, H) = \text{median}(\{x_{i,z(K)}\}_{i \in H(K)}) \) (i.e., the implemented policy is equal to the median majority legislator’s position).

Then, for any \( M \) in a neighborhood of zero and for small \( \gamma \), there exists a unique equilibrium that is symmetric across competitive districts, i.e., \( x_{D,i} = x_D(M) \) and \( x_{R,i} = x_R(M) \) for all districts \( i \) with \( k_D < i < 2n + 1 - k_R \).

1. \( x_D(M) \) and \( x_R(M) \) are independent of the number of competitive districts and independent of \( \gamma \). Furthermore, \( x_D(M) \leq x_R(M) \) for all \( M \).

2. \( x_D(0) = -\frac{m}{1+2\phi_0(0)m} \), and \( x_R(0) = \frac{m}{1+2\phi_0(0)m} \).

3. \( x_R(M) - x_D(M) < x_R(0) - x_D(0) \) for \( M \neq 0 \) in a neighborhood of zero.

4. The probability that \( R \) wins is strictly increasing in \( M \) for \( M \) in a neighborhood of zero.

5. Districts \( i \leq k_D \) are safe for Democrats, who get elected with position \(-m\), while districts \( 2n + 1 - k_R \) to \( 2n + 1 \) are safe for Republicans who get elected with position \( m \).

Proposition 2 has interesting implications for the effects of gerrymandering. Remember that we normalize such that the voter located at 0 is indifferent between the positions of the Democratic
and Republican primary voters at $-m$ and $m$, respectively. Start from a situation where all districts are identical and the expected median voter $M$ is located close to 0, and suppose now that district boundaries are redesigned such that there are some Democratic leaning districts in which the median voter is always to the left of zero, and some Republican districts where the median voter is strictly to the right of zero. According to Proposition 2 these districts are safe for the Democrats and Republicans, respectively. In the remaining competitive districts, there could be potentially three distinct effects.

First, the gerrymander may shift the distribution of median voters in competitive districts. For example, if both parties have the same number of safe districts, but the Republican safe districts are more moderate than the Democratic ones, then the expected median voter $M$ in the competitive districts shifts to the right, and Proposition 2 indicates how this impacts the elections there. The winning probability for the Democrat decreases, and if originally $M < 0$, then candidates are more polarized, while the reverse is true if $M > 0$.

Second, if one party has more of the safely-gerrymandered districts than the other party, then it has an obvious advantage in winning a majority of the legislature, since it needs to win fewer of the competitive districts. Potentially, this increased winning probability could affect the behavior in the competitive districts, but interestingly, Proposition 2 shows that it does not, and the same is also true in the case discussed below that representatives from the gerrymandered districts affect policy.

Third, more extreme legislators of the same party can “contaminate” the candidates in the swing districts, as discussed above. However, this effect is not present in Proposition 2 because, by assumption, there are only few gerrymandered districts, so that the members from those districts have no influence on national policy. In Proposition 3, we modify this assumption to analyze the case where the policy is determined by the candidates elected from safe districts.

In Proposition 3, we assume that the median voters in the safe districts are at $-m$ and $m$ respectively so that there is a unique equilibrium position for the winners in these districts at $-m$ and $m$, respectively. We do this to simplify the proof, and because the point is anyway only to provide a
possibility result.\footnote{It is clear that if the median district voters are close to $-m$ and $m$, then they will strictly prefer to vote a Democrat or Republican, respectively. As a consequence, the primary voters will select candidate at their ideal points $-m$ and $m$, respectively.}

**Proposition 3** Make the same assumptions as in Proposition 2, except that now $k_D, k_R > (2n+1)/3$, and that the district median voters are at $-m$ for $i \leq k_D$ and $m$ for $i \geq 2n + 1 - k_R$.

Then for any $M$ in a neighborhood of zero and for small $\gamma$, there exists a unique equilibrium that is symmetric across competitive districts, i.e., $x_{D,i} = x_D(M)$, and $x_{R,i} = x_R(M)$ for all districts $i$ with $k_D < i < 2n + 1 - k_R$.

1. The candidates nominated in the competitive districts are given by

\[
x_D = -\frac{\Phi_0(-M)m}{\Phi_0(-M) + \phi_0(-M)m}, \quad x_R = \frac{(1 - \Phi_0(-M))m}{(1 - \Phi_0(-M)) + \phi_0(-M)m}.
\]

2. There exists $\bar{m} < \infty$ such that for all $M \neq 0$, and $m > \bar{m}$ polarization is larger than in the case with few gerrymandered district from Proposition 2.

Proposition 3 shows that polarization in the other districts affects equilibrium positions in the competitive districts. Provided that the difference between party ideal positions is sufficiently large, candidate position are further apart than in the case with few gerrymandered districts (the only exception to this is the non-generic case in which the expected median $M$ is exactly zero).

This is a surprising result. A superficial intuition would suggest that, if party ideal points are far apart from each other, each party should be very concerned with the possibility of the other party taking over the majority in the legislature, and therefore should do its utmost in order to compete in the moderate swing districts, by nominating very moderate candidates there.

This is certainly true, but only one part of the intuition. The countervailing force is that, if party positions are far apart from each other, then the position of the cutoff voter in the general election (i.e., the one who is indifferent between the two local candidates) becomes very inelastic with respect to their positions, as he understands that the main potential effect of his local choice
is the chance that it affects the identity of the majority party. The less elastic the cutoff voter reacts to changes in local candidates’ positions, the more the parties have an incentive to nominate candidates who are close to their respective ideal positions. For $M = 0$, these two effects exactly cancel while for $M \neq 0$, the inelasticity effect actually outweighs the effect that winning the election is more important for parties.

Our results show that gerrymandering a particular district does not just affect that district, but other districts as well. In particular, if the more extreme legislators from gerrymandered districts determine the national policy, then we should observe increased polarization in the remaining competitive districts, exactly the behavior noted by the empirical literature. Of course, the same arguments also apply to the Senate where there is obviously no “gerrymandering,” but where increased regional preference differences have created an increasing number of safe seats for the parties. More extreme candidates elected in these safe states impose an externality on the remaining competitive states, creating increased polarization in those states as well.

7 Discussion

Much of the existing literature on electoral competition in legislative elections implicitly assumes that voters evaluate their local candidates based on their positions, but not on the party label under which they run. Such a model implies that both parties nominate candidates who are very close to the preferences of the respective district median voters. Therefore, even in districts with rather extreme preferences, both parties’ candidates should be competitive, and the position of Democratic and Republican Congressmen elected from similar districts should be very similar. It is safe to say that these predictions are not borne out in reality, and to understand why this is the case is of first-order importance for our understanding of the American democratic system.

In this paper, we have developed a theory of candidate nomination processes predicated upon the notion that majority party legislators collaboratively influence policy. This assumption is appears reasonable and yields fundamentally different results.

In our model, a candidate’s association with candidates of the same party that run in other
districts generates an incentive for voters to focus less on the candidates’ own position positions when deciding whom to vote for — local candidates are “contaminated” by their party association. This leads to less competitive local elections, providing the ideologically favored party with the leeway to nominate more extreme candidates who are nevertheless elected. As a consequence, the equilibrium of our model can explain how electoral competition can beget a very polarized legislature.

Our analysis has two additional important empirical implications. First, it can explain why a district’s ideological preferences have a smaller partisan effect in elections in which a candidate has a more autonomous policy influence, such as elections for executive leadership positions than in legislative elections. Of course, in reality, even executive leader positions are not entirely autonomous, so there will be some contamination in executive elections as well, but we would expect this effect to be smaller than in legislative elections, and this expectation is borne out in our empirical analysis of Senate and Gubernatorial elections in Section 3.

Second, much of the existing empirical analysis of the effects of gerrymandering on polarization in Congress is implicitly based on applying a naive model in which voters care only about the local candidates’ positions. Such a model may lead to incorrect inferences about the importance of gerrymandering. For example, the ideal position of the district median voter often does not affect the equilibrium position of candidates at the margin in our model, but the total effect of gerrymandering on polarization in Congress may nevertheless be substantial (and actually be much larger than in the naive model). Thus, one cannot infer that gerrymandering does not matter for polarization in Congress from showing that there is no marginal effect of changes in district medians on ideological positions of legislators, and that the difference in voting records of Republicans and Democrats representing the same or very similar districts has increased. In general, an implication of our model for empirical work is that legislator behavior in different districts is intricately connected rather than independent, and this implies that one needs to be very careful with claims that difference-in-difference approaches can identify causation.
8 Appendix (for online publication)

Proof of Proposition 1. See text. ■

Proof of Proposition 2. Consider a particular competitive district, and let \( p_k \) be the probability that \( k \) of the remaining \( 2n \) districts vote Republican. Suppose that the Republican in district \( i \) deviates to policy \( y \). Since \( k_R < (n - 1)/2 \) the median policy of if the Republicans win remains \( x_R \). The the payoff of a voter at \( M \) from the Democrat is

\[
-(1 - \gamma) \left( \sum_{k=0}^{n} p_k (M - x_D)^2 + \sum_{k=n+1}^{2n} p_k (M - x_R)^2 \right) - \gamma(M - x_D)^2,
\]

while the payoff from the Republican is

\[
-(1 - \gamma) \left( \sum_{k=0}^{n-1} p_k (M - x_D)^2 + \sum_{k=n}^{2n} p_k (M - x_R)^2 \right) - \gamma(M - y)^2.
\]

Thus, we can conclude that the cutoff voter is given by

\[
M_R(x_D, x_R, y) = \frac{1}{2} \frac{(1 - \gamma)p_n(x_R^2 - x_D^2) + \gamma(y^2 - x_D^2)}{(1 - \gamma)p_n(x_R - x_D) + \gamma(y - x_D)}.
\] (9)

It follows immediately that

\[
M_R(x_D, x_R, x_R) = \frac{x_D + x_R}{2}, \quad \left. \frac{\partial M_R(x_D, x_R, y)}{\partial y} \right|_{y=x_R} = \frac{1}{2} \frac{\gamma}{(1 - \gamma)p_n + \gamma}.
\] (10)

Suppose by contradiction that \( x_R(\tilde{M}) < x_D(\tilde{M}) \). In this case liberals vote for the Republican and conservatives for the Democrat. The cutoff voter is located at \( (x_D(M) + x_R(M))/2 \). Now suppose the Republican’s position is changed to \( x_{R,i} = -M \). Then (10) implies that the cutoff voter becomes more liberal. Hence the probability that the Democrat is elected in district \( i \) increases. Thus, the probability that Democrats receive a majority in the legislature and policy \( x_D(\tilde{M}) \) is implemented increase, making the Republican primary voter at \( m \) strictly better off, as long as \( \gamma \) is sufficiently
small, a contradiction. Hence $x_R(\bar{M}) \geq x_D(\bar{M})$.

Since $\Phi(M_R(x_D, x_R))$ is the probability that the Democrat gets elected, the Republican primary voter solves

$$\max_y -\Phi(M_R(x_D, x_R, y)) \left(1 - \gamma \left(\sum_{k=0}^{n} p_k (m - x_D)^2 + \sum_{k=n+1}^{2n} p_k (m - x_R)^2\right) + \gamma (m - x_D)^2\right)$$

$$- (1 - \Phi(M_R(x_D, x_R))) \left(1 - \gamma \left(\sum_{k=0}^{n-1} p_k (m - x_D)^2 + \sum_{k=n}^{2n-1} p_k (m - x_R)^2\right) + \gamma (m - y)^2\right),$$

(11)

The first derivative with respect to $y$ is given by

$$- \phi(M_R) \frac{\partial M_R}{\partial y} \left((1 - \gamma)p_n (m - x_D)^2 - (m - x_R)^2\right) + \gamma (m - x_D)^2 - (m - y)^2\right)$$

$$+ (1 - \Phi(M_R)) 2\gamma (m - y)$$

(12)

The second derivative is

$$- \left(\phi(M_R) \frac{\partial^2 M_R}{\partial y^2} + \phi'(M_R) \left(\frac{\partial M_R}{\partial y}\right)^2\right) \left((1 - \gamma)p_n (m - x_D)^2 - (m - x_R)^2\right) + \gamma (m - x_D)^2 - (m - y)^2\right)$$

$$- 4\gamma \phi(M_R) \frac{\partial M_R}{\partial y} (m - y) - 2\gamma (1 - \Phi(M_R)).$$

(13)

Equation (9) implies

$$\lim_{\gamma \to 0} \frac{1}{\gamma} \frac{\partial M_R(x_D, x_R, y)}{\partial y} = - \frac{x_D + x_R - 2y}{p_n(x_R - x_D)}$$

$$\lim_{n \to \infty} \frac{1}{\gamma} \left(\frac{\partial M_R(x_D, x_R, y)}{\partial y}\right)^2 = 0,$$

(14)

and

$$\lim_{n \to \infty} \frac{1}{\gamma} \frac{\partial^2 M_R(x_D, x_R, y)}{\partial y^2} = \frac{1}{p_n(x_R - x_D)}.$$

(15)

Dividing (13) by $\gamma$, taking the limit for $n \to \infty$ and using (14) and (15) yields

$$- \frac{\phi(M_R)}{p_n(x_R - x_D)} p_n (m - x_D)^2 - (m - x_R)^2 - (1 - \Phi(M_R)) 2\gamma < 0,$$

(16)
since \( x_D < x_R \). Thus, the second order condition is satisfied for all \( y \) when \( \gamma \) is small.

Evaluating the first order condition (12) at \( y = x_R \) yields (after canceling the common terms and multiplying by 2)

\[
-\phi \left( \frac{x_D + x_R}{2} \right) \left( (m - x_D)^2 - (m - x_R)^2 \right) + \left( 1 - \Phi \left( \frac{x_D + x_R}{2} \right) \right) 4(m - x_R) = 0. \tag{17}
\]

Similarly, the first order condition for a Democratic primary is (again after canceling and multiplying)

\[
-\phi \left( \frac{x_D + x_R}{2} \right) \left( (m + x_D)^2 - (m + x_R)^2 \right) - \Phi \left( \frac{x_D + x_R}{2} \right) 4(m + x_D) = 0. \tag{18}
\]

We now show that the first order conditions have a unique solution at \( M = 0 \).

Rearranging (17) and (18), we get

\[
\phi \left( \frac{x_D + x_R}{2} \right) = \frac{(1 - \Phi \left( \frac{x_D + x_R}{2} \right)) 4(m - x_R)}{(m - x_D)^2 - (m - x_R)^2} = \frac{\Phi \left( \frac{x_D + x_R}{2} \right) 4(m + x_D)}{-(m + x_D)^2 - (m + x_R)^2} \tag{19}
\]

Note that the denominator of the term in the middle of (19) can be written \((x_R - x_D)(2m - x_R - x_D)\), and the denominator of the right term of (19) can be written \((x_R - x_D)(2m + x_R + x_D)\).

Substituting this and canceling common terms, we get

\[
(1 - \Phi \left( \frac{x_D + x_R}{2} \right)) (m - x_R)(2m + x_R + x_D) = \Phi \left( \frac{x_D + x_R}{2} \right) (m + x_D)(2m - x_R - x_D) \tag{20}
\]

Suppose that \( x_R + x_D > 0 \). Since, for \( M = 0 \), \( \Phi \) is symmetric, this implies that \( \Phi \left( \frac{x_D + x_R}{2} \right) > \frac{1}{2} > 1 - \Phi \left( \frac{x_D + x_R}{2} \right) \). For (20) to hold, we must therefore have

\[
(m - x_R)(2m + x_R + x_D) > (m + x_D)(2m - x_R - x_D), \tag{21}
\]

which simplifies to \( x_R^2 < x_D^2 \), and hence \(|x_R| < |x_D| \). However, as shown above \( x_D < x_R \). Thus, \( x_D + x_R < 0 \), a contradiction. Similarly we get a contradiction if we assume that \( x_R + x_D < 0 \). Hence \( x_D = -x_R \).
The first order conditions together with the fact that $x_D = -x_R$ now imply we can simplify the equation to get

$$2\phi(0)x_Rm = 2(1 - \Phi(0))(m - x_R).$$

Since $\Phi(0) = 1/2$ we get

$$x_R = \frac{m}{1 + 2\phi(0)m}. \quad (22)$$

Now recall that $\Phi(x) = \Phi_0(x - M)$ and $\phi(x) = \phi_0(x - M)$. At $M = 0$ strategies are symmetric around zero and hence $x_D + x_R = 0$. We now take the derivatives of (17) and (18) with respect to $M$, evaluated at $M = 0$. To shorten the notation we write $x'_R$ and $x'_D$ for $x'_R(0)$ and $x'_D(0)$.

$$- \phi'_0(0) \left( \frac{x'_D + x'_R}{2} - 1 \right) \left( (m - x_D)^2 - (m - x_R)^2 \right) - \phi_0(0) \left(-2(m - x_D)x'_D + 2(m - x_R)x'_R\right)$$

$$- \phi_0(0) \left( \frac{x'_D + x'_R}{2} - 1 \right) 4(m - x_R) - (1 - \Phi_0(0))4x'_R = 0, \quad (23)$$

and

$$- \phi'_0(0) \left( \frac{x'_D + x'_R}{2} - 1 \right) \left( (m + x_D)^2 - (m + x_R)^2 \right) - \phi_0(0) \left(2(m + x_D)x'_D - 2(m + x_R)x'_R\right)$$

$$- \phi_0(0) \left( \frac{x'_D + x'_R}{2} - 1 \right) 4(m + x_D) - \Phi_0(0)4x'_D = 0. \quad (24)$$

If $M = 0$ we have the symmetric equilibrium characterized above where $x_R$ is given by (22). Thus, (23) and (24) imply

$$x'_D(0) = x'_R(0) = \frac{4\phi_0(0)^2m^2}{4\phi_0(0)^2m^2 + 1}. \quad (25)$$

The second derivatives of (17) and (18) with respect to $M$ evaluated at $M = 0$ are

$$-4\phi_0''(0)m_xR \left( \frac{x'_D + x'_R}{2} - 1 \right)^2 - 4\phi_0(0) \left( (m - x_R)x''_R - x_Rx''_D - 2 \left( \frac{x'_D + x'_R}{2} - 1 \right)x'_R \right) - 2x''_R = 0; \quad (26)$$

$$4\phi_0''(0)m_xR \left( \frac{x'_D + x'_R}{2} - 1 \right)^2 - 4\phi_0(0) \left( (m - x_R)x''_D - x_Rx''_R + 2 \left( \frac{x'_D + x'_R}{2} - 1 \right)x'_D \right) - 2x''_D = 0. \quad (27)$$

(26) and (27) imply $x''_D(0) = -x''_R(0)$. Let $S = 0.5(x'_D(0) + x'_R(0)) - 1$. Then (25) implies $S < 0$. We
can solve (26) for \( x''_R \) to get

\[
x''_R(0) = \frac{2\phi_0''(0)m x_R S^2 + 4\phi_0(0)x'_R(0)S}{1 + 2\phi_0(0)m}.
\] (28)

Thus, \( x''_R(0) < 0 \). Since \( x''_D(0) = -x''_R(0) \) it follows that \( x''_R(0) - x''_D(0) < 0 \). As a consequence \( x_R(M) - x_D(M) \) assumes a local maximum at \( M = 0 \). Therefore \( x_R(\tilde{M}) - x_D(\tilde{M}) < x_R(0) - x_D(0) \) for \( \tilde{M} \neq 0 \) in a neighborhood of 0.

Finally, note that \( x_D < 0 < x_R \) near \( M = 0 \), which implies that the median voters in districts 1 to \( k_D \) strictly prefer that the Democrats win, while Republicans in districts \( 2n + 1 - k_R \) to \( 2n + 1 \) strictly prefer that the Republicans win. As a consequence, districts 1 to \( k_D \) are safe for the Democrat, who gets elected with policy \(-m\), while districts \( 2n + 1 - k_R \) to \( 2n + 1 \) are safe for the Republicans who get elected with policy \( m \) if \( \gamma \) is not too large.

In particular, suppose that the median voter in district \( i < k_D \) deviates and elects the Republican. Then the probability that policy \( x_R \) is implemented increases, while the the probability of policy \( x_D \) decreases, which makes the median voter worse off as long as \( \gamma \) is small. Since the median voter is strictly better off with the Democrat, the primary voter will therefore propose a candidate with policy \( x_{D,i} = -M \). The argument that districts \( 2n + 1 - k_R \) to \( 2n + 1 \) are safe for the Republican is analogous.

**Proof of Proposition 3.** Since the median voters in the gerrymandered districts are at \(-m\) and \( m\), respectively, the resulting policies in these districts are \(-m\) and \( m\). By assumption the gerrymandered districts are at least \( 2/3 \) of all districts. As a consequence, the median legislature is at \(-m\) if the Democrat wins, and at \( m\) if the Republicans win.

Consider a particular competitive district, and let \( p_k \) be the probability that \( k \) of the remaining \( 2n \) districts vote Republican. Denote the Democrat’s and the Republican’s policies by \( x_D \) and \( x_R \). Then the payoff of a voter at \( M \) from the Democrat is

\[
-(1 - \gamma) \left( \sum_{k=0}^{n} p_k (M + m)^2 + \sum_{k=n+1}^{2n} p_k (M - m)^2 \right) - \gamma (M - x_D)^2,
\]
while the payoff from the Republican is

\[-(1 - \gamma) \left( \sum_{k=0}^{n-1} p_k (M + m)^2 + \sum_{k=n}^{2n} p_k (M - m)^2 \right) - \gamma (M - y)^2.\]

The cutoff voter, who is indifferent between the candidates, is therefore given by

\[M(x_D, x_R) = \frac{1}{2} \gamma \frac{x_R^2 - x_D^2}{x_R - x_D} + 2(1 - \gamma) p_n m.\]  

(29)

Note that

\[\lim_{\gamma \to 0} \frac{1}{\gamma} \frac{\partial M(x_D, x_R)}{\partial x_D} = -\frac{x_D}{2 p_n m}, \text{ and } \lim_{\gamma \to 0} M(x_D, x_R) = 0.\]  

(30)

The Democratic primary voter therefore solves

\[\max_{x_D} -\Phi (M_R(x_D, x_R)) \left(1 - \gamma \sum_{k=n+1}^{2n} p_k (2m)^2 + \gamma (m + x_D)^2 \right)\]

\[-(1 - \Phi (M_R(x_D, x_R))) \left(1 - \gamma \sum_{k=n}^{2n} p_k (2m)^2 + \gamma (m + x_R)^2 \right).\]

The first order condition is

\[-\phi(M) \frac{\partial M(x_D, x_R)}{\partial x_D} \left(\gamma ((m + x_D)^2 - (m + x_R)^2) - 4(1 - \gamma) p_n m^2 \right) - 2\Phi(M)\gamma (m + x_D)) = 0.\]  

(31)

Dividing both sides of (31) by \(\gamma\), then taking the limit for \(\gamma \to 0\), and using (30) yields

\[\phi(0)(-x_D)m = \Phi(0)(m + x_D).\]  

(32)

The Republican primary solves

\[\max_{x_D} -\Phi (M_R(x_D, x_R)) \left(1 - \gamma \sum_{k=0}^{n} p_k (2m)^2 + \gamma (m - x_D)^2 \right)\]

\[-(1 - \Phi (M_R(x_D, x_R))) \left(1 - \gamma \sum_{k=0}^{n-1} p_k (2m)^2 + \gamma (m - x_R)^2 \right).\]
The first order condition is

\[-\phi(M) \frac{\partial M(x_D, x_R)}{\partial x_R} \left( (1 - \gamma)4p_n m^2 + \gamma ((m - x_D)^2 - (m - x_R)^2) \right) + 2\gamma (1 - \Phi(M)) (m - x_R) = 0.\]

It follows that

\[\frac{\partial}{\partial x_R} M(x_D, x_R) \bigg|_{\gamma=0} = -\frac{x_R}{2p_n m}.\]

Again, dividing by \(\gamma\), setting \(\gamma = 0\) and using the fact that \(M = 0\) when \(\gamma = 0\), yields

\[\phi(0)x_R m = (1 - \Phi(0)) (m - x_R).\] (33)

This implies (8).

We now show that the objectives of the Democrats’ maximization problems is strictly concave. The derivative of (31) is

\[-\left( \phi(M) \frac{\partial^2 M(x_D, x_R)}{\partial x_D^2} + \phi'(M) \left( \frac{\partial M(x_D, x_R)}{\partial x_D} \right)^2 \right) \left( \gamma \left( (m + x_D)^2 - (m - x_R)^2 \right) - (1 - \gamma)4p_n m^2 \right)\]

\[-4\gamma\phi(M) \frac{\partial M(x_D, x_R)}{\partial x_D} (m + x_D) - 2\gamma\phi(M).\] (34)

Note that

\[\lim_{\gamma \to 0} \frac{\partial^2 M(x_D, x_R)}{\partial x_D^2} \frac{1}{\gamma} = -\frac{1}{2p_n m}, \quad \text{and} \quad \lim_{\gamma \to 0} \left( \frac{\partial^2 M(x_D, x_R)}{\partial x_D^2} \right) \frac{1}{\gamma} = 0.\] (35)

Dividing both sides of (34) by \(\gamma\), taking the limit for \(\gamma \to 0\), and using (30), and (35) yields

\[-2\phi(M)(m + 1) < 0.\]

Thus, for small \(\gamma\) the objectives is concave for every \(x_D\). Concavity of the Republican’s objective follows similarly.

If \(M = 0\) then \(\Phi(0) = 0.5\), hence the distance between the policies is the same as in Proposition 2, i.e., as in the case where all districts are symmetric.

Using the fact that \(\Phi\) is symmetric and hence \(\phi'(0) = 0\) and \(\Phi(0) = 0.5\), it is easy to verify that

\[\frac{\partial (x_R - x_D)}{\partial M} \bigg|_{M=0} = 0.\]
Hence, if \( \frac{\partial (x_R - x_D)}{\partial M} \bigg|_{M=0} < 0 \), \( M = 0 \) is a local maximum, and polarization, i.e., the distance between the policies is smaller in a neighborhood of \( M = 0 \). The reverse is true if \( \frac{\partial (x_R - x_D)}{\partial M} \bigg|_{M=0} > 0 \).

Again, using the fact that \( \phi'(0) = 0 \) and \( \Phi(0) = 0.5 \) it follows that

\[
\frac{\partial^2 (x_R - x_D)}{\partial M^2} \bigg|_{M=0} = -\frac{4m^2 \left( 8\phi(0)^3 + \phi''(0) + 2m\phi''(0)\phi(0) \right)}{(1 + 2m\phi(0))^3}.
\]

Thus, the second derivative is positive if and only if \( 8\phi(0)^3 + \phi''(0) + 2m\phi''(0)\phi(0) < 0 \), i.e., if \( m > \bar{m} \), where

\[
\bar{m} = -\left( \frac{4\phi(0)^2}{\phi''(0)} + \frac{1}{2\phi(0)} \right).
\]

Let \( x^G_R(\bar{M}) \), and \( x^G_D(\bar{M}) \) the policies in district \( n+1 \) in the gerrymandered model, and \( x_R(\bar{M}), x_D(\bar{M}) \) those in the symmetric model characterized in Proposition 2. Then for \( \bar{M} \) in a neighborhood of zero, \( \bar{M} \neq 0 \) we get \( x^G_R(\bar{M}) - x^G_D(\bar{M}) > x_R(\bar{M}) - x_D(\bar{M}) \), while item 3 in Proposition 2 implies \( x_R(\bar{M}) - x_D(\bar{M}) < x_R(0) - x_D(0) \). Since \( x^G_R(0) = x_R(0) \) and \( x^G_D(0) = x_D(0) \) it follows that the equilibrium policies in district \( n+1 \) are more polarized in the gerrymandered model for \( m \geq \bar{m} \).
References


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