Optimal Contracts when Enforcement is a Decision Variable

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Abstract

This paper analyzes choice-theoretic costly enforcement in an intertemporal contracting model with a differentially informed investor and entrepreneur. An intertemporal contract is modeled as a mechanism in which there is limited commitment to payment and enforcement decisions. The goal of the analysis is to characterize the effect of choice-theoretic costly enforcement on the structure of optimal contracts. The paper shows that simple debt is the optimal contract when commitment is limited and costly enforcement is a decision-variable (Theorem 1). In contrast, stochastic contracts are optimal when agents can commit to the ex-ante optimal decisions (Theorem 2). The paper also shows that the Costly State Verification model can be viewed as a reduced form of an enforcement model in which agents choose payments and strategies as part of a Perfect Bayesian Nash Equilibrium.

Keywords: contracts, enforcement, limited commitment, costly state verification, debt, stochastic monitoring, time consistency, renegotiation.

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1 Introduction

A contract is a set of promises individuals make today which they expect will be fulfilled in the future. Unfortunately, promises can be broken. This ability to renege on promises is important because it can break the links among individuals that are necessary for intertemporal trade.[1] One way to solve the problem is to use "choice-theoretic costly enforcement" as a commitment device. Consider an entity, such as a court, that has the ability to enforce an intertemporal contract between a differentially informed investor and entrepreneur upon request. We study the effect of costly enforcement on the structure of optimal contracts when enforcement is a decision-variable. When agents cannot commit ex-ante to request enforcement at a later time, they take this into account at the outset and write contracts that are time consistent.[2] We prove that when commitment problems of this type exist, debt is the optimal contract. Otherwise, stochastic contracts are optimal.

Our results are related to two literatures. First, there is a large literature on renegotiation with private information. This literature shows that fully anticipated renegotiation restricts the optimal ex-ante contract and that the implicit information revelation which can result from agents' actions is problematic. A key issue in these intertemporal problems is that contracts with full commitment to initial choices are desirable because they Pareto dominate contracts with renegotiation. Commitment is often difficult to achieve, and when it is not possible agents try to limit ex-post recontracting by revealing as little information as possible. This same force drives our results. If the entrepreneur revealed all information the investor would never enforce, but this undermines the credibility of enforcement (i.e., the court) and results in autarky. Stochastic contracts are not optimal when recontracting is possible because they reveal too much information which makes recontracting easier.

Second, in a series of papers designed to study debt use by firms, Townsend (1979), Gale-Hellwig (1985), and Williamson (1986) proved that when optimal contracts are deterministic, they resemble debt. Townsend (1979), Border-Sobel (1987), and Mookherjee-Png (1989) showed that stochastic contracts can also arise, and they Pareto dominate debt. Thus, deterministic and stochastic contracts do not co-exist in these costly state verification models. The simultaneous

[1] This is tantamount to the classic problem of attempting to trade with agents who will die or are unborn (cf., the overlapping generations model, Samuelson (1958)).

[2] A contract is time consistent if the ex-ante optimal agreement is such that agents do not wish to alter it ex post (cf., Kydland and Prescott (1977)).

existence of both contracts is of interest because debt is pervasive and stochastic contracts are observed frequently in insurance and taxation problems (i.e., audits are often stochastic; see Roth et al (1989)).[3,4]

2 The Model

Consider an economy with two risk neutral agents, a planning period, and three subsequent periods. Agents derive utility only from consumption in the final period. The investor has one unit of a consumption/investment good in the initial period and no endowment in any other period. The penniless entrepreneur owns a production technology which is described by a random variable with finitely many realizations $x_0, \ldots, x_n$. The technology transforms one unit of the input into x_i units of output. Agents share a common prior belief $\beta(x)$ about the possible realizations, and know that the entrepreneur will privately observe output. Because the entrepreneur has a technology but no input and the investor has an input but no technology, production occurs only if the investor can be persuaded to transfer the good to the entrepreneur. This is done by writing a *contract* which consists of payments by the entrepreneur in subsequent periods. In the first period nature chooses the outcome of the venture. In the second period the entrepreneur decides whether to make a voluntary payment chosen from a subset of project returns that includes the possibility of making no payment. In the last period the investor, after having observed the entrepreneur's payment but not the state, decides whether to enforce a final payment. Enforcement is provided by an outside agent such as a court.

In the contracting literature, a contract is usually defined as a pair M, V, where M is a set of messages m about the entrepreneur's realization and V is a payment schedule that depends on the message. Costless ex-post enforcement of the payment is assumed. Our model differs from the literature in two key respects.

[3] Debt is also of interest as theory often predicts complicated state contingent contracts (cf., Hart-Holmström (1987) or Allen-Winton (1995) for a review). Allen-Gale (1992) suggest that adverse selection can make it optimal to write contracts with few state contingencies and Matthews (1994) argues that moral hazard can make it optimal to write simple contracts that will be renegotiated. Enforcement problems are another reason.

[4] There is a large literature on commitment. Gale (1978, 1982) and Townsend (1980) discuss the importance of limited commitment for monetary theory (see also Huggett-Krasa (1996) and Chatterjee-Corbae (1996)); Becker-Chakrabarti (1995) show that capital can serve as a commitment device; and "control rights" (threatening to reassign control) give firms the incentive to adhere to contracts. Enforcement is another commitment device.

First, the entrepreneur decides whether to honor an unenforceable initial payment promise chosen from a set V. If v is zero no payment occurs, but if it is positive the transfer is made immediately. Second, we introduce an enforceable payment $F(\cdot)$. The investor has the ability to compel payment of F by choosing an action e, after the voluntary payment action has been observed. If $e = 1$ then F is enforced and if $e = 0$ it is not. Thus a contract in our model is a pair of payments with associated probabilities for choosing whether to make a voluntary payment or enforce.

Our model is a stylized version of the enforcement provided by courts. The assets available before enforcement are $x - v$, where x is the entrepreneur's output realization and v is the voluntary payment. The court's role, if enforcement is requested, is to determine the funds available after any voluntary payment, and then enforce payment F which is contingent on these funds. We assume that the court's enforcement technology is imperfect: the court cannot seize an amount $\bar{x}$.[5] The amount available for transfer is then $y = \max\{x - v - \bar{x}, 0\}$, where no funds can be transferred if the amount that can be hidden $\bar{x}$ exceeds $x - v$. We assume that enforcement is requested by the investor. When it occurs the investor pays a positive cost c_I and the entrepreneur pays a positive cost c_E. These costs are a deadweight loss, thus both agents have an incentive to minimize the court's use.

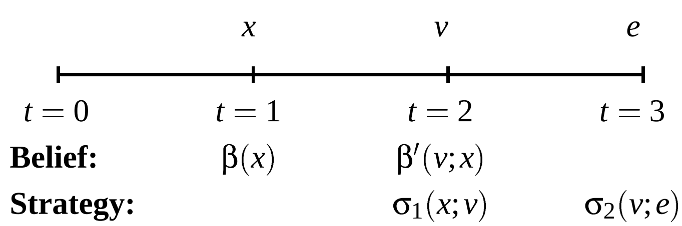
Payments V, F define a noncooperative game with incomplete information with associated strategies σ_1, σ_2. Strategy σ_1 is the probability the entrepreneur assigns to a particular voluntary payment v in the finite set V and σ_2 is the probability that the investor chooses to enforce payment F. The strategies are used to choose v and e optimally as part of a perfect Bayesian Nash equilibrium (PBNE). The optimal contract maximizes agents' expected utilities subject to resource and time consistency constraints, and a constraint that the strategies are a PBNE. We first specify a non-cooperative game and then maximize over all contracts and PBNE of the game. This may appear to differ from the standard optimal contracting approach of choosing a mechanism subject to an incentive constraint. Remember, however, that incentive constraints are motivated by a non-cooperative information revelation game. The revelation principle establishes that for problems without recontracting, any arbitrary (Bayesian) Nash equilibrium of the revelation game can be replaced by one in which agents tell the truth. The revelation principle cannot be invoked in our model due to subsequent opportunities to alter the contract (see Dewatripont-Maskin (1990)). Thus the PBNE constraint plays

[5] This can occur, for example, if the entrepreneur can abscond with or hide an amount $\bar{x}$ (Calomiris-Kahn (1991)).

the role of the incentive constraint. This will be apparent in Section 3.

Characterizing the form of the optimal contract amounts to asking the question—do agents choose deterministic or random (behavioral) strategies? We consider a game which permits random strategies because whether contracts are stochastic or deterministic is a property of the equilibrium of the game. A given contract defines a game that is summarized by a set of players, strategies, a production technology, beliefs, and payoffs.[6] Initial beliefs are given by a prior β over the outcomes. Figure 1 indicates that nature moves first and chooses $x \in X$ at time 1, which the entrepreneur privately observes. The game involves two actions: which voluntary payment v to make at time 2 and whether to go to court to enforce F at time 3 (yes if $e = 1$, and no if $e = 0$). The entrepreneur uses strategy σ_1 to choose v, and the investor uses strategy σ_2 to choose e. The investor uses publicly observable action v to update beliefs to β'.

Figure 1



Agents' payoff functions π_i, $i = I, E$ are given by:
$\pi_E(x,v,e) = x - v - e[F(y,v) + c_E]$ and
$\pi_I(x,v,e) = v + e[F(y,v) - c_I]$, where $y = \max\{x - v - \bar{x}, 0\}$.

The solution concept is the perfect Bayesian Nash equilibrium. Conditions (i) and (ii) in Definition 1 require strategies to be subgame perfect Nash equilibria given beliefs. Condition (iii) describes how the investor forms beliefs after observing the entrepreneur's voluntary payment action.

Definition 1. A collection of strategies σ_1, σ_2 and beliefs β, β' are a *Perfect Bayesian Nash Equilibrium* if and only if

(i) $\sigma_1 \in \Sigma_1$ maximizes $E_{\sigma_1,\sigma_2}\pi_E(x,v,e)$ for every x.
(ii) $\sigma_2 \in \Sigma_2$ maximizes $\sum_{x\in X}\beta'(v;x)E_{\sigma_2}\pi_I(x,v,e)$ for every v.
(iii) β' is derived using Bayes' rule whenever possible.

[6] Formally, $G^b = \{I, E, \Sigma_1, \Sigma_2, \mathrm{X}, \beta, \beta', \pi_\mathrm{I}, \pi_\mathrm{E}\}$, where Σ_1 is the collection of all functions $\sigma_1(x;v)$ that denote the probability payment v is made given x, and Σ_2 is the collection of all $\sigma_2(v;e)$ that denote the probability enforcement action e is chosen given v.

3 Optimal Contracts

We now characterize optimal contracts. Theorem 1 shows that when commitment is limited, simple debt is the optimal subgame perfect contract. Section 3.1 begins by stating the problem for which deterministic contracts are optimal. Three propositions are proved which make clear the relationship between deterministic contracts and debt. Theorem 1 follows immediately. In Section 3.2, Theorem 2 shows that when there is full commitment to the ex-ante optimal contract, the optimal contract in this "less constrained" problem is stochastic. All proofs are in the Appendix.

3.1 Optimality of Deterministic Contracts

In Problem 1 agents choose a contract in the initial period to maximize the investor's expected utility subject to four constraints. (1.1) is individual rationality which ensures that the entrepreneur gets an expected utility at least as great as reservation value $\bar{u}$ in every state. (1.2) requires payments to be feasible for all y, v, where non-negativity prohibits the entrepreneur from extorting further payments from the investor. (1.3) is the PBNE restriction in lieu of incentive compatibility, and (1.4) is time consistency.

Problem 1. At $t = 0$ choose $\sigma_1(x;v)$, $\sigma_2(v;e)$, V, F to maximize

$$\sum_x \beta(x) E_{\sigma_1,\sigma_2} \pi_I(x,v,e) \text{ subject to:}$$

(1.1) $\sum_x \beta(x) E_{\sigma_1,\sigma_2} \pi_E(x,v,e) \geq \bar{u}$.

(1.2) $0 \leq v \leq x$ and $0 \leq F(y,v) \leq y$ for all y, v.

(1.3) σ_1, σ_2, β, β' is a PBNE at $t = 1$.

(1.4) v, F, σ_2 is time consistent.

Agents have the opportunity to alter the initial contract at time 2, but (1.4) ensures they will choose $v' = v$, $F' = F(y,v)$, $\sigma_2' = \sigma_2$ where v', F', σ_2' is the optimal continuation contract which solves Problem 2 below and v, F, σ_2 is the original plan. (1.4) is similar to (4) in Dewatripont (1989, p. 599) as both state that agents cannot recontract in a future period to increase their expected payoffs. All possibilities for altering the initial contract are foreseen at the outset, and this constrains the choice of the initial contract. By modeling these future opportunities via (1.4) we use a cooperative approach rather than specifying a non-cooperative game. The goal of both approaches is to find Pareto superior allocations. The absence of Pareto improvements precludes renegotiation from actually occurring.

Problem 2 formalizes the time 2 problem from which the continuation contract is derived. This contract maximizes the investor's expected utility, given updated belief β', subject to three constraints. Individual rationality constraint (2.1) ensures that the entrepreneur's expected utility is at least as great as reservation value $\overline{u}'_x$ in almost every state x, when the investor uses alternative enforcement strategy σ'_2 and given updated investor belief β'.[7] (2.2) requires the choice of v' and F' to be feasible for all y. (2.3) requires the investor's enforcement choice to be optimal.

Problem 2. Let v, σ_1 be given. At $t = 2$ choose an optimal continuation contract v', F', σ'_2 to maximize $\sum_x \beta'(v;x) E_{\sigma'_2} \pi'_I(x,v,e)$ subject to:

(2.1) $E_{\sigma'_2} \pi'_E(x,v,e) \geq \overline{u}'_x$ for all x with $\beta'(v;x) > 0$;

(2.2) $v \leq v' \leq x$ with $\beta'(v;x) > 0$ and $0 \leq F'(y) \leq y$ for all y;

(2.3) σ'_2 fulfills (ii) of Definition 1.

The opportunity to alter the initial contract occurs after the voluntary payment action, but before the enforcement action is chosen. The timing of this payment is important because it implies limited commitment to v. In particular, the voluntary payment constitutes "money on the table" that cannot be retracted by the entrepreneur or court once the payment has been made. This payment presents two opportunities. First, the voluntary payment can be increased to $v' \geq v$ if this is mutually agreeable. Second, agents can change the enforceable payment to F'. The total payment ignoring enforcement costs is $v' + eF'$. By paying $v' > v$, however, the entrepreneur has the opportunity to induce the investor to refrain from enforcement or to enforce with lower probability. This saves deadweight enforcement costs and the surplus can be used to make Pareto improvements (i.e., higher payoffs net of enforcement costs).

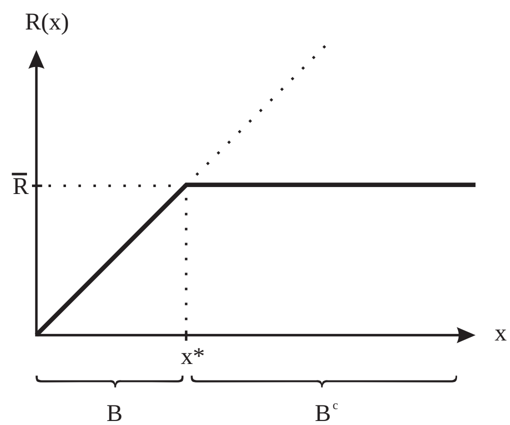
The main result in this section is Theorem 1 which states that simple debt is optimal when commitment is limited. More precisely, it states that debt is the optimal contract when agents have the ability to revise the payment and enforcement promises they agreed to at the outset in Problem 1. Before proceeding we wish to

[7] (2.1) ensures the entrepreneur receives in every state x at least $\overline{u}'_x = E_{\sigma'_2} \pi_E(x,v,e)$, the expected continuation payoff from the initial contract. Offers which make the entrepreneur better off in some states but worse off in others are not considered as this requires modeling a particular non-cooperative game. Our results would hold if the game had two features: (i) Assume there exist v, β' such that paying $v' > v$ in exchange for no enforcement makes the entrepreneur better off in all states, and the investor better off in expected utility given β'. Then the initial contract must be revised. (ii) Debt is time consistent.

make clear the relationship between deterministic contracts and debt. A deterministic contract is a contract where the choice of a particular v or e is deterministic (i.e., occurs with probability one or zero). In contrast, debt is often described as a pair $R(x), B$ where x is the realization of a random variable that describes the entrepreneur's privately observed project outcome, $R(x)$ is a payment function, and B is a lower interval set of "bankruptcy states" where assets are seized from the entrepreneur. In a debt contract bankruptcy occurs with probability one or zero as a deterministic function of the state. In other words, if the state x were known then bankruptcy occurs with probability one when the state is "bad" and with probability zero when the state is "good."

A simple debt contract $R(x), B$ is depicted below in Figure 2.

Figure 2



The lower interval bankruptcy set is given by $B = \{x < x*\}$ and indicates that bankruptcy occurs only for low project outcomes (i.e., those below $x*$). When bankruptcy occurs the entire realization $R(x) = x$ is seized from the entrepreneur in a simple debt contract. For all other sufficiently high realizations (those with $x \geq x*$), bankruptcy does not occur and the entrepreneur makes a fixed payment $R(x) = \overline{R}$ and retains $x - \overline{R}$.

When contracts are restricted to be deterministic a priori, Townsend (1979), Gale-Hellwig (1985) and Williamson (1986) have shown that debt is the optimal contract in a Costly State Verification (CSV) model. Like our model, this model has an entrepreneur with a technology but no input and an investor with an input but no technology. In addition, in the CSV model the investor has access to a monitoring technology that can be used to reveal the state at a cost. When monitoring is assumed to be deterministic simple debt is optimal because it minimizes the monitoring cost, but it is well known that debt is dominated by a contract with random monitoring when stochastic contracts are allowed. Thus in the CSV model debt is not robust to stochastic monitoring, is not ex-post efficient, and stochastic and deterministic contracts cannot co-exist.

In our model the environment and choice variables differ from the CSV model. As a consequence, we establish conditions under which debt is optimal but not subject to the problems noted above. Payment schedule $R(x) = v(x) + eF(y, v)$ allows for voluntary and enforceable payments, and limited commitment to initial promises. Problem 1 shows the decisions to make a voluntary payment or enforce F are chosen as part of the contract and hence are optimal each period by construction. We do not restrict these strategies to be deterministic at the outset. Rather, Theorem 1 proves that if commitment is limited (a friction imbedded in the primitives of the model), debt is optimal even when stochastic contracts are allowed. Proposition 1 first shows that enforcement is deterministic. Proposition 2 shows that when enforcement is deterministic we can appeal to the CSV model and identify suitably redefined payments $R(x)$ and a lower interval set of enforcement states B that resemble the debt contract depicted in Figure 2. Proposition 3 ensures that all constraints in Problem 1 are satisfied. Finally, Theorem 1 establishes that debt is the optimal solution to Problem 1.

The following assumptions are necessary for these results:

(A.1) $0 < \bar{x} - c_E < x_0$.

(A.2) $x_0 < \sum_{x<x*}(x - c_I)\beta(x \mid x < x^*)$.

A.1 and A.2 are conditions on the parameters that determine the entrepreneur and investor's minimal payoffs from enforcement, respectively. Recall that $\bar{x}$ is the amount of funds that the entrepreneur can hide, c_E is a deadweight enforcement cost paid by the entrepreneur, and x_0 is the lowest output realization. A.1 indicates that when the court seizes funds the entrepreneur's payoff after enforcement, net of costs, is small but positive. Most enforcement technologies are likely to have some degree of imperfection. Theorem 1 will show that even when this imperfection is very small and commitment is limited, simple debt is the optimal contract. We choose additive costs, $\bar{x} - c_E$, for simplicity. The results also hold for other cost structures (e.g., where costs are large or a percentage of total assets). A.2 indicates that the investor's expected payoff from a simple debt contract when bankruptcy occurs is larger than x_0, the gross payment the investor can receive with probability one in the worst state (ignoring $\bar{x}$ and c_E). In other words, when bankruptcy occurs, on average the investor can recover at least x_0 and her expected enforcement costs.

Proposition 1. *For a given V, F let σ_1, σ_2 be a PBNE. Assume that the contract is time consistent and that A.1 holds. Then σ_2 is deterministic.*

Proposition 1 is proved by way of contradiction.[8] Suppose the optimal contract is stochastic. Then the investor must be indifferent between the expected payoff from enforcement and no enforcement; the entrepreneur incurs deadweight loss c_E when enforcement occurs; and the entrepreneur retains amount $\bar{x} - c_E$ because the court's enforcement technology is imperfect (and because the strategies are a PBNE). These three facts give the entrepreneur the ability and incentive to "bribe" the investor to refrain from enforcement by altering the initial contract to $v' > v$, $F' = 0$, and $\sigma_2' = 0$. This is a contradiction since such a contract revision is not time consistent. Intuitively, stochastic contracts are not optimal when recontracting is possible because they reveal a lot of information. This occurs because the investor must be indifferent between the two enforcement options when σ_2 is stochastic.

Next consider Proposition 2 which shows that any arbitrary contract for which σ_2 is deterministic can be dominated by a simple debt contract, ignoring (ii) from Definition 1 and time consistency constraint (1.4). In our universe of contracts, V, F, σ_1, σ_2 is simple debt if there is a set of bankruptcy states and a critical value x^* where

(i) $B = \{x \in X \mid x < x^*\}$, and the investor's enforcement action is $e = 1$ for $x \in B$ and $e = 0$ for $x \in B^c$ with probability 1.

(ii) The entrepreneur pays $v = \bar{R}$ for $x \in B^c$ and $v = 0$ for $x \in B$. Moreover, $F(y, v) = \begin{cases} y & \text{for } x \in B; \\ 0 & \text{for } x \in B^c. \end{cases}$

This definition of simple debt corresponds to the contract depicted in Figure 2, with $R(x)$ and B modified to accommodate limited commitment to payment and enforcement decisions. Specifically, (i) requires enforcement to occur on a lower interval and (ii) requires all assets to be seized when bankruptcy occurs (which characterizes simple debt).

Proposition 2. *Consider an initial contract where σ_2 is deterministic and which fulfills (1.1), (1.2), and (i) of Definition 1. This contract is dominated by a simple debt contract that fulfills the same constraints.*

[8] The result that deterministic enforcement is optimal in Problem 1 also holds when agents are risk averse. This follows from the proof of Proposition 1 and results in Krasa and Villamil (1994).

The proof of Proposition 2 shows the relationship between the CSV model with deterministic monitoring and our costly enforcement model. Consider an arbitrary contract V, F, σ_1, σ_2 in which σ_2 is deterministic. This contract is mapped into a contract in the CSV model with deterministic monitoring. Without loss of generality one can assume that σ_1 is deterministic. As both strategies are deterministic, a set B exists where enforcement occurs. In addition, for each realization the payoff is given by $R(x)$. Thus a contract in our model corresponds to a contract in the CSV model and the expected payoffs are the same under both contracts. The Gale-Hellwig (1985) and Williamson (1986) argument (GHW) then shows that this arbitrary contract is dominated in payoffs by a simple debt contract in the CSV model where monitoring occurs on a lower interval solely as a function of x. This lower interval contract is then mapped back into a simple debt contract in our model, though (ii) of Definition 1 and (1.4) need not hold.

The proof of Proposition 2 considers contracts where no voluntary payments v are made in the bankruptcy case. Thus, the v payment reveals a minimal amount of information. In general there are many contracts with different v payments in the bankruptcy state that yield the same payoffs as a particular GHW debt contract. These contracts also reveal more information at $t = 1$. We map the GHW contract into a contract in our model where information revelation is minimal because this captures the idea that debt is informationally minimal.

The construction used in the proof of Proposition 2 may change β'. As a consequence, optimality of the investor's enforcement decision (ii) of Definition 1 and time consistency constraint (1.4) from Problem 1 need not hold. Proposition 3 shows that with assumption A.2 all the constraints of Problem 1 are satisfied. Thus simple debt is the optimal solution. Theorem 1 follows immediately. A.2 ensures there are enough funds to recover cost c_I when enforcement occurs and enough uncertainty about the enforcement states. Both prospects for fund recovery and uncertainty about x are crucial for making enforcement credible.[9] The credibility of costly enforcement, in turn, makes contracts time consistent.

Proposition 3. *Consider a simple debt contract where A.1, A.2 are satisfied and* $V = \{0, \bar{R}\}$. *Then the contract fulfills (ii) of Definition 1 and (1.4).*

Propositions 1–3 immediately imply Theorem 1, which states that simple debt

[9] To see that uncertainty matters, suppose that (i) two agents have signed a simple debt contract with face value $\bar{R}$, (ii) the actual realization is $\tilde{x} < \bar{R}$, and (iii) the entrepreneur can truthfully reveal $\tilde{x}$ to the investor. If agents go to court costs c_I and c_E are incurred, and if they do not this surplus can be split. Thus, enforcement would not occur. Once the entrepreneur knows that enforcement will not occur, she will reveal $\hat{x} < \tilde{x}$.

is optimal and subgame perfect (ex post efficient) even when agents have the opportunity to use stochastic decision rules. This debt result differs fundamentally from the CSV model where debt is not efficient ex post and not optimal when stochastic verification is possible.

Theorem 1. *Assume there exists a simple debt contract which satisfies A.1, A.2 and which gives the entrepreneur reservation utility $\bar{u}$. Then this contract solves Problem 1.*

3.2 The Optimality of Stochastic Contracts

We now show that stochastic contracts are optimal when commitment to the initial contract is possible. Agents solve the problem at the outset and have no opportunity to alter their actions. They are fully "committed to" their initial σ_1, σ_2, V, F. This reduces the subgame perfection conditions in Definition 1 to (i) only; (ii), (iii), and (1.4) are unnecessary as there is no opportunity to revise the contract. In addition, $F < 0$ is now possible under full commitment. That is, the enforcement technology can recover payment v from the investor. Problem 1' is the following "less constrained" analog of Problem 1:

Problem 1'. At $t = 0$ choose $\sigma_1(x; v)$, $\sigma_2(v; e)$, V, F to maximize

$$\sum_x \beta(x) E_{\sigma_1,\sigma_2} \pi_I(x, v, e) \text{ subject to:}$$

(1.1), (1.2), (i) of Definition 1, and $F(y, v) \geq -v + c_E$.

Now consider the CSV model with stochastic monitoring (cf., Border-Sobel (1987) or Krasa-Villamil (1994)). Let x be the true realization, x' be the reported realization, $p(x)$ be the probability that monitoring occurs when x is reported, t and f be payments that are linked to monitoring, and c be the monitoring cost. If monitoring does not occur the entrepreneur pays $t(x)$. If monitoring occurs the payment is $f(x, x')$. Implicit in this problem is the revelation principle. That is, if x is the true state then this state x is truthfully reported by the entrepreneur. For simplicity suppose that $\bar{x} = c_E = 0$. See the Appendix for the general case.

Problem 3. Choose $t(x)$, $f(x, x')$, and $p(x)$ to maximize

$$\sum_{x \in X} \Big[(1 - p(x))t(x) + p(x)(f(x, x) - c)\Big] \beta(x) \text{ subject to:}$$

(i) $\sum_{x \in X} \Big[x - (1 - p(x))t(x) - p(x) f(x, x)\Big] \beta(x) \geq \bar{u}$.

(ii) $x - [(1 - p(x))t(x) + p(x) f(x, x) \geq x - [(1 - p(x'))t(x') + p(x') f(x, x')$.

(iii) $0 \leq p(x) \leq 1$, $0 \leq t(x) \leq x$, $0 \leq f(x, x') \leq x$ for all $x, x' \in X$.

The optimal contract maximizes the investor's expected utility subject to (i) entrepreneur individual rationality, (ii) incentive compatible reports by the entrepreneur (i.e., truthfully reporting x is weakly better than reporting any other state x'), and (iii) feasibility.

Theorem 2 shows the equivalence between Problems 1' and 3. The proof shows how contracts in the costly enforcement model with full commitment and the CSV model can be mapped into each other without affecting payoffs and constraints. Intuitively, monitoring probability $p(x)$ corresponds to enforcement probability $\sigma_2(v;1)$ where v is the payment made in state x, v corresponds to $t(x)$, and F corresponds to $f-v$ (in the CSV model the total payment if monitoring occurs is f while in the enforcement model it is $F+v$). As a consequence of the equivalence, the result from the CSV model that stochastic contracts are optimal holds in the enforcement model with commitment.

Theorem 2. *Problem 1' and 3 are equivalent. Stochastic contracts are optimal.*

3.3 Concluding Remarks

Theorem 1 shows that simple debt contracts are optimal when there is limited commitment to initial decisions and enforcement is costly and imperfect. Theorem 2 shows that stochastic contracts are optimal in the "less constrained" model with full commitment to initial decisions. Problem 1' is less constrained because time consistency constraint (1.4) and condition (ii) of Definition 1 do not bind when there is full commitment to the ex-ante optimal contract. In Theorem 2 stochastic contracts Pareto dominate deterministic contracts because the weaker constraint set in Problem 1' permits agents to choose over a larger set of contracts.[10] The costly enforcement model thus yields both deterministic (debt) and stochastic contracts as a consequence of a "commitment friction." When commitment is limited, Problem 1 requires actions to be chosen optimally as part of a PBNE. Agents take into account any subsequent opportunity for contract revision at the outset (time consistency). It is precisely the inefficiency arising from the inability to commit that makes debt optimal in Theorem 1.

Our results show that the CSV model can be viewed as a "reduced form" of the costly enforcement model.[11] This is important because the CSV model has

[10]Lacker (1989) shows that costly enforcement can tighten incentive constraints, but contracts in his model need not be time consistent.

[11]Boyd-Smith (1995) calibrate a CSV model and study conditions under which the loss from using deterministic contracts is not too large (relative to stochastic contracts). As it is a CSV

been widely used as a model of financial intermediation,[12] but has been criticized as a model of debt for two reasons: First, debt is not ex post efficient in the model as the investor knows the true state. As a consequence, agents would prefer to revise the initial contract after the state has been announced rather than incur the monitoring cost. In contrast, in the costly enforcement model debt is time consistent and enforcement is chosen optimally by the investor as part of a PBNE. Second, debt is optimal only under the assumption of deterministic monitoring in the CSV model. Theorem 1 shows that the costly enforcement model justifies this assumption when commitment is limited and enforcement is costly and imperfect. In contrast when commitment is costless and perfect, Theorem 2 shows that stochastic contracts are optimal. Which type of contract is more "reasonable" depends on the underlying economic problem. In some environments it may be possible to commit to refrain from renegotiation (e.g., commit to audits by insurance companies, accounting firms, or tax authorities). In such cases Problem 1,' and hence stochastic contracts, seem natural. In environments where recontracting is possible (as in most loan contracts), Problem 1 and debt seem appropriate.

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model, debt is inefficient ex post.

[12]In addition to problems in financial intermediation, see Smith (1998) and the papers therein for recent applications of the CSV model to problems in growth, development and exchange rates (e.g., Antinolfi and Huybens (1998)) and real business cycle models (e.g., Cooley and Nam (1998)).

4 Appendix

Proof of Proposition 1. Assume by way of contradiction that there exists a v in Problem 1 such that the optimal initial contract entails stochastic enforcement ($0 < \sigma_2 < 1$). We first show that in the initial stochastic contract:

Claim. $x - v \geq \bar{x} - c_E$ for all x with $\beta'(v;x) > 0$.

Assume by way of contradiction that $x - v < \bar{x} - c_E$ for an x with $\beta'(v;x) > 0$. Since $\bar{x} - c_E < x_0 \leq x$ by A.1, v must be positive under the original σ_1. The entrepreneur's payoff under the original contract is $x - v - \sigma_2(v;1)[F(y,v) + c_E] \leq x - v$. Now assume the entrepreneur chooses $v = 0$ instead. The resulting payoff is $x - \sigma_2(0;1)[F(x - \bar{x}, 0) + c_E] \geq x - [F(x - \bar{x}, 0) + c_E] \geq x - (x - \bar{x} + c_E) \geq \bar{x} - c_E$, where the last inequality follows from $y = x - v - \bar{x}$ and $0 \leq F(\cdot) \leq y$. However, $x - v < \bar{x} - c_E$ implies that the entrepreneur is strictly better off choosing $v = 0$. This contradicts the optimality of σ_2 required by a PBNE, proving the claim.

We now show that constraint (1.4) does not hold, i.e., there exists a continuation contract which dominates the original contract. In particular, choose v' such that $v < v' < v + \min\{\bar{x} - c_E, \sigma_2(v;1)c_E\}$. Then the continuation contract v', $F' = 0$, $\sigma_2' \equiv 0$ increases the objective and fulfills all constraints of Problem 2. We first check the constraints:

(2.1) Since $F' = 0$ the entrepreneur's payoff in the alternative contract is $x - v'$. We must show that $x - v' \geq \bar{u}_x'$. Recall that $\bar{u}_x = x - v - \sigma_2(v;1)[F + c_E]$. As $F \geq 0$, it follows that $x - v - \sigma_2(v;1)c_E \geq \bar{u}_x$. Further, by definition $v' < v + \sigma_2(v;1)c_E$. Thus $x - v' \geq \bar{u}_x'$ and (2.1) holds.

(2.2) By construction $v' < v + \bar{x} - c_E$, and the claim implies that $v \leq x - (\bar{x} - c_E)$ for all x with $\beta'(v;x) > 0$. Thus $v' < x$ and (2.2) holds.

(2.3) follows because $\sigma_2' = 0$ is optimal if $F' = 0$.

We now show that the objective in Problem 2 is increased when switching from a stochastic to a deterministic contract. Stochastic enforcement implies that the investor's expected payoff from the two enforcement options must be the same given β': $\sum_{x \in X} \beta'(v;x)\pi_I'(x,v,0) = \sum_{x \in X} \beta'(v;x)\pi_I'(x,v,1)$. Thus, independent of the enforcement decision, the investor's expected payoff under the stochastic contract is v (because the payoff if no enforcement occurs is v). The payoff under the continuation contract is $v' > v$. Thus the stochastic contract does not solve Problem 2, a contradiction and σ_2 must be deterministic.

Proof of Proposition 2. Without loss of generality assume that σ_1 is deterministic.[13] Since σ_1, σ_2 are deterministic, whether enforcement occurs can be foreseen

[13] If the choice of v is random for given x, all v with $\sigma_1(x;v) > 0$ result in the same expected

at time 0 if x is known and is a function of x only. Let $R(x)$ be the payment and $B = \{x \mid \sigma_2(v;1) = 1 \text{ where } \sigma_1(x;v) = 1\}$ be the set of states where enforcement occurs. Since v and e are deterministic actions given x, then $R(x) = v + eF(x,v)$. The following hold.

(i) $R(x) = \bar{R}$ (constant) on B^c: If there exist x, $x' \in B^c$ with $R(x) < R(x')$, the entrepreneur is better off paying the same v in states x, x'.

(ii) $R(x) \le \bar{R} + c_E$ on B: Assume that $R(x) - c_E > \bar{R}$. Then it is better for the entrepreneur to make a voluntary payment corresponding to a state $x \in B$. Either payment contradicts σ_1 fulfilling (i) in Definition 1.

(iii) $0 \le R(x) \le x - \bar{x}$ for $x \in B$: The entrepreneur can always retain at least $\bar{x}$ in state x by choosing $v = 0$. Since σ_1 is optimal, $R(x) \le x - \bar{x}$.

(iv) $0 \le \bar{R} \le x - (\bar{x} - c_E)$ for all $x \in B^c$: Assume by way of contradiction that $\bar{R} > x - (\bar{x} - c_E)$ for all $x \in B^c$. Then $x - \bar{R} < \bar{x} - c_E$. By making a payment in B the entrepreneur can obtain at least $\bar{x} - c_E$ which is strictly higher than from paying $\bar{R}$, a contradiction.

Define a set of states by $X' = \{x - (\bar{x} - c_E) \mid x \in X\}$, and a new contract
$$R'(x') = \begin{cases} R(x' + (\bar{x} - c_E)) + c_E & \text{if } x \in B; \\ R(x' + (\bar{x} - c_E)) + (\bar{x} - c_E) & \text{if } x \in B^c. \end{cases}$$
Then $R(x), B$ fulfills (i)-(iv) if and only if $R'(x), B$ fulfills

(a) $R'(x') = \bar{R}'$ (constant) on B^c: This follows immediately.

(b) $0 \le R(x') \le x'$ for all $x' \in X'$: Let $x' \in B$. Then $R'(x') = R(x' + (\bar{x} - c_E)) + c_E \le (x' + \bar{x} - c_E) - \bar{x} + c_E = x'$. If $x \in B^c$ then $R'(x') = R(x' + (\bar{x} - c_E)) + (\bar{x} - c_E) \le x' + (\bar{x} - c_E) - (\bar{x} - c_E) + (\bar{x} - c_E) \le x + (\bar{x} - c_E) = x'$.

(c) $0 \le R'(x') \le \bar{R}'$ for all $x' \in B$: $R'(x') = R(x' + (\bar{x} - c_E)) + (\bar{x} - c_E) \le \bar{R} + (\bar{x} - c_E) = \bar{R}'$. The other direction of the proof is similar.

Now consider the CSV model with $x' \in X'$ where the investor pays all costs $c = c_I + c_E$. We have shown that any contract V, F, σ_1, σ_2 can be mapped into a GHW debt contract $R'(x), B$. The GHW debt result implies it is optimal to choose $R'(x) = x'$ for $x \in B$ and for the bankruptcy set to be a lower interval. It remains to prove that the GHW debt contract has the same payments in each state x. Let $V = \{0, \bar{R}'\}$, $F(y,0) = y$ and $F(y, \bar{R}') = 0$. Then $\sigma_1(x;0) = 1$ iff $x \in B$ and $\sigma_2(x; \bar{R}') = 1$, otherwise the investor enforces iff $v = 0$. Let $x \in B$. The entrepreneur gets $x - F(x - \bar{x}, v) - c_E = (\bar{x} - c_E)$. The investor gets $x - (\bar{x} - c_E)$ and pays $c_E + c_I$, or $x - \bar{x} - c_I$. Finally, σ_1 clearly fulfills (i) of Definition 1.

Proof of Proposition 3. We first prove that (ii) of Definition 1 holds. If $v = \bar{R}$ then $F(y,v) = 0$ by the definition of simple debt. Hence $e = 0$ is optimal. Assume that

payoff π_E. Among these v choose one which maximizes π_I and define $\sigma_1(x;v) = 1$.

$v = 0$. Then $\beta'(0;x) = 0$ if $x \geq x*$ and $\beta'(0;x) = \beta(x)/\beta(\{x|x < x*\})$. A.2 implies that the investor's expected payoff from enforcement is strictly greater than $x_0 - \bar{x} > 0$ (subtract $\bar{x}$ on each side of A.2). Since the payoff without enforcement is zero, (ii) of Definition 1 holds.

We now prove that (1.4) holds. Assume by way of contradiction that there exists v', F', σ_2' which increases the investor's payoff and fulfills the constraints of Problem 2. Then $v' \leq x_0 - \bar{x}$ by (2.2). Moreover probability $\sigma_2(v;1) < 1$. Otherwise, there would be no surplus and agents cannot be made strictly better off. (2.3) implies that the investor's expected payoff from enforcement is less than or equal to zero, thus the investor's expected payoff in the bankruptcy state does not exceed v'. Since $v' \leq x_0 - \bar{x}$, A.2 implies that the investor's payoff under the alternative contract is strictly less than $(1/\beta(\{x|x < x*\}) \sum_{x<x*}(x - \bar{x} - c_I)\beta(x)$, the expected payoff under the original contract given $v = 0$. The investor is strictly worse off under v', F', σ_2', a contradiction. Hence the original contract is time consistent.

Proof of Theorem 2. Fix the parameters of the economy X, β, c_I, c_E, $\bar{x}$. First assume that $\bar{x} = c_E = 0$. Problem 3 can written: Choose t, f, M, q, p to maximize $\sum_{x\in X}\sum_{m\in M}\Big[(1-p(m))t(m) + p(m)(f(m,x) - c_I)\Big]q(x;m)\beta(x)$ subject to:

(i) $\sum_{x\in X}\sum_{m\in M}\Big[x - (1-p(m))t(m) - p(m)f(m,x)\Big]q(x;m)\beta(x) \geq \bar{u}$.

(ii) $q(x;m) > 0$ if and only if $m \in \arg\max_m x - [(1-p(m))t(m) + p(m)f(m,x)]$.

(iii) $0 \leq t(m) \leq x$, for all m with $q(x;m) > 0$; $0 \leq p(m) \leq 1$, for all $m \in M$.

The entrepreneur now announces an arbitrary message $m \in M$ that need not coincide with the true state $x \in X$, where $p(m)$ is the probability of monitoring if m is announced, and $q(x;m)$ is the probability that a particular m is announced given that x is the true state. The revelation principle implies that the solutions of this problem correspond to those of Problem 3, i.e., without loss of generality we can choose $M = X$ and $q(x;m) = 1$ iff $x = m$ (when agents report truthfully, (ii) is the incentive constraint).

To prove that a contract from the enforcement model can be mapped into a contract from the CSV model it is sufficient to show that a contract V, F, σ_1, σ_2 can be mapped into a contract t, f M, q, p. The mapping is straightforward. Let $M = V$, $p(m) = \sigma_2(m;1)$, $q(x;m) = \sigma_1(x;m)$, $f(x,m) = F(x,m) - m$, and $t(m) = m$. Message m in the CSV model corresponds to payment v in the enforcement model. The CSV contract has the same payoffs as the original contract from the enforcement model.

To prove that a contract of the CSV model can be mapped into a contract of the enforcement model we appeal to the revelation principle. Assume without loss of generality that $M = X$ and that the entrepreneur announces the state truthfully. Let $f(x,x') = x$ if $x \neq x'$. First, assume that $t(x)$ is one-to-one. Define $V = \{t(x) \mid x \in X\}$, $F(x,v) = f(x,t^{-1}(v)) - v$, and $\sigma_2(x,v) = p(t^{-1}(v))$. Since the entrepreneur announces x truthfully in the CSV model, define $\sigma_1(x;v) = 1$ iff $v = t(x)$, and $\sigma_1(x;v) = 0$ otherwise. It is easy to see that all payoffs remain the same.

Now assume that $t(x)$ is not one-to-one. That is, in the CSV contract all states x are announced truthfully, but there are different states x, $\tilde{x}$ where $t(x) = t(\tilde{x})$. First assume that $f(x,x) = f(\tilde{x},\tilde{x}) = 0$. Then $p(x) = p(\tilde{x})$ must hold.[14] Choose $F(x,v) = F(x,\tilde{v}) = 0$ and $p(v) = p(x) = p(\tilde{x})$. Now consider the case where $f(\tilde{x},\tilde{x}) > 0$. Then we can assume that $p(x), p(\tilde{x}) > 0$.[15] Now increase $t(\tilde{x})$ by $\varepsilon/(1-p(\tilde{x}))$ and decrease $f(\tilde{x},\tilde{x})$ by $\varepsilon/p(\tilde{x})$, where $\varepsilon > 0$ but sufficiently small that f is strictly positive and t remains feasible. It is easy to see that incentive constraint (ii) in Problem 3 still holds for this alternative contract. Moreover $t(\tilde{x}) \neq t(x)$ for the alternative contract. Because the payments $t(x)$, $t(\tilde{x})$ are now different, we can again define $F(\cdot)$, $\sigma_2(\cdot)$ as in the first part of the argument where $t(\cdot)$ was one-to-one.

This concludes the proof of Theorem 2 for the case $\bar{x} = c_E = 0$. The arguments are similar for $\bar{x}, c_E > 0$ except let $0 \leq f(m,x) \leq x - (\bar{x} - c_E)$ and $F(x,m) \geq -v + c_E$ in the problem at the outset.

[14] Otherwise, the contract can be dominated by one for which the probability of monitoring is $\min\{p(x), p(\tilde{x})\}$.

[15] If for example $p(x) = 0$ and $p(\tilde{x}) > 0$, the entrepreneur would always announce x in state $\tilde{x}$, and similarly if $p(x) > 0$ and $p(\tilde{x}) = 0$. If $p(x) = p(\tilde{x}) = 0$ then one can choose without loss of generality $f(x,x) = f(\tilde{x},\tilde{x}) = 0$, which reduces it to the above case.

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