Informed Finance?*

Dan Bernhardt and Stefan Krasa
Department of Economics
University of Illinois
Champaign IL 61820
http://www.staff.uiuc.edu/~skrasa
danber@uiuc.edu

November 2005

Abstract

This paper investigates basic issues in contracting and information acquisition for entrepreneurial finance. We consider an environment in which it is costly for a financier to screen investment projects, and uninformed investors can compete to provide funding. If a financier does investigate, he must choose how carefully to investigate the project's quality, and the actions that maximize the project's payoff. We then determine how the possibility of outside funding affect the nature and quality of the information acquired by a financier, the equilibrium contracting terms, and the allocation of control rights.

We find that four distinct types of equilibria can exist. We categorize the equilibria by their real world counterparts: venture capital finance, angel finance, bank finance and no finance equilibria. We derive the project characteristics that support each equilibrium type, and fully characterize each equilibrium form, including signal choices, contract structure and welfare properties.

*We are grateful to Peter Norman, Mattias Polborn, Per Stromberg and Ján Zábojnik for helpful comments. We appreciate the audiences at the University of Chicago GSB, Carnegie-Mellon University, University of Toronto, University of Illinois, University of British Columbia, and the Western Finance Association Meetings in Tucson. Dan Bernhardt gratefully acknowledges financial support from NSF grant SES-0317700. Stefan Krasa gratefully acknowledges financial support from NSF grant SES-0318394. We are responsible for all errors.

1 Introduction

Three predominant forms of finance are used by entrepreneurs for initial funding of projects: bank finance, venture capital (VC) finance, and angel finance. These forms of finance differ dramatically with respect to ex-ante project screening and the allocation of control rights over managing the project. For example, both VCs and angels screen projects carefully, but while VCs have extensive control rights and participate actively in firm decision making, angels are hands-off. In contrast to the extensive screening done by VCs and angels, most ex-ante screening done by banks is cursory.

This paper develops a theoretical model that explains why and when these forms of finance emerge in equilibrium. In our environment, it is costly for a financier to screen an investment project, and uninformed investors can compete to provide funding. If a financier investigates a potential investment project, he chooses which aspects of the project to investigate, and how careful each investigation should be. In particular, the financier can acquire information about both project quality and about which strategic actions should be taken. If the financier acquires sufficient information, he becomes a better judge of what should be done than the entrepreneur. Finally, a financier who offers funding must design the contract offer in the face of potential outside competition.

Competition from additional outside funding sources generates the key friction in our model. To understand how and why competition can cause a financier to distort his project investigation, suppose that following a thorough evaluation. a financier does not offer funding. Such a decision conveys negative information that can reduce an entrepreneur's willingness to accept funding from less-informed investors. This allows other potential investors to free ride on the financier's screening and undercut contract offers that generate the informational rent necessary to support costly informed finance. In contrast, if a financier limits his investigation sufficiently, then even when the financier rejects the project, the entrepreneur may still be sufficiently uncertain about the project's payoff that he would accept funding from uninformed investors, thereby discouraging uninformed competition. More generally, to discourage uninformed competition and retain sufficient information rents to make costly information acquisition worthwhile, a financier can acquire less information, reduce claims to the project, and conceal in the contract how to best run the project.

Our paper is the first to investigate how such outside competition affects information acquisition and contracting, and we show that incorporating this competition can reconcile the empirical regularities characterizing venture capital and angel finance documented by Gompers and Lerner [18] and Kaplan and Stromberg [26] among others. That is, we provide a coherent, unifying way in which to understand the many empirical facets of informed finance. Equilibrium contract offers must find an optimal balance between distorting information acquisition, lowering the financier's share, and delegating control rights to the financier. Depending on project characteristics, four qualitatively different equilibria exist in our model.

- *Informed Financier with Control Rights:* In this equilibrium, the financier screens project quality and action choice, retains control rights and participates actively in decision making. This form of finance arises when (i) there is enough uncertainty about project outcomes (projects are neither too risky, nor too safe), (ii) the financier has sufficient expertise about evaluating and managing the project, and (iii) the entrepreneur's preferences over actions are not too strong. In equilibrium the financier acquires the socially optimal amount of information about action choice, and he may or may not limit information acquisition about project choice. We show that ex post, the entrepreneur may regret having ceded control rights to the financier.

- *Informed Financier without Control Rights:* In this equilibrium, the financier screens project quality, and possibly action choice, but does not retain control rights. The financier provides investment advice to entrepreneurs, but is not actively involved in managing the project. This form of finance arises when (i) the project is not too unlikely to pay off, nor too safe, (ii) the financier has sufficient evaluation expertise, and either (iii) the costs of choosing the entrepreneur's most preferred action are not too high, or (iv) given the financier's information, the entrepreneur and financier would agree about how best to manage the project.

- *Uninformed Finance:* If projects are too safe, and the financier's advice about the correct strategic action is not too valuable, then only uninformed finance is feasible, even though it may be socially optimal to acquire information.

- *No Finance:* A project that is too unlikely to payoff cannot obtain finance, even though it may be socially optimal to investigate the project and finance it following positive assessments.

The features of these equilibria accord well with their real world counterparts. The informed financier with control rights corresponds closely to a venture capitalist. Gompers and Lerner [18] emphasize that venture capitalists "concentrate investments in early stage companies and high-tech industries where [their information is valuable]", and where venture capital input on corporate strategy is crucial. This is precisely the prediction of our model—informed finance with control rights should arise when there is substantial uncertainty about project quality and hands-on decision making is crucial. Kaplan and Stromberg's [26] empirical analysis of venture capital contracts details the extensive control rights that are delegated to venture capitalists, especially for firms at early development stages (see also Sahlman [30], Gompers [17], Black and Gilson [7]). At early development stages, the venture capitalist holds an average of 65.8% of voting rights if the firm performs well, and even more if it does not. Venture capitalists force about one third of entrepreneurs out of their firms within five years (Hellmann and Puri [24]), suggesting that ex post many entrepreneurs regret ceding control rights to a venture capitalist. Also reflecting the importance of hands-on decision making, Gompers and Lerner [18] find that geographical proximity is important, and the New York Times (June 12, 2000) documents that venture capitalists spend 75-85% of their time working with ongoing investments, and only 15-25% of their time investigating new ventures. We prove that, ceteris paribus, increasing the variance of project payoffs raises the financier's payoff. The empirical evidence reveals that venture capitalists focus on high-risk projects: Cochrane [11] finds that the standard deviation of the venture capitalists return exceeds 100% and his data reveal that over 40% of project lose money ((Bernhardt and Krasa [5]). This high uncertainty leads venture capitalists to scrutinize serious projects intensively (Fried and Hisrich [15], Garmaise [16]), and to reject about 90% of those that they investigate (Bernhardt and Krasa [5]).

There is extensive evidence that venture capitalists are better judges of both the economic viability of entrepreneurial projects, and of how the project should be run (see Garmaise for a summary of the evidence). Most entrepreneurs typically develop only a few projects. In contrast, venture capitalists have extensive industry experience, and are exposed to a wide variety of projects. Their extreme specialized knowledge permits venture capitalists to distinguish winners from losers (Fenn, Liang and Prowse [13]). Indeed, Ljungqvist and Richardson [27] document both the narrow expertise of venture capital, targeting much of their funds to a single industry, and that venture capitalists are successful in picking winners, earning excess returns of 28% relative to the ex-ante cost of capital. While an entrepreneur's information may be fundamental for developing a novel product, venture capitalists are better-placed to evaluate it. That is, entrepreneurs typically have less information than a venture capitalist about the market for its product (and hence value), networking, or the product's likely competition. Not only does a venture capitalist's experience facilitate evaluation, but it also helps them identify the appropriate marketing strategies and key personnel (Byers [9], Bygrave and Timmons [10], Gorman and Sahlman [19], Helmann and Puri [24], and Sapienza [31]), and reduce the time to bring a product to market (Helmann and Puri [24]).)

Informed finance without control rights corresponds closely to angel finance. Angel finance is hands-off (Wong [35] documents this fact and the other empirical regularities below). Like VCs, angels tend to have expertise about the projects that they finance and are more qualified than entrepreneurs to judge a project's merits. Our model predicts that entrepreneurs prefer angel finance to venture capital finance only if choosing a particular management action matters more than maximizing profits, in which case the entrepreneur is very concerned about giving up control rights. This theoretical result is reflected in practice both by (a) the structure of angel finance contracts, and (b) an entrepreneur's decision of when to seek angel finance. In particular, under angel finance the firm's founders retain primary control over the firm's board and cash flows. Further, firms that generate enough revenues delay seeking angel finance on average by one year, indicating that they are concerned about giving up even the limited control rights required by angel finance.

Finally, uninformed finance corresponds closely to bank finance. Fiet and Fraser [14] and Hellmann [23] document that U.S. banks invest in safer projects. By law, U.S. banks (in contrast to European and Asian banks) cannot take an active hands-on role in the running of the firm, as long as the firm is solvent. Further, most U.S. banks devote minimal resources to evaluating (as opposed to monitoring) entrepreneurs, generally using credit scoring programs that use only readily available data (e.g., credit history, collateral, loan size) to determine whether to extend a loan (Akhavein, Frame and White [2], Astebro and Bernhardt [3]).

1.1 Related Literature

Our paper contributes to three research areas: (i) competition between financial intermediaries, (ii) contract design, and (iii) information acquisition decisions by financial intermediaries.

Broecker [8] exogenously endows each potential investor with a signal about the entrepreneur's project, and details conditions under which investors can earn strictly positive profits in the Bertrand equilibrium. In contrast, we endogenize the decision to become informed by a single financier, as well as the nature of the information that the financier acquires. For other aspects of competition between banks see Riordan [29], Dell'Ariccia et al. [12], Winton [34], Yosha [36], Villas-Boas and Schmidt-Mohr [33], Matutes and Vives [28], Smith [32].

The distortion in information acquisition that underlies our results has the flavor of the Grossman and Stiglitz [21] noisy rational expectations result. In their paper, if the equilibrium price is fully revealing then no information is acquired when information acquisition is costly. If, instead, there is added stochastic noise so that the competitive equilibrium price is partially revealing (c.f., Hellwig [25]), then information may be acquired. In what follows we determine the endogenous amount of noise that arises in equilibrium when the financier chooses the signal quality. In particular, we determine the characteristics of the economy for which endogenous noise can support costly information acquisition in equilibrium.

Beginning with Grossman and Hart [20], and continuing with Aghion and Bolton [1] and Hart and Moore [22], researchers have considered the optimal allocation of cash flow and control rights when complete contracts cannot be written and the interests of the contracting parties over action choices may not be aligned. In our paper, contracts are endogenously incomplete, and the allocation of control rights may also permit the financier to retain the informational rent necessary to make informed finance feasible.

Biais and Perotti [6] develop a related model of entrepreneurial finance in which the entrepreneur must aggregate complementary expertises from *multiple* experts to assess and implement a research project, but has to worry about the experts stealing/free-riding on his idea/information. In contrast, in our paper, it is the informed financier (the expert) who is concerned about free riding by uninformed parties.

There is a limited literature on endogenous information acquisition by a financial intermediary. Bernhardt and Dvoracek [4] distinguish between two types of information acquisition: the evaluation prior to a potential investment; and the monitoring of already-funded firms. They derive how the financier's information acquisition is affected by his stake in the firm and the liquidity of the after market for his claims. Bernhardt and Krasa [5] consider a model in which the entrepreneur proposes the funding terms to the investor, who must first decide whether or not to investigate, and whether to provide funding following an investigation. The reversed order of moves eliminates the lemons problem that is the focus of this paper.

2 The Model

Consider a potential entrepreneur with a project. The project requires one unit of external funding to be developed. If developed, the project either pays out 0 or $\bar{x}$. If the entrepreneur does not take on the project, he can work for a reservation wage of $w > 0$, which is public information. One can also interpret w as "entrepreneurial capital," which consists of the market value of patents and product ideas, personal capital, under-compensated and extensive time inputs (sweat equity), reputation, connections and expertise supplied by founders and key personnel if and only if the project is initiated.

The entrepreneur's project can be funded either by a financier who can acquire information about the project at a cost $c > 0$, or by investors who do not acquire information—whom we term uninformed. We consider a single financier who faces potential competition from uninformed investors at two stages. First, the entrepreneur can go straight to an uninformed investor (stage $t = 0$). Second, if the entrepreneur originally pursued informed finance, but was unhappy with contract terms or did not receive funding, the entrepreneur can again seek out uninformed finance (stage $t = 3$).

We consider two dimensions of information acquisition: the project's viability/intrinsic quality and the best method of managing the project. A viable project may pay off $\bar{x}$, while a project that is not viable always pays 0. Let p_X be the ex-ante probability that the project is viable. Whether a viable project pays $\bar{x}$ depends on how it is managed. We consider two payoff-relevant actions, a_1 and a_2. If the correct action is taken, a viable project pays $\bar{x}$. If, instead, the wrong action is chosen, the viable project pays $\bar{x}$ only with probability γ, where $0 \leq \gamma \leq 1$, and pays 0 otherwise. The ex-ante probability that action a_i is the correct action is p_{a_i}. We let $X(a_i)$ denote the random variable describing the project's payoff when action a_i is chosen.

The entrepreneur need not be indifferent between the action choices. For example, the action choices may correspond to whether or not to keep the entrepreneur as a manager of the firm, or which market the firm should target. We assume that action a_1 provides the entrepreneur a private benefit of $\epsilon_1 \geq 0$, and action a_2 provides the entrepreneur a private loss of $\epsilon_2 \leq 0$. These aspects of the economy are common knowledge to the entrepreneur and investors. To capture the fact that the entrepreneur's preferred action is more likely to be the correct action, we assume that $p_{a_1} \geq 0.5$. We assume that the entrepreneur's net opportunity cost is strictly positive, i.e., $w - \epsilon_1 > 0$, and that private benefits matter, i.e., $\epsilon_1 - \epsilon_2 > 0$.[1]

The financier can acquire distinct, independent signals about both the project's viability and the correct action choice. To emphasize the strategic incentives to limit information acquisition we assume that signals of arbitrarily high quality can be acquired, and that once the information acquisition cost $c > 0$ is paid, the marginal cost of more accurate signals is zero. Thus, the *only* reason not to acquire better information is if superior information adversely affects the equilibrium financial contracting terms.

A signal of quality $q_X \in [0, 1]$ reveals whether the project is viable with probability q_X, and is a random draw from the prior with probability $1-q_X$. A signal of quality $q_\Phi \in [0, 1]$ reveals the correct action choice with probability q_Φ, and is a random draw from the prior with probability $1-q_\Phi$. We define $\sigma_X(q_X) \in \{0, \bar{x}\}$ and $\sigma_\Phi(q_\Phi) \in \{a_1, a_2\}$ to be the realized signals about project quality and optimal action respectively, given signal qualities q_X and q_Φ. Where the context is clear, we write σ_X and σ_Φ instead of $\sigma_X(q_X)$ and $\sigma_\Phi(q_\Phi)$.

Timing of Decisions.

t=0 The entrepreneur chooses whether to seek informed or uninformed finance.

t=1 If informed finance was sought, then the financier decides whether to investigate the firm. If the financier investigates, he chooses a contingent contract that specifies signal qualities[2] q_X and q_Φ, an equity share k, and control rights that apply *only if* the financier provides funding. If, instead, uninformed finance was sought, then the uninformed investors offer contracts that specify the share of the firm's payout that they will receive, and the control rights allocation.

t=2 **(a)** The financier privately observes signals $\sigma_X(q_X)$, $\sigma_\Phi(q_\Phi)$ and decides whether to extend finance at the terms specified by the contingent contract; and

(b) Uninformed investors can offer contracts.

t=3 The entrepreneur either selects a contract, or rejects funding and pursues his alternative.

t=4 The financier can announce signals $\sigma_X(q_X)$ and $\sigma_\Phi(q_\Phi)$ (cheap talk).

t=5 Actions are chosen by the party with control rights and payoffs are realized.

[1]We only use this latter assumption to eliminate additional equilibria based on entrepreneur indifference.

[2]Signal qualities need not be contractible if they are observed by the entrepreneur and he can convey these signal qualities to uninformed investors.

2.1 Discussion of Model Assumptions

Why contingent contracts? It is advantageous for the investor to specify terms ex ante, thereby introducing commitment. Absent this commitment, the contract offer may signal information, which leads to multiple equilibria, including the equilibrium that we identify. The other equilibria are sub-optimal from the viewpoint of the investor because the investor gets a smaller share. Such equilibria are supported by the entrepreneur's beliefs that a larger share demand k indicates a bad signal realizations. By offering the contingent contract ex ante, such equilibria are precluded, maximizing the investor's payoff.

Venture contracts feature precisely such contingent terms. A representative contract provided to us by Per Stromberg consists of an initial contingent contract offer that specifies the financing terms (e.g., investment levels, equity shares, dividends). Funding is contingent on a positive project evaluation by the venture capitalist. If funding is extended, then the financing terms are those specified in the initial offer. The contract offer permits the venture capitalist to back out after due diligence in the next two months on his part: "These terms do not constitute any form of binding contract . . . nothing contained herein shall be considered binding until executed by *both* parties . . . The investment is contingent upon. . . "

Why a cheap talk stage? A key distinction between venture capital and angel finance concerns the allocation of control rights. In our model, the allocation of control rights corresponds to selecting the agent who makes the action choice. A venture capitalist has control rights, while an angel delegates action choice to the entrepreneur. An informed financier can discourage uninformed competition by concealing information about action choices prior to the entrepreneur's choice of financing source. Clearly, a venture capitalist can do this simply by not taking the action until after the entrepreneur accepts his funding offer. To avoid building an unfair advantage into venture capital over angel finance, we introduce a stage after the contract has been signed in which an angel can recommend actions to the entrepreneur, thereby communicating how the project should be managed. In practice, angels often provide advice to entrepreneurs—which they do not have to follow—and we want to capture this possibility.

Why competition only with uninformed investors? We provide the simplest model in which an informed investor faces competition. One could contemplate additional competition from informed parties at the ex-ante or ex-post stage. We will argue in Section 5 that the distortions in information acquisition and difficulty of supporting informed finance are only reinforced by competition at the ex ante stage, and that two sequential stages of informed competition lead to an outcome equivalent to the one we analyze, because the first investor will not acquire information.

In practice, for first stage finance, it is very rare for multiple venture capitalists to investigate the same project seriously. "Venture capitalists trade information quite freely and frequently" (Paul Keaton, http://www.i2m.org/ftp/freepubs/0501.pdf). To eliminate simultaneous competition, contracts feature no-shop provisions over the period where the venture capitalist is prepared to offer funding.[3] After this period, the entrepreneur can shop for other contracts, and this includes the uninformed finance that we model. The informed investor anticipates the threat of possible competition from uninformed sources, and designs his original contract in such a way that no competitor has an incentive to offer funding.

3 Equilibria with Hands-Off Finance

We first characterize equilibrium outcomes when action choices are not too important for outcomes, so that it is efficient to take the entrepreneur's preferred action. Formally, we suppose that γ is large enough that the expected output cost of taking the entrepreneur's preferred action a_1, even when it is "wrong," is less than the benefits that accrue to the entrepreneur from having his preferred action taken. As a result, it is not necessary for the financier to investigate which action maximizes the project's payoff, as it is optimal to set $q_\Phi = 0$ and delegate the action choice to the entrepreneur.

Theorem 1 proves that when taking the wrong action is not too costly, equilibria of the game must correspond to solutions of the following optimization problem:

Problem 1

$$\max_{q_X, k\in[0,1]} p_X\Big(E[kX(a_1)|\sigma_X(q_X)=\bar{x}]-1\Big)-c \tag{1}$$

subject to

1. $E[(1-k)X(a_1)|\sigma_X(q_X)=0]+\epsilon_1 \geq w$.

2. $E[kX(a_1)] \leq 1$.

The financier's objective is his ex-ante expected profit given that finance is extended if and only if the project viability signal is good, i.e. if and only if $\sigma_X(q_X)=\bar{x}$. The financier chooses the project viability signal quality q_X and equity share k to maximize these profits.

To understand why constraints 1 and 2 must hold in equilibrium, first note that if information acquisition is too accurate, i.e., if q_X is too close to one, then constraint 1 is violated as $E[(1-k)X(a_1)|\sigma_X(1)=0]+\epsilon_1=\epsilon_1<w$. The left-hand side of constraint 1 is the entrepreneur's expected payoff when the financier receives a bad signal, where $\epsilon_1 \geq 0$ is the private benefit that the entrepreneur receives when his preferred action is taken. The right-hand side is the entrepreneur's payoff when he does not pursue the project.

[3] In the representative contract, "the (firm) agrees to deal exclusively with (the venture capitalist) for a (two month) period." "The parties shall use their best efforts to close the transaction (within five weeks), but in no event, beyond (the two month period)." (During this two-month period) "the company agrees not to pursue or respond to competitive financing from other parties."

To see why the financier cannot acquire such accurate information, suppose that constraint 1 is violated. The informed financier will fund the project if and only if his signal is good. When the financier fails to offer funding, the entrepreneur realizes that the signal was bad and when constraint 1 does not hold, a share $1-k$ does not yield him an expected payoff that covers his opportunity costs, $w-\epsilon_1$. But then, even if uninformed investors offer slightly more favorable terms to the entrepreneur, $k' < k$, the entrepreneur would reject them whenever the financier fails to offer funding. That is, the offer k' by uninformed investors is only accepted when the informed financier received a good signal. As a result, the financier's gross profit must be zero and he cannot cover his investigation costs $c > 0$.[4]

Constraint 1 ensures that if an uninformed investor offers a contract with a share $k' < k$, then the entrepreneur would always accept the contract. Constraint 2 is then necessary for the profits from offering k' to be negative for all $k' < k$. Theorem 1 documents that when γ is sufficiently large, all equilibria with informed finance are described by Problem 1.

Theorem 1

1. *If, ceteris paribus, γ is sufficiently large and informed finance is offered, then the equilibrium k and q_X solve Problem 1.*

2. *Conversely, if the k and q_X that solve Problem 1 generate a non-negative financier payoff, then for γ sufficiently large, there exists an equilibrium with informed hands-off finance in which the financier's demands a share k and acquires a project viability signal of quality q_X. In equilibrium, the entrepreneur strictly prefers informed finance to uninformed finance when constraint 2 of Problem 1 is slack, but he is indifferent if constraint 2 binds.*

The key intuition for Theorem 1 is as follows. If the loss due to taking the wrong action is small compared to the entrepreneur's private benefit from selecting his preferred action, a_1, then social surplus is maximized by taking action a_1, even when the action signal recommends a_2. Because the financier has all bargaining power, the financier maximizes his own payoff by ensuring that action a_1 is taken. He does this by delegating the action choice to the entrepreneur.[5] Note that Theorem 1 implies that entrepreneurs with projects that ex ante are less likely to pay off gain more from having a financier investigate their merits, as these are the projects for which constraint 2 is slack.

[4]There is also no mixed strategy equilibrium with excessively good information acquisition: if the financier mixed between funding a good project and not, then indifference demands that he make zero gross profits (the profits when he chooses not to fund). But then again the financier does not cover his investigation costs. Finally, this result generalizes to arbitrary realizations of x as long as w is sufficiently large, or parties are not restricted in the nature of the contracts that they offer.

[5]Theorem 1 also rules out other potential equilibria. For example, in one of these potential equilibria, the financier could extend funding only when $\sigma_X(q_X) = \bar{x}$, and $\sigma_\Phi(q_\Phi) = a_1$.

It is important to observe that along the equilibrium path, uninformed investors do not need to make offers. Nevertheless, the threat of competition from uninformed investors influences the financier's actions. In particular, the financier's contingent contract is designed so that uninformed investors cannot profitably compete; and in equilibrium, the investors need not bother.

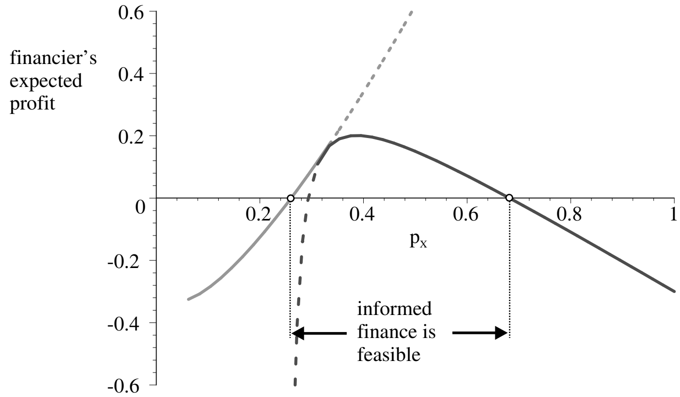


Figure 1: The profit of an informed financier

The graph shows that financier's ex-ante expected profit for the following parameter values: $\bar{x} = 4$, $w - \epsilon_1 = 0.1$, $c = 0.3$, $\gamma_1 = 1$.

We next characterize the solution to Problem 1. Figure 1 illustrates how the financier's equilibrium payoffs varies with the probability p_X that the project is viable. Theorem 2 proves that the key features illustrated in Figure 1 hold generally. Figure 1 reveals that if p_X is too small or too large, then informed finance is infeasible. This is obvious if p_X is small. When p_X is large, uninformed investors would face little risk from competing if the financier tried to retain too much surplus. As a result, uninformed investors would be willing to compete so aggressively that the financier cannot recover the costs of investigation, even when it is socially efficient to acquire information. Such safe projects still have a positive ex-ante NPV, and hence receive uninformed finance.

As Figure 1 reveals, for small values of p_X where only constraint 1 binds, profits are a convex, increasing function of p_X.[6] In this region, $E[kX(a_1)] < 1$ so that any investor who offers the same contract as the financier would lose money. Therefore, it is sufficient for the financier to generate a lemons problem—k and q_X are not limited in any other way. The intuition for the convexity is as follows. When the project is unlikely to pay off, it is hard to make uninformed finance attractive to the entrepreneur after a negative project evaluation—the financier must both take a small share of the firm and choose a low signal quality. Fixing k and q_X, the financier's ex-ante payoff would increase linearly in p_X, as the objective (1) would be

[6] In the figure the solid line is the actual payoff. The dotted portion of the convex curve shows the financier's profit were we to ignore the fact that constraint 2 binds, so that $E[kX(a_1)] > 1$. The dotted part of the concave curve assumes that constraint 2 binds, so that $E[kX(a_1)] = 1$, even when p_X is small enough that it is optimal for the financier to choose a smaller share, and to acquire better information, instead.

the product of p_X and a constant. But, in addition, raising p_X makes it is easier to induce a lemons problem. As a result, the financier can increase both q_X and k without inviting uninformed competition. This increase of k and q_X raises ex-ante payoffs from linear to convex.

The financier's ex-ante payoff function becomes concave when constraint 2 binds so that $E[kX(a_1)] = 1$. Once constraint 2 binds (at the tangency point of the convex and concave curves), raising p_X further allows uninformed investors to compete more aggressively. As a result, the financier must lower k to prevent investors from undercutting his contract, i.e., to retain $E[kX(a_1)] = 1$. Therefore, the linear increase in ex-ante payoff due to the increase of p_X is dampened by the reduction of k, causing payoffs to be concave. Finally, because the slope of ex-ante payoffs is positive at the tangency point, it follows that the p_X that maximizes the financier's ex-ante payoff is on the concave portion. Theorem 2 summarizes the key features.

Theorem 2 *Indexing projects by the ex-ante probability p_X that a project is viable, there exists a $\hat{p}_X$ such that for $p_X < \hat{p}_X$ only constraint 1 binds; and for $p_X > \hat{p}_X$, both constraints 1 and 2 of Problem 1 bind. Information acquisition about the project's viability is always distorted, so that $q_X < 1$. Further,*

1. *Financier profits are strictly positive if and only if $p_X \in (\underline{p}_x, \bar{p}_X)$, where $0 < \underline{p}_x \leq \bar{p}_X < 1$. The interval $(\underline{p}_x, \bar{p}_X)$ is non-empty if and only if the information cost c is sufficiently small.*
2. *The financier's payoff is a strictly convex and strictly increasing function of p_X for $p_X < \hat{p}_X$.*
3. *The financier's payoff is a strictly concave function of p_X for $p_X > \hat{p}_X$ with a strictly interior maximizer, p_X^*. At p_x^*, the project has a strictly positive ex ante NPV.*
4. *Projects with $p_X > \bar{p}_X$ have a strictly positive ex ante NPV and hence receive uninformed finance.*

The proof to Theorem 2 details the equilibrium values of q_X and k, that underlie the financier's ex-ante expected profits. Theorem 2 suggests that the financier prefers riskier projects. Still, we cannot yet draw this conclusion as p_X affects both a project's mean return and its risk. We now show that the value of information acquisition is greater if, ceteris paribus, the project's variance is higher. It follows that a risk-neutral informed financier prefers riskier projects. Positive NPV projects that are too safe can only receive uninformed bank finance. Thus, we can reconcile empirical findings in Fiet and Fraser [14] and Hellmann [23], who document that banks invest in safer projects. We now formally state this result.

Theorem 3 *If the equilibrium outcomes are characterized by the solution to Problem 1 then for a given expected project payoff $p_X\bar{x}$, increasing the project variance raises the financier's ex-ante expected profit. Informed finance is infeasible if the project's variance is too low.*

4 Equilibria with Hands-On Finance

We now characterize equilibrium outcomes when information about actions matters. Formally, this means that $\gamma < 1$ and the entrepreneur's private loss, $\epsilon_1 - \epsilon_2$, from taking action a_2 rather than a_1 is not too large. The financier's information acquisition choice is now multi-dimensional: both the project viability signal quality, and the action signal quality matter. Further, the financier's decision about whether to extend funding may conceivably reveal information about both the project's viability and the optimal action choice.

Theorem 4 below details that equilibria of the game can again be characterized by the solution to a constrained optimization problem for the financier. In equilibrium, the financier extends funding if and only if he receives a good signal about the project's viability. The choice to fund a project therefore reveals information about the project's viability to the entrepreneur, but does not reveal information about the optimal action. The financier "hides" his information about the right action by offering a hands-on contract that gives him control rights. The cost of hiding this information is that the entrepreneur does not know whether his preferred action will be chosen. The benefit is that neither uninformed investors nor the entrepreneur can extract information about the action choice. As a result, uninformed investors cannot compete by simply offering a share k' that is smaller than the share k offered by the financier. Rather, an investor's share must account for these costs and benefits. We now state the optimization problem and then explain the constraints.

Problem 2

$$\max_{q_X, k, \tilde{k} \in [0,1]} p_X \Big(E[kX(a_i)|\sigma_X(q_X) = \bar{x}, \sigma_\Phi(1) = a_i] - 1 \Big) - c \tag{2}$$

subject to

1. $E[(1-\tilde{k})X(a_1)|\sigma_X(q_X) = \bar{x}] + \epsilon_1 = E[(1-k)X(a_i)|\sigma_X(q_X) = \bar{x}, \sigma_\Phi(1) = a_i] + \sum_j p_{a_j}\epsilon_j$

2. *If* $q_X < 1$ *then*

   (2a) $E[(1-\tilde{k})X(a_1)|\sigma_X(q_X) = 0] + \epsilon_1 \geq w$

   (2b) $E[\tilde{k}X(a_1)] \leq 1.$

3. *If* $q_X = 1$ *then*

   (3a) $E[\tilde{k}X(a_1)|\sigma_X(q_X) = \bar{x}] \leq 1.$

   (3b) $E[(1-k)X(a_i)|\sigma_X(q_X) = \bar{x}, \sigma_\Phi(1) = a_i] + \sum_j p_{a_j}\epsilon_j \geq w.$

The argument of Problem 2 is the financier's ex-ante expected profit. Because the contingent contract conveys no information about the best action, uninformed investors cannot free ride on a financier's acquired expertise about actions. As a result, the financier chooses $q_\Phi = 1$. Via constraint 1, the share k and signal quality q_X determine a share $\tilde{k}$ with the following property: were an uninformed investor who does not know the best action to offer $\tilde{k}$ and the financier were to offer funding at share k, then the entrepreneur would be indifferent between informed and uninformed finance.[7] An investor who wishes to undercut the financier's contingent contract must therefore ask for a share $\tilde{k}' < \tilde{k}$.

The financier has two options. One option is to generate a lemons problem for uninformed investors as in Problem 1. That is, the financier can choose signal qualities so that if an investor undercut the financier's contract terms, then the entrepreneur would accept the investor's offer even when the financier did not offer funding. The financier's other option is to exploit his knowledge about the correct action and acquire full information about project viability. Because uninformed investors cannot free ride on information about action choice, they may not compete even if they can infer that the entrepreneur would not pursue funding whenever the financier does not extend funding.

The financier then chooses whether or not to induce a lemons problem to maximize profits. Constraints 2(a) and 2(b) apply when the financier generates a lemons problem for uninformed investors, and are hence the analogues to constraints 2 and 3 of Problem 1. Together, the constraints imply that an uninformed investor who undercuts by offering a share $\tilde{k}' < \tilde{k}$ faces a lemons problem that results in losses.

Constraints (3a) and (3b) of Problem 2 apply when the financier chooses not to create a lemons problem for uninformed investors. In this case, the financier chooses $q_X = q_\Phi = 1$ because there is no longer a reason to distort information acquisition. Then, if the financier does not extend funding, the entrepreneur infers that his project is certain to fail. As a result, the entrepreneur will not accept funding from other sources. Constraint (3a) ensures that no investor wants to offer funding even knowing that the entrepreneur would only accept their offer when the project is viable. Formally, if an investor undercuts with $\tilde{k}' < \tilde{k}$ then (3a) implies $E[\tilde{k}' X(a_1)|\sigma_X(q_X) = \bar{x}] < 1$, i.e., the investor loses money. Finally, constraint (3b) ensures that the entrepreneur receives at least his outside payoff.[8]

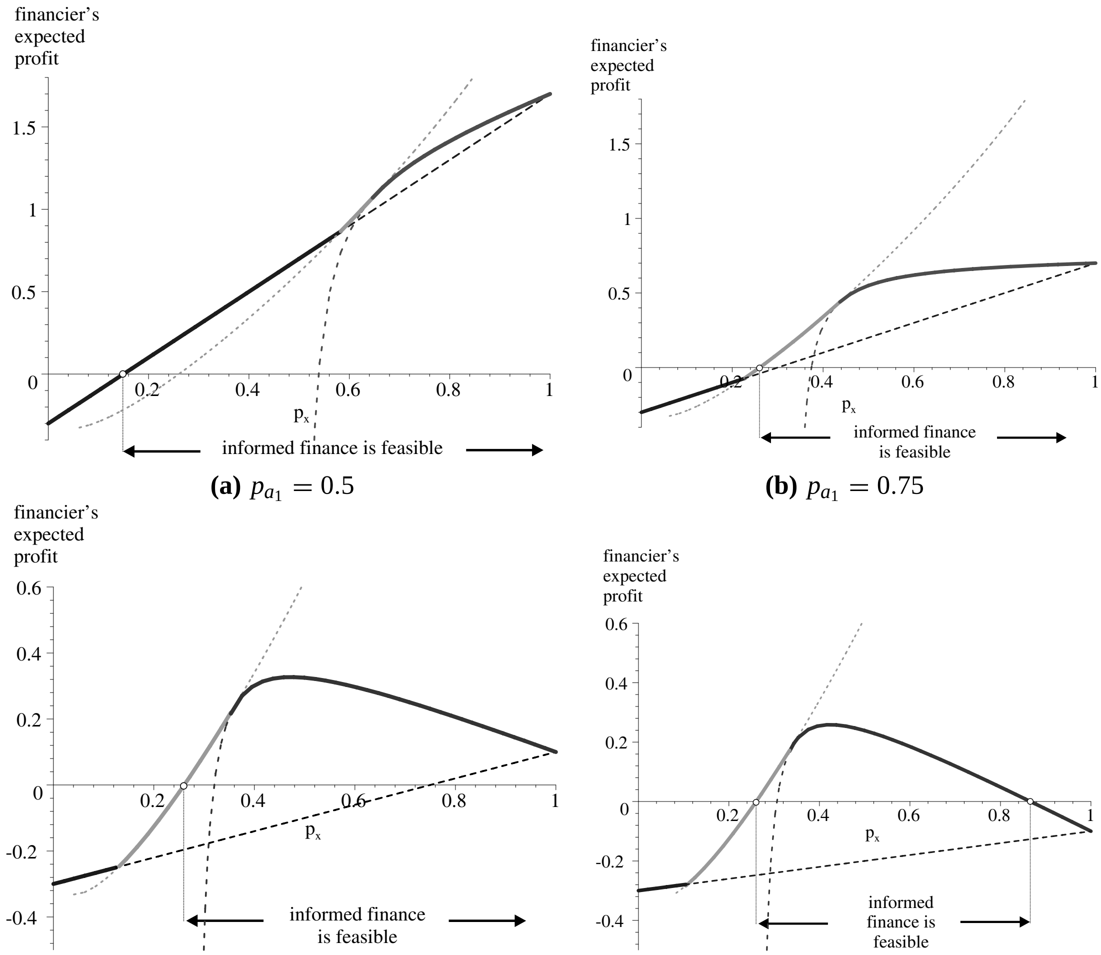
Theorem 4 provides sufficient conditions for all equilibria to correspond to solutions of Problem 2.

Theorem 4 *1. If, ceteris paribus, γ and $\epsilon_1 - \epsilon_2$ are not too large, and informed finance is offered, then $q_\Phi = 1$, and k and q_X solve Problem 2.*

2. Conversely, if the k and q_X that solve Problem 2 generate a non-negative financier payoff, then for γ and $\epsilon_1 - \epsilon_2$ not too large, there exists an equilibrium with informed hands-on finance. In this

[7] The left-hand side of constraint 1 is the entrepreneur's expected payoff if he accepts the investor's contract instead of the financier's when the financier offers funding. Thus, the expectation is conditioned on $\sigma_X = x$. The right-hand side is the entrepreneur's expected payoff from accepting the financier's contract. The expectation is also conditioned on $\sigma_\Phi(1) = a_i$, because the financier knows the correct action. Because both actions are chosen in equilibrium, the entrepreneur's ex-ante expected private benefit is $p_{a_1}\epsilon_1 + p_{a_2}\epsilon_2$.

[8] As we detail in the proof of Theorem 4, constraint (3b) can only bind when uninformed finance is infeasible. This is the only situation in which the entrepreneur's payoff can be driven down to his outside payoff.



(a) $p_{a_1} = 0.5$ **(b)** $p_{a_1} = 0.75$ **(c)** $p_{a_1} = 0.9$ **(d)** $p_{a_1} = 0.95$

Figure 2: The profit of an informed financier

The solid line in the graph shows that financier's ex-ante expected profit for the following parameter values: $\bar{x} = 4$, $w = 0.1$, $\epsilon_1 = \epsilon_2 = 0$, $c = 0.3$. Four different values of p_{a_1} are shown.

equilibrium, the financier offers a share k and acquires a signal of quality q_X about the project's viability, and acquires a signal of quality $q_\Phi = 1$ about the correct action choice.

Intuitively, if it is socially optimal to choose action a_2 when recommended, then it is in the financier's interest to select a hands-on contract and always select the recommended action. The assumptions that γ and $\epsilon_1 - \epsilon_2$ are not too large—i.e., taking the wrong action is sufficiently costly and the entrepreneur does not prefer action a_1 by too much—simply ensure that this is so.

We now solve for the optimal shares k, $\tilde{k}$, and for q_X that characterize the solutions to Problem 2. Figure 2 illustrates how equilibrium payoffs vary with p_{a_1}, which captures the level of uncertainty about how to manage the project optimally. The concave and convex portions of the financier's payoffs correspond to when the financier induces a lemons problem. The reasoning underlying their curvature is identical to that for Figure 1. The linear portion of the financier's payoff applies when the financier chooses not to induce a lemons problem. In this case $q_X = q_\Phi = 1$ and k do not depend on p_X, so that the financier's ex-ante expected payoff (2) rises linearly with p_X. The financier does not induce a lemons problem when p_X is small because he must then choose q_X and k to be small; otherwise, the offer of an investor who undercuts is only attractive to the entrepreneur when the financier also extends funding. When p_X is higher, the financier finds it more attractive to induce a lemons problem, as he can do so with a larger share k and a more accurate project viability signal. Contrasting Figures 3(a)-3(d) reveals how the financier's choices vary with p_{a_1}. Note that the attraction of not inducing a lemons problem is higher when p_{a_1} is smaller, and is maximized by $p_{a_1} = 0.5$. This is because when p_{a_1} is closer to 0.5, uninformed investors are more likely to select the wrong action, raising the costs of uninformed finance to the entrepreneur.

Theorem 5 proves that the features of the financier's payoffs depicted Figure 2 hold true generally.

Theorem 5 *Indexing projects by the ex-ante probability that a project is viable, there exist $\hat{p}_X^2, \hat{p}_X^3$ such that*

1. *For all $p_X < \hat{p}_X^3$, constraint 3 applies. In this region, $q_X = q_\Phi = 1$, i.e., information acquisition is not distorted, and the financier's expected payoff is a linear, strictly increasing function of p_X.*
2. *If $\hat{p}_X^3 < \hat{p}_X^2$, then constraint (2a) binds but constraint (2b) does not, for all p_X with $\hat{p}_X^3 < p_X < \hat{p}_X^2$. In this region, the financier's expected payoff is a strictly convex, strictly increasing function of p_X, and the financier selects $q_X < 1$ and $q_\Phi = 1$.*
3. *If $\hat{p}_X^3 < 1$, then for $p_X > \max\{\hat{p}_X^2, \hat{p}_X^3\}$, both constraints (2a) and (2b) bind. In this region, the financier's payoff is a strictly concave function of p_X, and the financier selects $q_X < 1$ and $q_\Phi = 1$.*
4. *Financier profits are non-negative for $p_X \in [\underline{p}_x, \bar{p}_X]$, where $\underline{p}_x > 0$ and $\bar{p}_X \leq 1$. The region is non-empty if and only if c is not too large.*

Clearly, an equilibrium with informed finance only exists if the financier's payoff is non-negative. Again, projects will not be funded if the probability p_X that the project is viable is too small. However, in contrast to hands-off finance, projects with $p_X = 1$ could receive funding, as Figures 2 (a)–(c) indicate, because the financier's expertise about how to manage the project may give him a sufficient informational advantage over uninformed investors. For uninformed investors, a project with $p_X = 1$ is only safe when p_{a_1} is close to one, because the project pays $\bar{x}$ only with probability $p_{a_1} + (1 - p_{a_1})\gamma$. Thus, p_{a_1} captures uncertainty about how to manage the project optimally. For uninformed investors, this uncertainty translates into uncertainty about the project's payoff. It follows immediately that the financier's ex-ante expected payoff falls as projects become safer in the sense that p_{a_1} is closer to 1. That is, the financier can derive greater profits from projects where his expertise about actions matters. For example, if we interpret a_1 as the action of retaining the entrepreneur in charge of the firm, and action a_2 as replacing the entrepreneur, then venture capitalists should be more likely to finance projects where p_{a_2} is significantly larger than 0. Consistent with this, Hellmann and Puri [24] document that venture capitalists force about one third of entrepreneurs out of their firms within five years.

We have set up the game to allow the financier to convey information to the entrepreneur about the appropriate action choice once the entrepreneur has accepted the contract. By delaying the revelation of the correct action until after the entrepreneur accepts the financier's funding offer, the financier makes it less attractive for uninformed investors to compete. Clearly, if at this stage the entrepreneur's and the financier's interests are aligned with regard to action choice, then the financier would want to convey the correct action to the entrepreneur, and the entrepreneur would follow the financier's advice. As a result, the same outcomes could also be implemented by a contract in which the financier tells the entrepreneur the correct action *after* the entrepreneur has accepted the contract terms. In this case, either angel or venture capital finance can be used. If, instead, interests are not aligned because the entrepreneur would prefer to take his preferred action a_1 even when action a_2 maximizes expected project payoffs, then the hands-on equilibrium outcome can only be implemented by giving the financier control rights. Thus, venture capital finance is necessary. That is, the following is immediate.

Theorem 6 *Suppose that a hands-on equilibrium exists. Then the hands-on equilibrium outcomes cannot be implemented by giving the entrepreneur control rights if and only if the entrepreneur would strictly prefer action a_1 when a_2 maximizes project payoffs, i.e., the entrepreneur and financier's interests are not aligned.*

Clearly, interests are less likely to be aligned the more the entrepreneur dislikes action a_2. However, if the entrepreneur's disutility from action a_2 is too large, then hands-on finance is no longer optimal, and the financier may instead offer the hands-off contract characterized in section 3. The following theorem characterizes projects for which hands-on finance is optimal, but interests are not aligned ex post. More specifically, the theorem shows that interests are only aligned if the project has a sufficient ex-ante NPV.

Theorem 7 *Consider a project with $E[X(a_1)|\sigma_X(1) = 1] > 1$. Then if w, $\epsilon_1 - \epsilon_2$, and c are not too large:*

1. *An equilibrium with hands-on finance exists. In this equilibrium the financier generates a lemons problem for investors.*

2. *Ex post the interests of the entrepreneur and the financier with respect to action choice are not aligned if and only if the project's ex ante expected NPV is not too positive.*

The condition that w, $\epsilon_1 - \epsilon_2$, and c are not too large simply ensure that informed finance is provided and the contract is characterized by the solution to Problem 2 subject to constraints 2(a) and 2(b). The condition that w is small implies that it is easy to produce a lemons problem; if the financier does not induce a lemons problem, then he chooses $q_X = q_\phi = 1$, in which case interests are always aligned. The intuition underlying Theorem 7 is that raising a project's ex-ante NPV raises the opportunity cost to the entrepreneur of taking the wrong action, so that eventually interests are aligned. Thus, as long as the financier can freely transmit his information to the entrepreneur, the allocation of control rights to the venture capitalist is vital only for sufficiently risky projects that have low ex-ante NPVs. The reason that the entrepreneur may seek venture capital finance even though interests may not be aligned ex post is that it permits the venture capitalist to exploit his expertise about action choice. In practice, interests are not aligned even when the project has a "large" ex-ante NPV and w, $\epsilon_1 - \epsilon_2$, and γ are not "small". For example, if $\bar{x} = 4$, $\gamma = 0.7$, $w = 0.15$, $\epsilon_1 = 0.1$, $\epsilon_2 = -0.3$, and $q_{a_1} = 0.75$, then interests are not aligned as long as $p_X < .417$, or equivalently for projects with ex ante expected returns (*net* of $w - \epsilon_1$) that are less than 49.4%.

Inspection of constraint 1 of Problem 2 reveals that the entrepreneur does not gain directly from the financier's superior information about the correct action. This is because equilibrium contracting terms are driven by the threat of competition from uninformed investors who cannot free ride on this information. That is, unless information about the correct action choice causes the financier to acquire more information about project viability, the rents to knowing the correct action choice accrue solely to the financier. Calculations of q_X for Problems 1 and 2 reveal that the financier acquires at least as much information about project viability with hands-on finance (Problem 2) as he would with hands-off finance (Problem 1). Further, he acquires strictly more information as long as constraints (2a) and (2b) of Problem 2 do not both bind. Inspection of constraint 1 of Problem 1, and constraint (2a) of Problem 2 reveal that the entrepreneur's payoff is monotonically increasing in q_X.[9] Thus, the entrepreneur gains indirectly from information acquisition about action choice.

We now derive the analogue to Theorem 3 for hands-on finance. That is, consistent with the empirical findings that we have highlighted, we now show that venture capitalists find riskier projects more attractive.

Theorem 8 *Suppose it is optimal for the financier to extend hands-on informed finance. Then, fixing the expected project payoff, increasing the project variance raises the financier's ex-ante expected profit.*

[9]In Problem 1 the entrepreneur's payoff is $E[(1-k)X(a_1)|\sigma_X(q_X) = \bar{x}] = E[(1-k)X(a_1)|\sigma_X(q_X) = 0]\frac{q_X + (1-q_X)p}{(1-q_X)p} = (w - \epsilon_1)\frac{q_X+(1-q_X)p}{(1-q_X)p}$, which is increasing in q_X. The argument for Problem 2 is similar, except we must also use constraint 1.

5 Discussion and Further Research Questions

We now discuss some of our assumptions and how altering them opens interesting directions for future research.

Action choices. We only consider two action choices, but, in practice, a broader range of action choices may matter. Obviously, additional actions do not change our qualitative findings if the financier does not acquire information about actions, as the entrepreneur's most preferred action would then be taken. The primary difference is that, typically, with many actions, interests are not aligned for every action. For example, interests may be aligned for all but the action of forcing the entrepreneur out of the firm. Then, the equilibrium outcome with informed finance is either venture capital finance, in which control rights are allocated to the financier and there is ex-post regret when the entrepreneur is fired; or angel finance, in which the entrepreneur retains control, and the angel provides more limited advice about action choice. This accords with common practice: although angels do not have control rights, they provide guidance to entrepreneurs. The question becomes: How do angels design the advice optimally?

Project realizations. We consider two project realizations, 0 and $\bar{x}$, to circumvent issues about contract design. With two realizations and limited liability, there are no differences between debt and equity. With more states, debt and equity continue to be equivalent if no information is acquired and agents are risk neutral, but they are not equivalent with informed finance. Qualitatively, our results extend if we restrict attention to equity finance, which is the form of finance that angels and venture capitalists adopt. It is interesting that with many possible realizations, the entrepreneur's and financier's interests may cease to be aligned over whether to pursue the project, even when action choice does not matter. For example, with debt finance, the financier would extend funding if he were sure to be repaid, but the entrepreneur would only want to pursue the project if he would cover his opportunity cost, w. The questions become: What are the advantages or disadvantages of informed debt versus informed equity? When are interests over funding aligned?

Other Informed Competition. We assume that there is no competition from other informed financiers, which accords with the observation that venture capitalists rarely compete with each other at the investigation stage. Even so, introducing another informed financier at the investigation stage only reinforces our qualitative findings. In particular, in order to generate a lemons problem, the entrepreneur has to be willing to pursue funding when he is rejected by all informed financiers. As a result, information acquisition is further distorted by competition at the ex-ante stage, and it is more difficult to support the informational rent necessary for informed finance. Note that modeling competition of informed financiers sequentially does not alter our predictions, because the first financier would not investigate, and the second would behave as in our model.

Bargaining power. We give the financier most of the bargaining power, subject only to the threat of outside competition. In Bernhardt and Krasa [5] we allow the entrepreneur to choose among incentive compatible informed equity, informed debt and uninformed finance, where the entrepreneur can set the terms of funding given that (i) it must be incentive compatible for the financier to investigate, and (ii) extend funding. We characterize how the form of finance depends on the project's characteristics, and show that, even with all bargaining power it is not optimal for the entrepreneur to extract all surplus from venture capitalists.

Outside payoff. We assume that w is common knowledge. In practice, the entrepreneur's outside opportunity may vary with the entrepreneur, and be private information to the entrepreneur. For example, suppose that the entrepreneur's opportunity cost is either high, w_h, or low, w_ℓ. Then the contracting terms can be chosen so that the types are either pooled or separated, with resulting implications for the lemons problem that uninformed investors face. With separation, one type receives either no finance or uninformed finance, and the other type may receive informed finance. The question then becomes: How does the financier design the contracts to separate or pool types optimally?

6 Conclusion

This paper shows how potential competition affects the type of funding that an entrepreneur can expect to receive. Very generally, we characterize which ex-ante project types may receive venture capital finance, angel finance, or bank finance at the initial stage of funding, and which projects are unable to obtain finance. We also derive the consequences for the nature of the information acquisition decisions of the financier and the contract that he offers. As we have emphasized, the resulting equilibrium outcomes can reconcile many empirical regularities. Further, our basic model can be extended to consider important questions about the relationship between project characteristics and the form of finance.

Finally, we observe that the economic principles that we identify are germane to other economic environments. For example, in competitive labor environments, an employer may be reluctant to investigate its employees' abilities in order to place them efficiently in job assignments to the extent that competing firms can free ride on the information that is revealed by the ultimate job assignment.

7 Appendix

Proof of Theorem 1. First, note that constraints (1) and (2) of Problem 1 are necessary. Otherwise, there would be no lemons problem for uninformed investors, and the financier could not recover costs c. At date $t = 1$ the financier selects a contingent contract that maximizes his expected profit. Thus, the equilibrium values k and q_X solve Problem 1.

Conversely, assume that k^I and q_X^I solve Problem 1 and that financier profits are non-negative. Then, given k^I and q_X^I, investors cannot make a profitable offer. Next, it is immediate that the entrepreneur prefers informed to uninformed finance at $t = 0$: Constraint (2) implies that $k^U \geq k^I$, where equality holds only if constraint (2) binds. Note that the entrepreneur receives w when funding is not offered.

There are two possible signals with informed finance: $\sigma_X(q_X^I) = \bar{x}$ and $\sigma_X(q_X^I) = 0$. When $\sigma_X(q_X^I) = \bar{x}$, comparing informed and uninformed finance, if $k^U > k^I$, the entrepreneur is strictly better off with informed finance, and if $k^U = k^I$, he is indifferent. Now consider $\sigma_X(q_X^I) = 0$. With informed finance, the entrepreneur would not be funded, and hence would receive w. With uninformed finance, $k^U \geq k^I$ and the fact that constraint (1) binds for k^I implies that with uninformed finance, the entrepreneur's expected payoff is less than or equal to w. Hence, the entrepreneur is indifferent to informed and uninformed finance if $k^U = k^I$, and he stricly prefers informed finance if $k^U > k^I$.

It remains to prove that the financier cannot improve by offering hands-on finance when γ is large enough and that finance is offered if and only if $\sigma_X(q_X) = \bar{x}$. Assume the financier offers a hands-on contract where funding is provided if and only if $\sigma_X(q_X) = \bar{x}$ and the recommended action $\sigma_\Phi(q_\Phi) = a_i$ is chosen. Let k be the financier's share. Then investors can compete with a hands-off contract with share $\tilde{k}$ that fulfills

$$E[(1-\tilde{k})X(a_1)|\sigma_X(q_X) = \bar{x}] + \epsilon_1 > E[(1-k)X(a_j)|\sigma_X(q_X) = \bar{x}, \sigma_\Phi(1) = a_j] + \sum_j p_{a_j}\epsilon_j. \quad (3)$$

First, assume there is no lemons problem for investors, i.e.,

$$E[(1-\tilde{k})X(a_1)|\sigma_X(q_X) = 0] + \epsilon_1 < w. \quad (4)$$

Then if $E[\tilde{k}X(a_1)|\sigma_X(q_X) = \bar{x}] > 1$, investors would be prepared to offer this contract, and the entrepreneur would choose it if and only if he receives an offer from the financier, i.e., if and only if $\sigma_X(q_X) = \bar{x}$. But then the financier's equilibrium profits are $-c$, a contradiction to equilibrium. Thus, $E[\tilde{k}X(a_1)|\sigma_X(q_X) = \bar{x}] \leq 1$. For γ close to 1, (3) and $\epsilon_1 - \epsilon_2 > 0$ imply $\tilde{k} > k$, so that $E[kX(a_1)|\sigma_X(q_X) = \bar{x}] < 1$, i.e., the financier would lose money, a contradiction.

We can therefore assume that there is a lemons problem for investors, i.e.,

$$E[(1-\tilde{k})X(a_1)|\sigma_X(q_X)=0]+\epsilon_1 \geq w. \tag{5}$$

For investors not to find undercutting profitable, $E[\tilde{k}X(a_1)] \leq 1$ for all $\tilde{k}$ that fulfill (3), and therefore also for $\tilde{k}$ that solves

$$E[(1-\tilde{k})X(a_1)|\sigma_X(q_X)=\bar{x}]+\epsilon_1 = E[(1-k)X(a_j)|\sigma_X(q_X)=\bar{x},\sigma_\Phi(1)=a_j]+\sum_j p_{a_j}\epsilon_j. \tag{6}$$

Equation (6) implies that the entrepreneur's payoff from the hands-on contract is the same as that from a hands-off contract with share $\tilde{k}$ (and the same q_X). But for γ sufficiently high, total financier-plus-entrepreneur surplus is lower if action a_2 is chosen when $\sigma_\Phi(q_\Phi)=a_2$. Because the entrepreneur's payoff is the same for both contracts, it follows that the financier's payoff must be lower under the hands-on contract.

It is immediate that hands-on finance cannot be offered independent of σ_X: when γ is close to 1, total surplus is increased by choosing a_1 even when $\sigma_\Phi(q_\Phi)=a_2$. Then, for any k that would earn the financier positive expected profit, uninformed investors would be willing to offer a contract with share $\tilde{k}<k$ that would be preferred by the entrepreneur, so that the financier cannot cover his information costs.

Now, we consider equilibria with hands-off finance. First, assume that finance is extended if and only if $\sigma_X(q_X)=\bar{x}$ and $\sigma_\Phi(q_\Phi)=a_1$. Then $E[kX(a_1)|\sigma_X(q_X)=\bar{x},\sigma_\Phi(q_\Phi)=a_1]>1$. Otherwise, the financier could not recover costs c. Therefore, if γ is sufficiently large, we get $E[kX(a_1)|\sigma_X(q_X)=\bar{x},\sigma_\Phi(q_\Phi)=a_2]>1$. Hence the financier would also extend finance when $\sigma_\Phi(q_\Phi)=a_2$, a contradiction. Second, assume that finance is extended if and only if $\sigma_\Phi(q_\Phi)=a_1$. Then for the financier to recover costs c, it must be that $E[kX(a_1)|\sigma_\Phi(q_\Phi)=a_1]>1$. Thus, $E[kX(a_1)|\sigma_\Phi(q_\Phi)=a_2]>1$ for γ sufficiently large. Hence, an investor could profitably undercut, by offering a share that is marginally less than k, a contradiction. Third, the same argument applies to the case where finance is offered except when $\sigma_X(q_X)=0$, $\sigma_\Phi(q_\Phi)=a_2$. Fourth, it is not possible to have informed finance where finance is extended independent of the signals, as investors could profitably undercut any share k that covers the financier's costs c. Finally, note that if it is optimal to offer finance when $\sigma_X(q_X)=y$ and $\sigma_\Phi(q_\Phi)=a_2$ then it must be optimal to offer finance when $\sigma_X(q_X)=y$ and $\sigma_\Phi(q_\Phi)=a_1$. Therefore, we have exhausted all cases, i.e., we have shown that only the only form of informed finance that could arise is hands-on finance where funding is offered if and only if $\sigma_X(q_X)=\bar{x}$.

■

Proof of Theorem 2. To prove the theorem we solve Problem 1. Note that constraint 1 must always bind. Otherwise, we could increase q_X thereby increasing the financier's payoff. If only constraint 1 binds, then we can solve constraint 1 for k and substitute k into the objective. The optimal q_X can then be derived by taking the derivative with respect to q_X. For $k, q_X > 0$, their optimal values are

$$k = 1 - \sqrt{\frac{(1-p_X)(w-\epsilon_1)}{p_X \bar{x}(p_{a_1} + \gamma p_{a_2})}}, \quad \text{and} \quad q_X = 1 - \sqrt{\frac{w-\epsilon_1}{p_X \bar{x}(1-p_X)(p_{a_1} + \gamma p_{a_2})}}.$$

Substituting k and q_X into the objective of Problem 1 yields the financier's maximized payoffs,

$$(1-p_X)(w-\epsilon_1) + p_X x(p_{a_1} + \gamma p_{a_2}) - c - p_X - 2\sqrt{p_X \bar{x}(p_{a_1} + \gamma p_{a_2})(1-p_X)(w-\epsilon_1)}. \tag{7}$$

Taking the second derivative with respect to p_X yields

$$\frac{\sqrt{\bar{x}(w-\epsilon_1)(p_{a_1} + \gamma p_{a_2})}}{2p_X^{3/2}(1-p_X)^{3/2}} > 0,$$

which implies that the financier's payoff is convex in p_X. We next show that the profit function is strictly increasing. Note that $q_X < 1$. Otherwise, constraint 1 is violated. Now fix the k and q_X that are optimal given p_X. Then constraint 1 is slack for the same k and q_X if we raise p_X to p_X'. This follows because $E[(1-k)X(a_1)|\sigma_X(q_X) = 0] = (1-q_X)p_X\bar{x}(1-k)$ is a strictly increasing function of p_X. We can therefore increase k and q_X, which strictly increases the financier's expected payoff.

Next, note that k is increasing in p_X. Therefore, there exists $\bar{p}_X$ such that constraint 2 binds for all $p_X > \bar{p}_X$. If both constraints of Problem 1 bind, then the optimal values of q_X and k are determined solely by the constraints. Substituting these values of q_X and k into the argument yields the financier payoff

$$-\frac{(1-p_X)(w-\epsilon_1)}{p_X \bar{x}(p_{a_1} + \gamma p_{a_2}) - 1} + 1 - c - p_X. \tag{8}$$

The second derivative with respect to p_X is

$$-\frac{2\bar{x}(w-\epsilon_1)(p_{a_1} + \gamma p_{a_2})(\bar{x}(p_{a_1} + \gamma p_{a_2}) - 1)}{(p_X \bar{x}(p_{a_1} + \gamma p_{a_2}) - 1)^3}.$$

Constraint 2 implies that the denominator and the last factor in the numerator are both strictly positive. Because $w - \epsilon_1 > 0$ the financier's payoff is therefore concave in p_X. Therefore, the maximum financier payoff is obtained when both constraints bind. Now note that constraint 1 implies $E[(1-k)X(a_1)] + \epsilon_1 > w$. Adding this to constraint 2 yields $E[X(a_1)] > w - \epsilon_1 + 1$, i.e., the project has a positive ex-ante NPV. Finally, substituting $p_X = 1$ into the solution yields financier profits of $-c$. Intuitively, were gross financier profits positive, uninformed investors could risklessly undercut. Hence, sufficiently safe projects cannot obtain informed finance. ■

Proof of Theorem 3. Fix the expected project payoff, $\text{EP} = p_X \bar{x}(p_{a_1} + p_{a_2}\gamma)$. Then decreasing p_X while keeping EP fixed increases the variance. It follows from Theorem 1 that Problem 1 applies.

Assume first that only constraint 1 of Problem 1 binds. Substituting EP into (7) yields

$$(1 - p_X)(w - \epsilon_1) + \mathrm{EP} - c - p_X - 2\sqrt{\mathrm{EP}(1 - p_X)(w - \epsilon_1)}. \tag{9}$$

The second derivative of (9) with respect to p_X is $\frac{\sqrt{(w-\epsilon_1)\mathrm{EP}}}{2(1-p_X)^{3/2}} > 0$. Therefore, the financier's payoff is convex in p_X if we maximize subject to constraint 1 only. Next, assume that both constraints bind. Then the financier's payoff is given by (8). Substituting EP yields

$$-\frac{(1 - p_X)(w - \epsilon_1)}{\mathrm{EP} - 1} + (1 - p_X) - c, \tag{10}$$

which is a linear function of p_X. Financier optimization implies that this line is tangent to (9). Note that if $p_X = 1$ then the payoff (10) is $-c$, i.e., informed finance is not feasible. Moreover, if $p_X = 1$ then constraint 2 binds. Therefore, constraint 2 binds for all p_X between the tangency point p_X^t and $p_X = 1$. If the slope of (10) is negative, then convexity implies that the slope of (9) is negative for all $p_X \le p_X^t$. Therefore, the financier's payoff is a declining function of p_X and hence an increasing function of the project's variance. Finally, assume that (10) has a positive slope. Then informed finance is not feasible if both constraints of Problem 1 bind. At the tangency point p_X^t both the value of (9) and its derivative with respect to p_X are negative. Therefore, if finance is feasible at $p_X < p_X^t$, i.e. if (9) is positive, then convexity of (9) implies that there is a $\hat{p}_X$ such that (9) is declining in p_X for $p_X < \hat{p}_X$. Again, this implies that increasing the project's variance raises the financier's payoff.

Finally, note that if the variance is small enough then p_X is near 1. Theorem 2 therefore implies that informed finance is infeasible. ■

Lemma 1 *Consider a hands-on equilibrium in the stage 3 subgame and suppose that in this subgame the financier selects the recommended action and offers funding independently of $\sigma_X(q_X)$. Then the financier's payoff would be strictly higher in a stage 3 subgame in which $q_X = q_\Phi = 1$ and the financier offers funding only when $\sigma_X(q_X) = \bar{x}$.*

Proof. If finance is offered independent of $\sigma_X(q_X)$ then we can assume without loss of generality that $q_X = 0$. It is also optimal for the financier to choose $q_\Phi = 1$. We will show that the financier earns higher expected profits setting $q_X = q_\Phi = 1$ and financing if and only if $\sigma_X(q_X) = \bar{x}$.

Define $\tilde{k}_i$ implicitly by

$$E[(1 - \tilde{k}_i)X(a_i)] + \epsilon_i = E[(1 - k)X(a_j)|\sigma_\Phi(q_\Phi) = a_j] + \sum_j p_{a_j}\epsilon_j. \tag{11}$$

Note that in equilibrium

$$E[\tilde{k}_i X(a_i)] \leq 1 \tag{12}$$

$$E[(1-k)X(a_i)|\sigma_\Phi(q_\Phi) = a_i] + \sum_i p_{a_i}\epsilon_i \geq w, \tag{13}$$

where one of these inequalities must hold as an equality. If not, the financier could increase k marginally and raise his expected profits.

Assume first that (12) holds with equality. The financier's expected profit is $p_X k\bar{x} - 1 - c$. Using (11) and (12) we can solve for k. Substituting this value of k into the financier's expected profit yields

$$(1 - p_{a_1})\big[p_X\bar{x}(1-\gamma) - (\epsilon_1 - \epsilon_2)\big]. \tag{14}$$

Now assume that the financier chooses $q_X = q_\Phi = 1$. Then finance is only offered when the project pays $\bar{x}$, earning the financier a profit of

$$(1 - p_{a_1})\big[p_X\bar{x}(1-\gamma) - p_X(\epsilon_1 - \epsilon_2)\big]. \tag{15}$$

Because $\epsilon_1 - \epsilon_2 > 0$ it immediately follows that (15) is greater than (14). Finally, note that the share $\tilde{k}$ is higher in the original equilibrium. The right-hand side of (11) is the entrepreneur's expected payoff when finance is extended. Therefore, if (13) holds for the original equilibrium then it holds also when $q_X = q_\Phi = 1$.

Now assume that (12) holds with a strict inequality and that (13) holds with equality for the contract offered when $q_X = 0$.

When $q_X = 1$, if k' is the share offered by the financier, then the contingent contract must give the entrepreneur at least his outside payoff,

$$\sum_i p_{a_i}\Big(E[(1-k')X(a_i)|\sigma_\Phi(q_\Phi) = a_i, \sigma_X(q_X) = \bar{x}] + \epsilon_i\Big) \geq w. \tag{16}$$

If (16) holds with equality then $k' > k$. Therefore, the financier is strictly better off. If (16) does not hold with equality, then (12) must hold as an equality, and the first part of the argument again implies that the financier is strictly better off. ■

Proof of Theorem 4. *Step 1: If an equilibrium with informed finance exist under the conditions of Theorem 4, then hands-on contracts are offered and finance is extended if and only if $\sigma_X(q_X) = \bar{x}$.*

Lemma 1 excludes hands-on contract where finance is offered independent of σ_X. It is therefore sufficient to show that equilibria with hands-off finance do not exist.

Step 1.1: Finance is offered if and only if $\sigma_X(q_X) = \bar{x}$.

The contingent contract must solve Problem 1. Let k_1, q_X be the solution. Let $\tilde{k} = k_1$, and define k by constraint 1 of Problem 2. Then q_X, k, and $\tilde{k}$ fulfill the constraints of Problem 2. The entrepreneur's payoff remains unchanged. However, if $\epsilon_2 - \epsilon_1$ is sufficiently small then the total surplus is increased because the recommended action is always chosen, so that, the financier, as the residual claimant, must have a strictly higher payoff.

Step 1.2: Finance is offered if and only if $\sigma_X(q_X) = \bar{x}$ and $\sigma_\Phi = a_1$.

Assume the financier offers a share k_2. For the financier to be able to recover costs c, there must be a lemons problem for investors, i.e.,

$$E[(1 - k_2)X(a_1)|(\sigma_X(q_X), \sigma_\Phi(q_\Phi)) \neq (\bar{x}, a_1)] + \epsilon_1 \geq w, \tag{17}$$

and

$$E[k_2 X(a_1)] \leq 1. \tag{18}$$

Moreover,

$$E[k_2 X(a_1)|\sigma_X(q_X) = 0, \sigma_\Phi(q_\Phi) = a_1] \leq 1 \tag{19}$$

must hold. Otherwise, the financier would offer funding also when $\sigma_X(q_X) = 0$ and $\sigma_\Phi(q_\Phi) = a_1$. Keeping the financier's expected payoff $E[k_2 X(a_1)|\sigma_X(q_X) = \bar{x}, \sigma_\Phi(q_\Phi) = a_1]$ fixed, reduce q_X to q'_X and increase q_Φ to q'_Φ such that (19) remains satisfied. Then we get either $q'_\Phi = 1$ or

$$E[k_2 X(a_1)|\sigma_X(q'_X) = 0, \sigma_\Phi(q'_\Phi) = a_1] = 1. \tag{20}$$

First assume that $q'_\Phi = 1$. Then if γ is small,

$$E[(1 - k_2)X(a_1)|\sigma_X(q'_X) = \bar{x}, \sigma_\Phi(1) = a_2] + \epsilon_1 < w. \tag{21}$$

By construction $E[k_2 X(a_1)|\sigma_X(q_X) = \bar{x}, \sigma_\Phi(q_\Phi) = a_1] = E[k_2 X(a_1)|\sigma_X(q'_X) = \bar{x}, \sigma_\Phi(1) = a_1]$, and hence,

$$E[(1 - k_2)X(a_1)|\sigma_X(q_X) = \bar{x}, \sigma_\Phi(q_\Phi) = a_1] = E[(1 - k_2)X(a_1)|\sigma_X(q'_X) = \bar{x}, \sigma_\Phi(1) = a_1]. \tag{22}$$

Also, note that the ex-ante probability that $\sigma_X(q_X) = \bar{x}$ and $\sigma_\Phi(q_\Phi) = a_1$ is $p_X p_{a_1}$ and is therefore independent of q_X and q_Φ. Thus,

$$\begin{aligned}
&p_X p_{a_1} E[(1 - k_2)X(a_1)|\sigma_X(q_X) = \bar{x}, \sigma_\Phi(q_\Phi) = a_1] \\
&\qquad + (1 - p_X p_{a_1})E[(1 - k_2)X(a_1)|(\sigma_X(q_X), \sigma_\Phi(q_\Phi)) \neq (\bar{x}, a_1)] \\
&= E[(1 - k_2)X(a_1)] \\
&= p_X p_{a_1} E[(1 - k_2)X(a_1)|\sigma_X(q'_X) = \bar{x}, \sigma_\Phi(1) = a_1] \\
&\qquad + (1 - p_X p_{a_1})E[(1 - k_2)X(a_1)|(\sigma_X(q'_X), \sigma_\Phi(1)) \neq (\bar{x}, a_1)],
\end{aligned}$$

which implies

$$E[(1-k_2)X(a_1)|(\sigma_X(q_X), \sigma_\Phi(q_\Phi)) \neq (\bar{x}, a_1)] = E[(1-k_2)X(a_1)|(\sigma_X(q'_X), \sigma_\Phi(1)) \neq (\bar{x}, a_1)]. \tag{23}$$

Then (17), (21), and (23) imply

$$E[(1-k_2)X(a_1)|\sigma_X(q'_X) = 0] + \epsilon_1 \geq w. \tag{24}$$

Now let $\tilde{k} = k_2$. Define k implicitly by constraint 1 of Problem 2. Then $(\tilde{k}, k, q'_X, q'_\phi = 1)$ satisfy the constraints of Problem 2. Moreover, the payoff to the entrepreneur remains unchanged. However, as above, the financier's payoff is increased because when $\epsilon_1 - \epsilon_2$ is small, total surplus is increased by taking the correct action choice.

Now assume that (19) holds with equality. Then we claim that

$$E[(1-k_2)X(a_1)|\sigma_X(q'_X) = 0, \sigma_\Phi(q'_\Phi) = a_1] \geq w - \epsilon_1. \tag{25}$$

To see this, suppose that (25) is violated, i.e.,

$$E[(1-k_2)X(a_1)|\sigma_X(q'_X) = 0, \sigma_\Phi(q'_\Phi) = a_1] < w - \epsilon_1. \tag{26}$$

Note that

$$E[k_2 X(a_1)|\sigma_X(q'_X) = \bar{x}, \sigma_\Phi(q'_\Phi) = a_2] < E[k_2 X(a_1)|\sigma_X(q_X) = \bar{x}, \sigma_\Phi(q_\Phi) = a_2] \leq 1, \tag{27}$$

where the first inequality follows because $q'_X < q_X$ and $q'_\Phi > q_\Phi$, and a_1 is not the recommended action; while the second inequality follows because the financier does not offer funding when $\sigma_X(q_X) = \bar{x}$ and $\sigma_\Phi(q_\Phi) = a_2$. Then (20) and (27) imply $E[(1-k_2)X(a_1)|\sigma_X(q'_X) = \bar{x}, \sigma_\Phi(q'_\Phi) = a_2] \leq E[(1-k_2)X(a_1)|\sigma_X(q'_X) = 0, \sigma_\Phi(q'_\Phi) = a_1]$. Thus, (26) implies

$$E[(1-k_2)X(a_1)|\sigma_X(q'_X) = \bar{x}, \sigma_\Phi(q'_\Phi) = a_2] < w - \epsilon_1. \tag{28}$$

Thus, (26) and (28) imply that (17) cannot hold. This establishes inequality (25).

Let $\tilde{k} = k_2$ and define k by

$$E[(1-\tilde{k})X(a_1)] + \epsilon_1 = E[(1-k)X(a_j)|\sigma_\Phi(q_\Phi) = a_j] + \sum_j p_{a_j}\epsilon_j. \tag{29}$$

Assume that the financier offers a hands-on contract with share k independent of σ_X and that this contract gives the entrepreneur at least his outside payoff. If $\epsilon_2 - \epsilon_1$ is small then $\tilde{k} < k$. This and the fact that the recommended action is chosen implies that the financier's payoff is strictly increased when $\sigma_X(q_X) = \bar{x}$. Because (19) holds as an equality, $\tilde{k} < k$ implies that the financier's payoff is also increased when $\sigma_X(q_X) =$

0. Finally, assume that given share k, the entrepreneur receives less than his outside payoff w. If $\epsilon_1 - \epsilon_2$ is small then (25) implies that a small reduction of $\tilde{k}$ and k is sufficient to guarantee the entrepreneur w. Therefore, the financier is again better off. Lemma 1 above proves that the financier can improve further by offering an alternative hands-on contract if and only if $\sigma_X = \bar{x}$.

Step 1.3: Finance is offered if and only if $\sigma_\Phi(q_\Phi) = a_1$.

Let k_3 be the share offered by the financier. Then for the financier to cover information costs c, there must be a lemons problem for investors, i.e., $E[(1-k_3)X(a_1)|\sigma_\Phi(q_\Phi) = a_2] + \epsilon_1 \geq w$ and $E[k_3 X(a_1)] \leq 1$. Choose $q_\Phi = 1$, $q_X = 0$. Let $\tilde{k} = k_3$ and define k by (29). If $\epsilon_1 - \epsilon_2$ is small then $\tilde{k} \leq k$. This and the fact that the correct action is chosen makes the financier strictly better off. If the contingent contract does not give the entrepreneur his outside payoff w then because $\epsilon_1 - \epsilon_2$ is small, a slight reduction of k and $\tilde{k}$ gives the entrepreneur his requisite outside payoff. The financier's payoff under the hands-on contract remains strictly higher than under the hands-off contract.

Step 1.4: Other Hands-off contracts.

Finally, note that if it is optimal to extend finance when $\sigma_\Phi(q_\Phi) = a_2$, then it is optimal to extend finance when $\sigma_\Phi(q_\Phi) = a_1$. Therefore, we have exhausted all cases.

Step 2: The optimal k and q_X solve Problem 2. It is immediate that the constraints of Problem 2 are necessary. Therefore, at stage 2, the financier selects the contract that maximizes his expected profit subject to these constraints.

Step 3: Sufficiency of Problem 2. It remains to prove that the entrepreneur chooses informed finance at stage 1. If constraints 2(a) and 2(b) apply then the argument is identical to that for Theorem 1. Now suppose that $q_X = 1$ so that constraint 3(a) and 3(b) apply. Let k^U be the share that an uninformed investor would offer were the entrepreneur to select uninformed finance at stage 1, i.e., $E[k^U X(a_1)] = 1$. If 3(a) applies then it follows that $\tilde{k} < k^U$, because in 3(a), expectations are conditioned on the project being viable. If the project is not viable, then under uninformed finance, the entrepreneur receives ϵ_1. Under informed finance a project that is not viable is not funded and the entrepreneur receives w. Because $w - \epsilon_1 > 0$, the entrepreneur strictly prefers informed finance.

Finally, suppose that 3(a) is slack and that 3(b) binds. Then $k^U > \tilde{k}$ implies that the entrepreneur would receive strictly less than w under uninformed finance. Under informed finance the entrepreneur receives w. Therefore the entrepreneur is willing to take informed finance. ■

Proof of Theorem 5. If constraint 3 in Problem 2 applies then $q_X = q_\Phi = 1$. Next note that either (3a) or (3b) must be slack. In both cases the financier's payoff is a linear function of p_X. To see, this note that $\tilde{k}$ and k in (3a) and (3b), respectively, are independent of p_X because $q_X = 1$. The same argument implies that the k defined implicitly in constraint 1 is independent of p_X. As a consequence, the term $E[kX(a_i)|\sigma_X(q_X) = \bar{x}, \sigma_\Phi(1) = a_i]-1$ in the objective is also independent of p_X. Therefore, the financier's payoffs are linear in p_X.

Next, assume that constraint (2a), but not (2b) applies. Then the financier's payoff is determined algebraically as follows. Use constraint 1 to solve for k as a function of $\tilde{k}$ and the remaining parameters. Then use (2a) to eliminate $\tilde{k}$ and substitute the resulting k into the objective of Problem 2. The first order condition with respect to q_X reveals that $q_X = 1 - \frac{\sqrt{w-\epsilon_1}}{\sqrt{p_X(1-p_X)\bar{x}}}$, which in turn determines k. Substituting k and q_X into the objective determines the financier's payoff. The second derivative of this payoff with respect to p_X is

$$\frac{\sqrt{(w-\epsilon_1)\bar{x}}}{2(p_X(1-p_X))^{3/2}} > 0.$$

Therefore, the financier's payoff is convex.

To show that the financier's payoff when only (2a) binds is strictly increasing in p_X, let q_X, $\tilde{k}$, and k, be the optimal values given p_X. Then constraint (2a) becomes slack for the same values of $\tilde{k}$ and q_X if p_X is increased. Next, fixing $\tilde{k}$ in constraint 1 and raising p_X increases the implied solution for k. Therefore the financier's expected payoff strictly increases.

If both (2a) and (2b) bind then q_X, $\tilde{k}$, and k are computed directly from these constraints. Specifically, (2b) determines $\tilde{k}$. Substituting this value of $\tilde{k}$ into (2a) yields q_X. Finally, substituting both of these values into constraint 1, determines k. Substituting q_X and k into the objective determines the financier's expected payoff. The second derivative of this payoff with respect to p_X is

$$-\frac{2\bar{x}(w-\epsilon_1)\big(E[X(a_1)|\sigma_X(1)=\bar{x}]-1\big)}{\big(E[X(a_1)]-1\big)^3}. \tag{30}$$

Constraint (2a) implies that $\tilde{k} < 1$. This, and constraint (2b) then imply that the denominator of (30) is strictly positive. This, in turn, implies that $E[X(a_1)|\sigma_X(1) = \bar{x}] > 1$. Therefore, (30) is negative.

It follows from the above that the two payoff functions must be tangent at some value $\hat{p}_X^2$. Because the convex part is strictly increasing, it follows that (2b) binds only if $p_X > \hat{p}_X^2$.

Note that when (3b) binds, constraint (2a) can never be satisfied. In this case $\hat{p}_X^3 = 1$. Algebra also reveals that at $p_X = 1$, the financier's payoff is the same when (2a) and (2b) bind to determine q_X and k, as when $q_X = 1$ and (3a) binds to determine k.

Next we show that if (3a) applies for some value of p_X then (3a) also applies for all smaller values of p_X. First, consider the case where the slope of the financier's payoff at $p_X = 1$ assuming that (2a) and (2b) bind is less than or equal to the slope of the payoff function when (3a) applies. Then $\hat{p}_X^3 = 1$. In particular, concavity of the financier's payoff function when (2a) and (2b) bind, immediately implies that the financier prefers to set $q_X = 1$, i.e., constraint (3a) binds. Next, note that at $p_X = 0$ the financier's expected payoff when constraint (3a) binds is $-c$.

Above, we have shown that there exists a p_X sufficiently small, such that the solution of Problem 2 subject only to constraints (2a) and 1 yields a financier payoff of less than $-c$. It follows that for even smaller values of p_X, constraints 4 and (2a) cannot be simultaneously satisfied. It follows that the solution to Problem 2 when (3a) applies gives strictly higher payoffs than when (2a) applies.

Next, consider the case where the slope of the financier's payoff at $p_X = 1$ when (2a) and (2b) both apply/bind exceeds the slope of the payoff function when (3a) applies. The same reasoning as above implies that there exists a $\hat{p}_X^3$ such that it is optimal for the financier to maximize subject to (3a) when $p_X < \hat{p}_X^3$, and to maximize subject to constraint 2 when $p_X > \hat{p}_X^3$.

The final statement follows immediately, as c does not affect the choice of q_X, k, and $\tilde{k}$ in Problem 2. ■

Lemma 2 *In an equilibrium with hands-on contracts, interests are not aligned if and only if* $\tilde{k} > k$.

Proof. In particular, $\tilde{k} > k$ and constraint 1 of Problem 2 imply

$$\begin{aligned} &\sum_j p_{a_j} E[(1-k)X(a_1)|\sigma_X(q_X) = \bar{x}, \sigma_\Phi(q_\Phi) = a_j] + \epsilon_1 \\ &= E[(1-k)X(a_1)|\sigma_X(q_X) = \bar{x}] + \epsilon_1 > E[(1-k)X(a_j)|\sigma_X(q_X) = \bar{x}, \sigma_\Phi(q_\Phi) = a_j] + \sum_j p_{a_j}\epsilon_j. \end{aligned} \tag{31}$$

Subtracting $p_{a_1}\big(E[(1-k)X(a_1)|\sigma_X(q_X) = \bar{x}] + \epsilon_1\big)$ from both sides of (31) and dividing by p_{a_2} yields

$$E[(1-k)X(a_1)|\sigma_X(q_X) = \bar{x}, \sigma_\Phi(q_\Phi) = a_2] + \epsilon_1 > E[(1-k)X(a_2)|\sigma_X(q_X) = \bar{x}, \sigma_\Phi(q_\Phi) = a_2] + \epsilon_2,$$

i.e., the entrepreneur prefers action a_1 when action a_2 is recommended. Because $q_\Phi = 1$, the financier always prefers choosing the recommended action. Therefore, interests are not aligned. ■

Proof of Theorem 7. First suppose that $E[X(a_1)] \leq 1$. Therefore, constraint (2b) of Problem 2 can never apply, as $k < 1$. Since $E[X(a_1)|\sigma_X(1) = \bar{x}] > 1$, there exist, $0 < k < 1$ and $q_X < 1$ such that $E[kX(a_1)|\sigma_X(q_X) = \bar{x}] > 1$. Hence, financier profits are strictly positive if c is small. If w and ϵ_1 are sufficiently small then constraint (2a) is satisfied. Next, we verify that constraint 3 does not apply.

Assume that $q_X = 1$. Then if $w - \epsilon_1$ is small, we claim that constraint (3a) is tighter than (3b). To see this, note that the financier's payoff subject to constraints (3a) or (3b) is linear in p_X. At $p_X = 0$ the payoffs are the same. At $p_X = 1$ the payoff difference is $1 + w - \epsilon_1 - \bar{x}(p_{a_1} + (1 - p_{a_1})\gamma) =$

$1 + w - \epsilon_1 - E[X(a_1)|\sigma_X(1) = \bar{x}]$. By assumption, $E[X(a_1)|\sigma_X(1) = \bar{x}] > 1$, so that this payoff difference is negative. Therefore, constraint (3b) is slack.

Computing the difference in financier payoffs between using constraint (2a) and constraint (3a), as $w - \epsilon_1$ converges to 0 yields $p_X(\bar{x}(p_{a_1} + (1 - p_{a_1})\gamma) - 1) > 0$. Therefore, choosing $q_X < 1$ is strictly better when $w - \epsilon_1$ is small.

Computing $k - \tilde{k}$ when constraint (2a) applies, for $w - \epsilon_1 \to 0$ yields $(\epsilon_1 - \epsilon_2)(p_{a_1} - 1)/\bar{x} < 0$. Therefore, $\tilde{k} > k$ when $w - \epsilon_1$ is small. Lemma 2 then implies that interests are not aligned.

Now suppose that $E[X(a_1)] > 1$. First, assume that it is optimal to choose $q_X < 1$. Then both constraint (2a) and (2b) apply. In particular, assume by contradiction that (2b) is slack. If $w - \epsilon_1$ is sufficiently small, then k is close to 1. Thus, $E[X(a_1)] > 1$ implies $E[kX(a_1)] > 1$.

If $q_X = 1$ is optimal then the argument of the proof of Theorem 7 shows that only constraint (3a) applies. If we compute the financier's payoff from using $q_X < 1$ minus the financier's payoff from $q_X = 1$ and let $w - \epsilon_1$ converge to 0 then we get $1 - p_X$. Therefore, if $w - \epsilon_1$ is small it is optimal to choose $q_X < 1$.

Lemma 2 implies that interest are aligned if and only if $k - \tilde{k} \geq 0$. As $w - \epsilon_1$ converges to 0,

$$k - \tilde{k} = ((1 - p_{a_1})\gamma + p_{a_1})(p_X\bar{x}(1 - \gamma) - (\epsilon_1 - \epsilon_2)) - (1 - \gamma).$$

Re-arranging we see that this is positive if and only if

$$E[X(a_1)] = ((1 - p_{a_1})\gamma + p_{a_1})p_X\bar{x} > 1 + \frac{(\epsilon_1 - \epsilon_2)}{1 - \gamma}(1 - p_{a_1})\gamma + p_{a_1}).$$

Since γ and w are small, interests are only aligned if the project has a sufficiently large ex-ante NPV. ■

Proof of Theorem 8. The proof follows that of Theorem 3. ■

References

[1] P. Aghion and P. Bolton. An incomplete contracts approach to financial contracting. *Review of Economic Studies*, 59:473–494, 1992.

[2] J. D. Akhavein, W. S. Frame, and L. J. White. The diffusion of financial innovations: An examination of the adoption of small business credit scoring by large banking organizations. working paper, 2001.

[3] T. Astebro and I. Bernhardt. Financing, owner characteristics, and survival. University of Waterloo, 2001.

[4] D. Bernhardt and V. Dvoracek. Monitoring and evaluation by financiers. University of Illinois, 2000.

[5] D. Bernhardt and S. Krasa. The simple analytics of informed finance. working paper: University of Illinois, 2005.

[6] B. Biais and E. Perotti. Entrepreneurs and new ideas. working paper: CEPR, 2002.

[7] B. Black and R. Gilson. Venture capital and the structure of capital markets: Banks versus stock market. *Journal of Financial Economics*, 47:243–277, 1998.

[8] T. Broecker. Credit-worthiness tests and interbank competition. *Econometrica*, 58:429–452, 1990.

[9] B. Byers. Relationship between venture capitalist and entrepreneur. In *Pratt's Guide to Venture Capital Sources*. Wellesley Hills, MA, 1997.

[10] W. Bygrave and J. Timmons. *Venture Capital at the Crossroads*. Harvard University Press, Boston, MA, 1992.

[11] J. H. Cochrane. The risk and return of venture capital. working paper: GSB Chicago, 2003.

[12] G. Dell'Ariccia, E. Friedman, and R. Marquez. Adverse selection as a barrier to entry in the banking industry. *Rand Journal of Economics*, 30:515–534, 1999.

[13] G. Fenn, G. N. Liang, and S. Prowse. The economics of private equity markets. Staff Study 168, Board of Governors of the Federal Reserve System, 1995.

[14] J. Fiet and D. Fraser. Bank entry into the venture capital industry. *Managerial Finance*, 20:31–42, 1994.

[15] V. H. Fried and R. D. Hisrich. Toward a model of venture capital investment decision making. *Financial Management*, 23:28–37, 1994.

[16] M. Garmaise. Informed investors and the financing of entrepreneurial projects. working paper: GSB Chicago, 2001.

[17] P. Gompers. An examination of convertible securities in venture capital investments. Harvard Business School, 1998.

[18] P. Gompers and J. Lerner. *The Venture Capital Cycle*. MIT Press, Cambridge, MA, 1999.

[19] M. Gorman and W. Sahlman. What do venture capitalists do. *Journal of Business Venturing*, 4:231–248, 1989.

[20] S. Grossman and O. Hart. The costs and benefits of ownership: A theory of vertical and lateral integration. *Journal of Political Economy*, 94:691–719, 1986.

[21] S. Grossman and J. E. Stiglitz. On the impossibility of informationally efficient markets. *American Economic Review*, 66:393–408, 1980.

[22] O. Hart and J. Moore. A theory of debt based on the inalienability of human capita. *Quarterly Journal of Economics*, 9:841–879, 1994.

[23] T. Hellmann. Venture capital: A challenge for commercial banks. *The Journal of Private Equity*, 1:49–55, 1997.

[24] T. Hellmann and M. Puri. The interaction between product market and financing strategy: the role of venture capital. *Review of Financial Studies*, 13:959–984, 2000.

[25] M. Hellwig. On the aggregation of information in competitive markets. *Journal of Economic Theory*, 22:279–312, 1980.

[26] S. Kaplan and P. Stromberg. Financial contracting theory meets the real world: An empirical analysis of venture capital contract. forthcoming *Review of Economic Studies*, 2003.

[27] A. Ljungqvist and M. Richardson. The cash flow, return and risk characteristics of private equity. working paper: NYU, 2003.

[28] C. Matutes and X. Vives. Imperfect competition, risk taking, and regulation in banking. *European Economic Review*, 44:1–34, 2000.

[29] M. Riordan. Competition and bank performance: a theoretical perspective. In *Capital Markets and Financial Intermediation, Handbooks in OR&MS Vol. 9*. Elsevier Science BV, 1995.

[30] W. Sahlman. The structure and governance of venture-capital organizations. *Journal of Financial Economics*, 27:473–521, 1990.

[31] H. Sapienza. When do venture capitalists add value? *Journal of Business Venturing*, 7:9–27, 1992.

[32] T. Smith. Banking competition and macroeconomic performance. *Journal of Money, Credit and Banking*, 30:793–815, 1998.

[33] M. Villas-Boas and U. Schmidt-Mohr. Oligopoly with asymmetric information: Differentiation in credit markets. *Rand Journal of Economics*, 30:375–396, 1999.

[34] A. Winton. Competition among financial intermediaries when diversification matters. *Journal of Financial Intermediation*, 6:307–364, 1997.

[35] A. Wong. Do you believe in angels? GSB, University of Chicago, 2001.

[36] O. Yosha. Diversification and competition: Financial intermediation in a large cournot-walras economy. *Journal of Economic Theory*, 75:64–88, 1997.