Competition between heterogeneously capable candidates

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Abstract

We develop a formal model in which the government provides public goods in different policy fields for its citizens. We start from the basic premise that two office-motivated candidates have differential capabilities in different policy fields, and compete by proposing how to allocate government resources to those fields.

The model has a unique equilibrium that differs substantially from the standard median-voter model. While candidates compete for the support of a moderate voter type, this cutoff voter differs from the expected median voter. Moreover, no voter type except the cutoff voter is indifferent between the candidates in equilibrium. The model also predicts that candidates respond to changes in the preferences of voters in a very rigid way. We also analyze under which conditions candidates choose to strengthen the issue in which they have a competence advantage, and when they rather compensate for their weakness.

Keywords: Issue ownership, differentiated candidates, policy divergence.

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1 Introduction

In his seminal work “An economic theory of democracy,” Anthony Downs (1957) develops a model of two-party electoral competition and shows that candidates’ platforms converge to the preferences of the median voter. This median voter model has become the standard framework through which scholars typically study electoral competition; even analyses set to show that candidates platforms diverge typically use the Downsian setting as their starting point.

An implicit assumption in nearly all of the formal works based on Downs standard model is that candidates have equal ability to implement the policies proposed during the electoral campaign. Yet, candidates in elections frequently differ in their personal background, professional expertise, and the set of key aides whom they would rely on if elected, and consequently, the electorate perceives each candidate as having different strengths and weaknesses in certain policy areas relative to his opponent.

We develop a model of electoral competition between candidates who have different abilities in implementing two different public goods such as national security and education. In our model, each candidate proposes how to allocate a fixed total amount of money (or effort) between the two goods, and the resource allocation together with the winner’s exogenous ability determine the amount of public goods provided to the voters. Voters differ in the intensity with which they care about the relative provision of the goods and vote for their respective preferred candidates.

The model advances our understanding of electoral competition by providing a framework that captures the following observations: First, candidates/parties compete fiercely for the support of certain “moderate” voters, by proposing platforms designed to appeal to these voters. Second, in contrast to these moderate voters who are almost indifferent between parties, most voters have strict preferences for one of the two candidates. Third, competing candidates often do not just differ in exogenous characteristics, but also choose policy platforms that differ from their opponent’s platform. Fourth, from an ex-ante perspective, candidates often have substantially different winning probabilities. Fifth, candidate platforms sometimes display a surprisingly strong rigidity with respect to new information about the distribution of voter preferences.

The paper proceeds as follows. The next section contains an informal discussion of the causes of differences in candidate ability, and an informal and intuitive description of our results. The relation of our paper to previous literature is detailed in Section 3. Section 4 describes our formal model, and Section 5 contains the analysis of the equilibrium which can be done for the most part using graphics. In Section 6, we discuss several extensions; in particular, we provide some arguments showing that parties have an incentive to field candidates with differentiated expertise. Section 7 concludes. Some proofs and more general theorems are in the Appendix.
2 Causes and Consequences of Heterogeneous Candidate Abilities

2.1 Policy areas and differences between candidates.

Our model focuses on policy areas that Stokes (1963) calls *valence issues*. That is, all voters in our model agree that a higher output in both public goods (say, a low crime rate, the quality of schooling etc.) is desirable. However, they differ on which trade-offs they are willing to make between these different political goals. The key departure of our model from previous literature is that we assume that candidates have differential abilities in the different policy areas. To keep our model simple, we abstract from horizontal issues that Stokes (1963) calls “position issues”, i.e. policy questions like abortion, gay marriage or gun control in which voters disagree over the desired outcome, and in which the issue of “implementation ability” is evidently less important.

There are several intuitive reasons why candidates (or parties, depending on the unit of analysis) have different abilities in different policy areas. First, individual candidates already have a background (e.g. education, experience, personal interests) when they enter politics and such personal background may tilt their interests toward some policy issues rather than others. When business leaders run for elected office, they usually highlight their management experience as a reason to expect a competent management of government from them. Likewise, candidates with a military background often leverage their experience on military and foreign affairs issues, and focus their policy proposals on this area.

Second, once in office, individuals may choose to work on those issues in which they are more capable and such self-selection further reinforces whatever initial competence the candidate brings on those specific policy issues. For example, it is plausible that Franklin Delano Roosevelt, after having started the New Deal program in his first term, was considered more competent in managing a more active government involvement in the economy than any Republican challenger. Similarly, George W. Bush successfully leveraged his experience in fighting the “war on terror” in his 2004 reelection campaign.¹

Third, based on different backgrounds and preferences, self-sorting of citizens into parties may occur: Prior to becoming candidates, individuals start out as citizens with certain policy preferences. If an individual has a stronger than average preference for national defense, it is natural that he will be more interested in foreign relations or defense technology (relative to another individual who is more concerned with, say, education). Over time, his competence on defense-related matters will increase, while his competence on education-related matters will be weaker than that of an individual who cares more about education. Moreover, it is not only natural that these individuals will join different parties that are composed from individuals with similar preferences — very likely, there is also a career-incentive for aspiring politicians to self-select into the appropriate party because an individual’s specific competence for, say, national defense is more appreciated by the members and primary voters of the Republican party.

¹Note that the decisive point for electoral success is always the level of competence perceived by the electorate rather than the actual level.
Forth, and complementary to the third explanation, parties can be seen as networks whose members cooperate in the provision of government services. If, for whatever reasons, one party has attracted most individuals with specific knowledge about one policy area, then an elected candidate from this party can draw on this network of knowledgeable party members to provide both specific new ideas and to recruit key personnel for government positions. In contrast, if the competitor from the other party is elected, then he cannot draw on these network resources, and his ability to optimally implement policy in this policy field is diminished. In this way, the ideological predisposition of party members may influence what policies their candidate is able to offer (in the sense of capability), and, in addition, which policy (i.e., budget allocation) their candidate will choose.

In the standard model, competence is sometimes incorporated as an additive “valence” component. However, since valence enters voters’ utility functions in a way that is separable from which policy is implemented, it does not capture the notion of issue-specific ability, which lies at the core of our model. For example, military experience is electorally more valuable for a candidate when international conflict is a serious concern for voters than when they are mostly concerned with economic issues.

2.2 Heterogeneous candidates competing for voters.

In our model, the executive has to choose which proportion of the government budget to spend on two public goods, such as education and law enforcement, or domestic and foreign policy. The resources spent on each field in combination with the executive’s skill in each field translate into amounts of public goods 0 and 1 provided. During the election campaign, each candidate proposes a budget allocation (or, equivalently, a combination of goods 0 and 1 from his own production possibility set). There is a continuum of voters who care differentially about the different goods, ranging from those who almost only care about good 0 over those who care about both to ones that care almost exclusively about good 1.

If both candidates have identical abilities in our framework, then the median voter (i.e., the voter who has the property that half of the electorate cares more about good 1 relative to good 0 than this voter, and the other half of the electorate cares less about good 1 relative to good 0 than this voter) is decisive for the election outcome, and both candidates propose a policy that maximizes the utility of the median median voter. (Candidates in our model are uncertain about the exact preferences of the median voter so that they have to cater to the “median median” voter, that is, the median realization of the median voter). Since both candidates propose the same policy, all voters are indifferent between the candidates.

The assumption that candidates have heterogeneous abilities substantially changes the nature of electoral competition relative to the standard model. Each candidate has a natural target audience, in the sense that he has an advantage in appealing to these voters. Assume for the moment that candidates do not

\[2\]While we assume that candidates are exogenously differentiated in the basic model, in Section 6.1, we allow parties to choose the characteristics of their nominees and show that they have very robust incentives to choose a candidate whose capabilities differ significantly from those of the opposition candidate.
“overcompensate” with their equilibrium budget allocations, i.e., the candidate who has a productivity advantage in the production of good 0 ends up producing more of good 0 than his competitor, and vice versa for good 1 (our formal model shows that “no overcompensation” is in fact a property of equilibrium). In this case, voters who care primarily about Candidate 0’s strong good vote for Candidate 0, and those voters who care primarily about Candidate 1’s strong good vote for Candidate 1. In between the extremes is a moderate “cutoff” voter type who is indifferent between candidates. We show that both candidates choose their platforms so as to maximize the cutoff voter’s utility.

The equilibrium has a number of appealing properties that distinguish it from the standard model and other existing models such as the probabilistic voting model and the citizen-candidate model.

**Competition for the cutoff voter’s support.** Just as in the Downsian model with office motivated candidates, in our model, there exists one voter type whose utility both candidates maximize. However, the similarity between the models ends there: In the Downsian model, this type is always the median (or median median) voter. In our model, the location of this voter, which we refer to as the cutoff voter, depends solely on the differential abilities of the candidates (and will in general not coincide with the median). Furthermore, unlike in the Downsian model, our candidates offer different policies and only the cutoff voter is indifferent while all other voters have a strict preferences. All other voters have a strict preference for one of the two candidates. Thus, our model can reconcile the notion that candidates in an election campaign compete fiercely for the support of some “moderate” voters with the observation that, in most elections, many voters feel passionately that there is a significant difference between candidates.

**Distribution independence and rigidity.** In the standard model, the position of the candidates’ platforms depend decisively on the distribution of voter preferences and, in particular, the likely position of the median voter. In contrast, the equilibrium platforms of candidates in our framework depend exclusively on candidate skills and properties of the utility function, but not on the voter type distribution. Thus, there is a marked contrast between the standard model and ours with respect to the effect of changes in the voter distribution (or, of better information about the voter distribution).

**Policy divergence.** In our model, there is always policy divergence in terms of outcomes (i.e., in terms of the bundle of goods that the two candidates would provide), but the actual “output” may often be impossible to measure. For example, the “level of national security” provided by a candidate is difficult to measure objectively by an outside observer, while spending on national security related items is clearly defined. Thus, another empirically relevant concept of divergence is platform divergence, i.e., whether the two candidates propose a different budget allocation. Except for special cases, the equilibrium also features platform divergence in our model.³

³The first exception is if both candidates have exactly the same abilities, which renders the model equivalent to the standard model with convergence to the expected median’s preferred position; the second exception is if voters have an elasticity of
2.3 Predictions and comparison with existing models

The assumption that candidates are purely office-motivated generates policy convergence and indifference of all voters between the candidates in the Downsian framework. It is thus surprising that our model, which shares the assumption of office-motivated candidates, leads to policy divergence and strict voter preferences. Moreover, while some superficially similar results can be obtained in variations of the standard framework, our model offers qualitatively new insights.

The nature of policy divergence. The platform choice of candidates for political office is one of the major areas of interest in formal models of politics. There is a huge literature on the topic of policy convergence or divergence in one-dimensional models (or models with one policy dimension and one valence dimension). For excellent reviews of this area, see, e.g., Osborne (1995), Roemer (2001) and Grofman (2004).

Policy divergence can be obtained in the Downsian framework by assuming that candidates are policy-motivated (Wittman (1983); Calvert (1985); Roemer (1994); Groseclose (2001); Martinelli (2001)). In this type of model, divergence away from the opponent’s position reduces a candidate’s winning probability, but increases his utility in case of a victory. This trade-off should be affected by a number of exogenous factors. First, better information quality about the position of the median voter translates into less policy divergence. Second, information arrival (for example, opinion polls conducted during the campaign) should induce candidates to adjust their positions. Third, policy divergence should be less pronounced when winning office becomes more attractive (say, a more prestigious office, or an office in which the office holder receives a higher salary).

In contrast, in our model, the candidates choose a level of policy divergence that maximizes their respective winning probabilities, and none of the factors discussed above would change the candidates’ equilibrium policy positions. For example, candidates in our model “stick to their guns” in the face of shifting voter preferences, because adjusting their platform in the direction of their opponent is not going to help them electorally: “Moderation”, in the sense of moving closer to the opponent’s proposed budget allocation, would lose rather than win votes.

The fact that further moderation is a losing electoral strategy does not necessarily imply that it would be unpopular. It is well possible that a majority of the electorate would “sincerely” prefer that the weaker candidate’s position becomes more moderate. However, these voters with whom moderation is popular have a strict preference for the opponent that the weaker candidate cannot overcome. In equilibrium, candidates focus on voters who are close to indifferent between the candidates, and the preferences of substitution exactly equal to one, i.e. a logarithmic utility function.

4 By “sincerely”, we mean that these voters would prefer to be ruled by the weaker candidate with a more moderate position to being ruled by him with his equilibrium position. What we don’t mean is a voter’s “strategic” preference for the disliked candidate to take a less electable position.
these “swing voters” (rather than the majority’s) are decisive for the positions that candidates take.

In our model, candidates care only about their probability of winning, and they are perfectly free to choose any budget allocation. Thus, the forces that drive divergence in the model cited above (e.g., policy motivation) are not present in ours, so that the nature of divergence differs substantially. First, policy divergence in our model increases rather than decreases a candidate’s probability of winning. Second, from a normative standpoint, policy divergence is usually negative in standard models, in the sense that a majority of the population could be made better off if candidates chose platforms that are marginally closer to the policy of their opponent. In contrast, our Proposition 3 shows that candidates converge too much in our model.

**Rigidity.** If policy divergence arises in a Downsian world, the losing candidate always regrets his position choice: He could have done better (and maybe even won the election) if he had just chosen a different policy position. In contrast, if a candidate loses in our model because too many voters cared strongly about the good in which his opponent had an advantage, then there is really nothing that could have changed the outcome of the vote in a favorable way.

This result corresponds very well to the argument of Petrocik (1996) that “A Democrat’s promise to attack crime by hiring more police, building more prisons and punishing with longer sentences would too easily be trumped by greater GOP enthusiasm for such solutions. […] Candidates respond thus because […] to do otherwise would advantage their opponent.” In other words, the weak candidate in a particular policy area cannot benefit electorally by simply copying the platform of the strong candidate in this area.

**Medium-run stability of party dominance.** Our model also provides a framework in which one party can sustain dominance over the opposition party for an extended period of time. For example, it is generally acknowledged that the Democrats were the dominant political party from 1932 to 1968. This result is very difficult to obtain in the standard model: If the position of the median voter shifted towards the left in 1932, then the standard model implies that candidates of both parties choose more liberal positions than previously, but a voter preference shift would not translate into more power for the Democrats in the sense that they win substantially more seats in Congress. To run on a losing platform over and over again requires a remarkable extent of slow-wittedness in the Downsian framework. Even policy-motivated candidates (or rather, particularly policy-motivated candidates) should moderate their position in order to increase their chance of winning, as even the ideologically purest election program is of no use when the opposition wins. In contrast, in our model, if Republicans cannot successfully imitate the Democrats’ policy position, then sticking with their old platform and hoping for a reversal of the preference shift is the best (from an electoral perspective) that Republicans can do in the short to medium run. If the preference shift is substantial and persistent, a party would have to “re-invent” itself, in the sense of changing its perceived strengths and weaknesses (i.e., its production function).
There is currently a lively discussion in the Republican party about whether the party should reposition itself after having lost the 2006 and 2008 national elections. Some party leaders are adamant that Republicans should not move toward the center. In our model, it is certainly a plausible strategy for Republicans not to become “Democrats light”, at least until it is clear whether the preference shift is persistent.

As a further example, consider the Labour Party in the UK which lost power in 1979, plausibly due to a fundamental and persistent change in the preference distribution of voters (say, more emphasis on economic growth relative to social justice). In the interpretation of our model, the party is initially stuck with its previously successful leaders who are specialists in social justice. During this time, we would expect that party platforms change very little, and the party just hopes that the voters return to their previous preferences. If this does not happen, popular support for the party is correspondingly reduced. Over the longer term, however, the party fostered the development of new leaders who specialized more in being able to deliver on economic growth (while being weaker on social justice). Only when these new leaders were in place, a corresponding adjustment of the party platform was implemented that eventually brought the party on track for a return to power.

Valence vs. position issues  As we have argued above, our model applies for valence issues (i.e., in settings where candidates can have differentiated abilities), while the Downsian model with identical candidates is a more useful framework for thinking about position issues such as gun control or gay marriage in which differences in implementation ability are more-or-less immaterial. This creates a useful testable implication. Shifts of the voter preference distribution in valence issues should affect candidates’ positions much less than shifts of the voter distribution in position issues.

Differences to other models of candidate competition.  The properties of equilibrium in our model also differ from the citizen-candidate model and the probabilistic voting model. In the two-candidate equilibrium of the citizen-candidate model (see Osborne and Slivinski (1996), Besley and Coate (1997)), the two candidates locate at the same distance at opposite sides of the median voter’s ideal point. Thus, while there is equilibrium policy divergence and most voters have strict preferences for one of the candidates, the equilibrium platforms shift with the distribution of voter preferences, and both candidates win with probability 1/2.

In the probabilistic voting model (Lindbeck and Weibull (1987), Lindbeck and Weibull (1993), Coughlin (1992), Dixit and Londregan (1996)), voter (groups) differ in their preferences over policy chosen by the candidates, and individual voters, in addition, have “ideological” preferences for candidates’ immutable characteristics. In equilibrium, both candidates choose a platform that maximizes a weighted sum of the utilities of all voters, where the weights reflect how “movable” certain voter groups are. Thus, in contrast to our model, the equilibrium of the probabilistic voting model displays policy convergence. Like in our model, the candidates choose policy not to maximize the utility of the median
voter; however, in contrast to our model, the determination of the voter type whose utility is maximized depends crucially on the voter preference distribution.

### 2.4 Welfare implications.

Our model is important for our understanding and interpretation of the results of electoral competition. In the standard model, policy convergence to the policy preferred by the median appears efficient in the sense that there is no other policy that a candidate could propose that increases the utility of a majority of voters. Moreover, to the extent that policy divergence arises in the standard framework (for example, in the citizen-candidate model), moderation would be beneficial in the sense that, if the winning candidate implements a policy that departs from his election platform in the direction of his opponent’s platform, a majority of the electorate benefits. This result has been influential in shaping the point of view of a large segment of “moderate” political pundits that moderation and bipartisanship is inherently beneficial for society. This school of thought is (sometimes satirically) called *Broderism* (after David Broder of the Washington Post), defined by the Urban Dictionary as “the worship of bipartisanship for its own sake.”

In contrast, in our model, a majority of voters appreciates if the winning candidate *further accentuates* the policy differences to his beaten competitor that were the reason for why the winner won the election in the first place. For example, suppose that the Democrat wins the election. This happens if and only if a majority of voters has preferences that are “more liberal” than those of the cutoff voter (in the sense of having stronger preferences for those goods that the Democrat has an advantage supplying). In other words, in his attempt to maximize the set of voter distribution for which he wins the election, the Democrat caters during the election campaign to a cutoff type that has more conservative preferences than those of a strict majority of the electorate. A majority of the electorate would therefore be better off ex-post if the Democrat provided a *more* partisan policy than promised in his election platform. Of course, a symmetric argument shows that, when the Republican wins the election, then a majority of the electorate prefers a more partisan Republican policy than the cutoff voter.

Clearly, which model is correct has also important implications for our interpretation of institutions that encourage to “moderation”. Suppose, for example, that the filibuster rule in the U.S. Senate prevents Democrats from implementing the strong health insurance reform that they promised during the election campaign of 2008 and forces them to accept a more watered-down version that is palatable to the most conservative Democrats and/or the most liberal Republicans. Interpreted in a standard framework, such enforced moderation may be beneficial as the preferences of the median voter are likely to be somewhere between the Democratic and the Republican election platforms. In contrast, in our framework, the reason why Democrats won the elections is that a majority of the electorate favored their platform (and would be happy with an even more radical reform). Any institutional constraint that prevents Democrats from implementing their election platform would be detrimental in our framework.

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3 Related Literature

Differentiated candidate models. A key ingredient of our model is that candidates differ in their productivities in different policy fields. Krasa and Polborn (2009a) analyze general models of political competition when candidates have some exogenously differentiated characteristics, but are free to choose policy positions on a number of different issues. Our model here is built on this general framework, in which candidates have characteristics and choose policy to maximize their winning probability. Voters have general preferences over candidate characteristics and policy. Krasa and Polborn (2009a) characterize a class of uniform candidate ranking (UCR) voter preferences that generically lead to policy convergence in equilibrium, even if candidates have differentiated characteristics. While almost all models in the existing literature have voters with UCR preferences, voter preferences implied by the candidates’ differentiated production possibilities in our model do not satisfy UCR, as each candidate has a range of potential policies in which he is more productive than his opponent. This is the fundamental reason for equilibrium divergence in the present paper.6

This general framework is also applied in Krasa and Polborn (2009b), which is complementary to the present paper. In Krasa and Polborn (2009b), two office-motivated candidates are differentiated with respect to their ability to supply small and large quantities of a single public good. Voters differ in both their income and their taste for public goods, and both parameters influence their preferences over tax rates. The main focus is on the determination of equilibrium tax rates and how they vary with the productivity of the candidates; however, an analysis of spending allocations to different policy fields is intractable in this framework. In contrast, the present paper focuses on the allocation of money to different policy fields while taking the level of taxation as exogenous and equal between candidates.

Dixit and Londregan (1996) (henceforth DL) analyze a probabilistic voting model in which candidates choose which transfers to promise to different interest groups. Within each group, voters differ in their ideological predisposition towards the candidates. DL’s main interest is to analyze which factors determine how successful an interest group will be in attracting transfer payments from candidates. The part of their model which is related to ours is that they also analyze the effect of differential candidate ability to transfer money to different interest groups; this is modeled as a candidate- and group-specific parameter that determines how much of each dollar transferred gets lost in the process. If both candidates have the same ability to transfer money to each group, they propose the same vector of transfers to the different interest groups. In contrast, if a candidate has an advantage in transfers to a given interest group (in the sense that the loss rate is lower than his opponent’s loss rate), then candidates choose to give higher transfers to the groups for which they have a higher transfer expertise.

6One of the few existing papers with non-UCR preferences and office-motivated candidates is Adams and Merrill (2003), in which voters have, in addition to preferences over policy positions from the [0, 1] interval, a “non-policy” or partisan preference for one of the two candidates. Moreover, voters may abstain due to “alienation” (if their preferred candidate does not provide them with sufficient utility). The possibility of abstention generates incentives for equilibrium policy divergence.
DL also consider heterogeneous candidates, but their main emphasis and results differ substantially from ours. First and foremost, their focus is on the determinants of a group’s success in competing for transfer payments, while they assume that general interest policies, i.e., actions that influence the utility of all voters, are exogenously fixed (they are subsumed in the exogenous “ideological” component of voters’ utilities); in contrast, we ignore any redistribution and focus on general interest policies (the provision of different public goods). Consequently, the dimension of electoral competition is different. In our model, candidates choose their platforms to compete for the support of a cutoff voter type who is moderately interested in both public goods, and a main interest of our model is how the cutoff voter is determined and how his preferences over policy influence the platforms offered by the candidates. Moreover, voters split their support according to their preferences over positions chosen by the candidates. In contrast, voters in DL split (entirely or predominantly) according to the exogenous “ideological” dimension. Because there is no cutoff voter in the transfer dimension (as all voters only care about transfers to their own group), the way the policy vector is determined in DL is also completely different and depends crucially on the distribution of ideological preferences in each group. In contrast, in our model, equilibrium policies are independent of the distribution of voter preferences.

Also, the marginal effects of changing policy in favor of a particular group differ starkly. In DL, if a candidate offers higher transfers to a group, then the candidate will always pick up more votes from this group; this effect is, in equilibrium, balanced by vote losses in the other groups whose transfers have to be reduced accordingly. In contrast, consider the candidate who has an advantage in providing good 0 in our model. If this candidate provides slightly more of good 1 than in equilibrium, this would not win over any of the supporters of his opponent (who, while they like the direction of the move, still prefer the opponent), while losing some moderate previous supporters.

Valence models. In most models with exogenously differentiated candidates, the fixed characteristic enters additively in all voters’ utility functions and therefore does not change a voter’s ideal policy. When candidates are office-motivated as in our model (e.g., Ansolabehere and Snyder (2000), Aragones and Palfrey (2002)), the analysis focuses on a strategic dilemma: On the one hand, the weaker candidate must avoid to be too close to the location of the stronger candidate, because otherwise all voters prefer the stronger candidate. On the other hand, the stronger candidate could guarantee a victory if he takes the same position as his weaker competitor. Thus, in contrast to our model, one candidate has an incentive to differentiate, while the other candidate has an incentive to converge to his opponent’s position.

Groseclose (2001) provides an influential theoretical model in which candidates are policy-motivated, and provides an explanation for the contradictory results in empirical studies of the “marginality hypothesis” that posits that weaker candidates “moderate” their policy position in order to increase their reelection probability. We discuss this paper in more detail in Section 5.3.
**Issue ownership.** After completing this paper, we learned of the independent work of Soubeyran (2009) who analyzes a special case of our basic model. Like in our model, candidates have differentiated production functions and allocate money to the production of two different goods. Voters are assumed to have logarithmic utility functions (which is the special case in our model that leads to both candidates choosing the same observable budget allocation).\(^7\)

To our knowledge, the only other formal model of issue ownership is Egan (2008). While he focuses primarily on the empirical side of issue ownership and its implications for the responsiveness of candidates to shifts in the preference distribution of the electorate, he also develops a short alternative theory of issue ownership. In his one-dimensional model, the “issue owner” can implement the policy he promises, while the other candidate can implement his promised policy only with an error. Since voters are assumed to be risk averse, this implementation error lowers the utility that voters receive from the issue owner’s competitor. Thus, the policy-motivated issue owner can set his promised policy closer to his preferred position and still win (the competitor chooses to propose the median voter’s ideal policy, but loses, since his implementation would be subject to an additional error term).

In a setting where candidates each “own” certain issues, we would expect that candidates focus their campaign rhetoric on their strong issue and rarely talk about the issue in which they are weaker than their opponent. This corresponds to what William Riker called the “Dominance Principle” in campaign rhetoric: “When one side dominates the volume of rhetorical appeals on a particular theme, the other side abandons appeals on that theme” (Riker (1996), p.6). As a consequence, candidates rarely engage in “dialogue” in a campaign (Simon (2002)).

### 4 The Model

A polity provides for its citizens two public goods \(x_0\) and \(x_1\) (e.g., schooling or law enforcement), which are produced by the administration of the candidate who wins the election. The two candidates \(j = 0, 1\) are differentially productive in providing the two goods and have to choose how much of the government’s fixed budget (normalized to 1) to allocate to the production of each public good. Specifically, if Candidate \(j\) uses a fraction \(a^j\) of the budget for the production of good 0, then he provides the following level of the two public goods:\(^8\)

\[
\begin{align*}
    x_0 &= G_0^j(\gamma_0^j, a^j) = \gamma_0^j a^j \\
    x_1 &= G_1^j(\gamma_1^j, a^j) = \gamma_1^j (1 - a^j),
\end{align*}
\]

Candidates have different areas of expertise. We assume that \(\gamma_0^0 > \gamma_1^0\) and \(\gamma_1^1 > \gamma_0^1\), so that Candidate 0

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\(^7\)More peripherally related is Soubeyran and Gautier (2008). They analyze a dynamic model in which candidates have differential abilities and in which public goods are somewhat durable, but in which candidates do not compete for the support of a cutoff voter (they are instead assumed to maximize the utility of the deterministic median voter in each period).

\(^8\)Generally, we use superscripts to denote the candidate and subscripts to denote the good.
has an advantage in the providing good 0, and Candidate 1 has an advantage in the providing good 1. As shown in the left panel of Figure 1, the two candidates’ production possibility sets overlap, with Candidate 0’s production possibility frontier being flatter than that of Candidate 1.

Voters differ in their utility functions, which depend on the amounts of public goods provided. The utility function of a type \( t \in [0, 1] \) voter is given by \( v(x_0, x_1, t) \), where \( t \) parameterizes voters’ preferences for good 0 versus good 1, with low types putting more emphasis on good 0 and high types on good 1. For example, utility functions of the form \( v(x_0, x_1, t) = (1 - t)v_0(x_0) + tv_1(x_1) \), where \( v_0(\cdot) \) and \( v_1(\cdot) \) are the same concave functions for all voters, satisfy this property.\(^9\)

The role of \( t \) is to parameterize the relative importance of the two goods. Voters with a low value of \( t \) care primarily about the provision of good 0, and not so much about the provision of good 1. Conversely, voters with a high value of \( t \) care primarily about good 1. Graphically, the indifference curve of a high \( t \) voter is flatter than the indifference curve of a low \( t \) voter through the same point \((x_0, x_1)\).

There is a continuum of voters,\(^10\) and the distribution of voter types in the population is uncertain. Formally, nature draws a state \( \omega \) that defines a distribution of voter types in state \( \omega \). The median of the voter types in state \( \omega \) — which we denote by \( t_m(\omega) \) — will be shown to be decisive for the election outcome in that state. Recall that, if \( t_m(\omega) \) is the median voter type, then 50\% of the electorate in state \( \omega \) is to the left and 50\% to the right of \( t_m(\omega) \). It is useful to denote the cumulative distribution function of the median voter type \( t_m(\omega) \) by \( F(\cdot) \).

Including uncertainty about the voter distribution has two objectives. First, it appears quite realistic to assume that the location of the median voter is not precisely known and that candidates have to make their choices under some uncertainty. Second, if there is uncertainty over \( t_m(\omega) \), then in our setup, maximizing winning probability and maximizing vote share are typically identical objectives for candidates. Thus, the assumption helps us to refine the set of equilibria.\(^11\)

The timing of the game is as follows: Candidates \( j = 0, 1 \) simultaneously announce policies \( a^j \in [0, 1] \). Each citizen votes for his preferred candidate, or abstains when indifferent.\(^12\) The candidate who receives more votes than his opponent wins the election. In case of a tie between the candidates, each wins with probability 1/2. The winning candidate receives a payoff of 1, while the loser gets 0 (i.e., candidates are office-motivated).

\(^9\)In the appendix, we show that our qualitative results hold for a large class of preferences that satisfy a single-crossing condition such that the marginal rate of substitution between goods 0 and 1 is decreasing in \( t \).

\(^10\)Nothing of importance would change if, instead, there are finitely many voters.

\(^11\)If, instead, the distribution of voters was known with certainty, then (generically) one candidate wins for sure, and thus, the policy choice of his opponent is indeterminate and the better candidate can also win with a whole set of policies. Therefore, many strategies could be part of an equilibrium when candidates care only about the probability of winning. A natural focal point of all these equilibria is the one in which both candidates maximize their vote share, and this again corresponds to the equilibrium in our model.

\(^12\)If a voter is indifferent, he could in principle vote for any candidate or abstain. However, abstention is quite natural (e.g., in the presence of even very small voting costs).
A final word of interpretation is in order concerning the setup. While there are two public goods, there is a fixed budget constraint, and thus the policy variable $a^j$ is one-dimensional. In this regard, the model exactly mirrors the standard one-dimensional spatial model that dominates most of the literature. It is well-known that pure strategy equilibria generically fail to exist in a multidimensional version of the spatial model, and the same reasons that lead to this result also apply in ours.

5 Results

Throughout this section, we concentrate on intuitive (often, graphical) arguments. Detailed formal proofs are in the appendix.

5.1 Equilibrium

We argue first that, in any equilibrium, Candidate 0 locates at a point that is to the right of the intersection $\hat{x}$ of the two production possibility lines in the left panel of Figure 1, and Candidate 1 locates to the left of that intersection point. It is easy to see that candidates cannot locate in equilibrium at points where their opponent is strictly superior. For example, if Candidate 0 were instead to locate at $x^0$ strictly to the left of the intersection point, then Candidate 1 could just choose a point such as $x^1$ in which Candidate 1 provides more of both public goods than Candidate 0, and consequently, all voters vote for Candidate 1 (remember that both candidates spend the same amount of money, so voter preferences are based only on the two candidates’ public good provisions). Thus, Candidate 0’s choice was not optimal.

![Figure 1: Production possibility sets and non-equilibrium choices](image)

Next, we show that in equilibrium the level of public goods provided cannot be at the intersection point $\hat{x}$.
Denote by $t_m$ the voter type such that a candidate wins with probability 50%, if either all types $t \leq t_m$ or all types $t \geq t_m$ vote for him. In analogy to the standard model, we call this type the “median”. Voter $t_m$’s indifference curve is drawn in the right panel of Figure 1. As the graph indicates, Candidate 0 could instead move to $x^0$, which is strictly preferred by type $t_m$, thereby increasing Candidate 0’s winning probability to more than 50%. Similarly, Candidate 1 could move to $x^1$ and increase his winning probability.

We now know that, in a pure strategy equilibrium, the candidates’ public goods bundles are differentiated such that Candidate 0 provides more of good 0 than Candidate 1, and vice versa for good 1. Consequently, voters whose type $t$ is low (i.e., who care primarily about good 0) strictly prefer Candidate 0, and voters whose type $t$ is high (i.e., who care primarily about good 1) strictly prefer Candidate 1. There is some intermediate type $\bar{t}$ who is indifferent between the candidates, and whom we call the cutoff voter. The exact location of $\bar{t}$ of course depends on the platforms of both candidates, so that we sometimes write this dependence as $\bar{t}(a^0, a^1)$.

![Figure 2: Equilibrium choices](image)

Consider first the left panel in Figure 2 in which candidates offer $x^0$ and $x^1$, respectively. The solid indifference curve that runs through both of these bundles is that of the cutoff voter type $\bar{t}(a^0, a^1)$. Voters with types $t < \bar{t}$ have indifference curves that are steeper, and they strictly prefer $x^0$ to $x^1$ — in the graph, such an indifference curve is indicated by the dashed curve through $x^0$, where the arrow points is the “better” direction. Consequently, voters with $t < \bar{t}(a^0, a^1)$ strictly prefer Candidate 0. Conversely, voters with types $t > \bar{t}$ have indifference curves that are flatter than $\bar{t}$’s, and they strictly prefer $x^1$ to $x^0$ — the dashed indifference curve through $\bar{x}^1$ represents one such voter. Consequently, all voters $t > \bar{t}(a^0, a^1)$

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13Formally, type $t_m$ is the “median median” in the following sense. Remember that $t_m(\omega)$ is the median type in state $\omega$. This generates a distribution of median voters for different states $\omega$, and $t_m$ is the median of this distribution.

14There are two nongeneric cases in which $t_m$’s indifference curve is tangent to one of the production possibility lines. Even in these cases, the other candidate can deviate and improve his winning probability, showing again that both candidates locating at $\hat{x}$ is not an equilibrium.
strictly prefer Candidate 1.

Note that Candidate 1’s choice in the left panel in Figure 2 does not maximize the utility of the cutoff voter. If Candidate 1 instead shifts his proposed bundle to \( \bar{x}^1 \), then the previous cutoff voter and even some voters who have slightly steeper indifference curves now prefer Candidate 1. Thus, the set of voters who vote for Candidate 1 increases, and Candidate 1’s winning probability increases. It therefore follows that \((x^0, x^1)\) is not an equilibrium. In an equilibrium, neither candidate can further increase the set of voters who support him. For this, it is necessary that the cutoff voter’s indifference curve is tangent to both production possibility frontiers, and that candidates locate at the respective points of tangency \((x^0, \bar{x}^1)\) as in the right panel of Figure 2.

We summarize our results in the following proposition.

**Proposition 1** Let \((\bar{a}^0, \bar{a}^1, \bar{t})\) denote the solution of the following equation system.

\[
\begin{align*}
\gamma_0^0 \frac{\partial v(\gamma_0^0 a^0, \gamma_0^0 (1 - a^0), \bar{t})}{\partial x_0} - \gamma_1^0 \frac{\partial v(\gamma_0^0 a^0, \gamma_0^0 (1 - a^0), \bar{t})}{\partial x_1} &= 0 \\
\gamma_0^1 \frac{\partial v(\gamma_0^1 a^1, \gamma_1^1 (1 - a^1), \bar{t})}{\partial x_0} - \gamma_1^1 \frac{\partial v(\gamma_0^1 a^1, \gamma_1^1 (1 - a^1), \bar{t})}{\partial x_1} &= 0
\end{align*}
\]

If a pure strategy Nash equilibrium exists, it is given by candidates choosing \((\bar{a}^0, \bar{a}^1)\), and all voters with types \( t < \bar{t} \) voting for Candidate 0, and all voters with types \( t > \bar{t} \) voting for Candidate 1.

The equation system in Proposition 1 has a straightforward interpretation. Equation (3) specifies that the cutoff type \( \bar{t} \) is determined as the voter who is indifferent between the candidates. Equations (4) and (5) specify that the candidates choose their platforms to maximize the utility of voter type \( \bar{t} \). Of course, this equation system corresponds to the fact that \( x^0 \) and \( x^1 \) are on the same indifference curve of the cutoff voter, and that they are both at points of tangency.

There is always a unique solution to the equation system (3)–(5), as we prove in Theorem 2 in the Appendix. Intuitively, suppose that \((\bar{a}^0, \bar{a}^1, \bar{t})\) is a solution of (3)–(5), and suppose that there was a second solution \((\tilde{a}^0, \tilde{a}^1, \tilde{t})\) to the equation system (3)–(5), with \( \tilde{t} > \bar{t} \). We know that type \( \tilde{t} \) has indifference curves that are everywhere flatter than type \( \bar{t} \)'s indifference curves. In the first solution \((\bar{a}^0, \bar{a}^1, \bar{t})\), type \( \bar{t} \) prefers \( x^1 \) to every bundle of public goods that Candidate 0 can offer. A fortiori, this is true if Candidate 1 offers the optimal bundle for type \( \bar{t} \). Thus, if \( \bar{t} \) satisfies (5), then (3) cannot hold. A similar argument shows that \( \tilde{t} < \bar{t} \) cannot hold either. Finally, for a given value of \( \bar{t} \), there are unique values of \( a^0 \) and \( a^1 \) that satisfy (4) and (5).

It should be clear that the strategy profile characterized in Proposition 1 is at least a local (strict) equilibrium, in the sense that small deviations by a candidate would always decrease the set of voters who vote for him, and therefore his winning probability. This is true because small deviations always
decrease the utility of the cutoff voter $\tilde{t}$, and therefore the deviating candidate loses the support of the cutoff voter and the set of voter types who support the deviating candidate is smaller than before. No matter how the type distribution is, this decreases both the vote share and the winning probability of the deviating candidate.\(^\text{15}\) This argument also shows that our modeling assumption that candidates are uncertain about the distribution of voters’ preferences does not drive the result in Proposition 1 in any significant way — the same result would hold in a setting where candidates know the distribution of voters and aim to maximize their vote share.

Now consider large deviations. We say that Candidate 0 *outflanks* Candidate 1 if he deviates to offer a bundle that offers more of good 1 than the bundle proposed by Candidate 1; and analogously for outflanking by Candidate 1. In other words, an outflanking candidate tries to appeal to those voters who care most about goods for which his opponent has a production advantage.

We will present two types of conditions that guarantee existence of equilibrium. The first one, detailed in Theorem 1 in the Appendix, makes sure that candidates do not have a deviation available that allows them to outflank their respective opponent. This type of condition depends only on properties of the utility functions and the two candidates’ production possibility sets, but is independent of the specific distribution of voter types.

The second type of existence condition applies in situations in which candidates can, in principle, outflank their opponent. An outflanking move means that a candidate specializes extremely (i.e., more strongly than his opponent does in equilibrium) on the public good in whose production he has a disadvantage. For this reason, the outflanking candidate is in a very precarious position that often makes this an unattractive strategy (so that the original profile is, in fact, an equilibrium).

Note first that any deviation from $(\bar{a}^0, \bar{a}^1)$ that is not outflanking cannot increase a candidate’s winning probability. This follows from essentially the same arguments as above: Suppose, for example, that Candidate 0 deviates to $\tilde{a}^0$, but that this is not outflanking. Thus, both before and after the deviation, low $t$ types vote for Candidate 0, and the decisive issue is only how the deviation changes the cutoff voter type who is indifferent between candidates. But the deviation away from $\bar{a}^0$ means that voter type $\tilde{t}$ would be worse off with Candidate 0 than before, and now strictly prefers Candidate 1. Consequently, the new cutoff voter must be to the left of $\tilde{t}$, and Candidate 0’s probability of winning decreases. In summary, any deviation that is not outflanking decreases a candidate’s set of voters.

If candidates have sufficiently different expertise, then no candidate has any outflanking deviation. Such a scenario is depicted in the left panel of Figure 3. Here, $\tilde{x}^0$ is such that Candidate 1 cannot provide more of good 0 than Candidate 0 does in equilibrium, even if he puts all resources in the production

\(^{15}\)A decrease of the set of a candidate’s supporter voter types translates into a decrease of the candidate’s winning probability if and only if the density of possible median voter types is positive at $\tilde{t}$. If $F’(\tilde{t}) = 0$, i.e. the density of possible median voter types is zero at $\tilde{t}$, then there are, in addition to the equilibrium we characterize, other equilibria (all of which have the same winning probabilities for the candidates). A sufficient condition to exclude all other equilibria is to assume that $F’(t) > 0$ for all $t \in (0, 1)$. 

16
of good 0 (i.e., $a^1 = 1$). Similarly, $\bar{x}^1$ is such that Candidate 0 cannot provide more of good 1 than Candidate 1 does in equilibrium, even if he puts all resources in the production of good 1 (i.e., $a^0 = 0$). In this case, candidates cannot appeal successfully to their opponent’s core supporters. Graphically, it is clear that this case is more likely to arise if the equilibrium platforms are far apart from each other, and this in turn is more likely if the curvature of the cutoff voter’s indifference curve is small. In Theorem 1 in the Appendix, we provide a formal condition that guarantees that equilibrium platforms are such that no outflanking deviations are possible. Consequently, this provides a sufficient condition for the strategy pair identified in Proposition 1 to be the equilibrium.

The right panel of Figure 3 depicts an outflanking deviation for Candidate 1. In the $(\bar{x}^0, \bar{x}^1)$ configuration, Candidate 1 attracts the votes of all types with indifference curves flatter than those of type $\bar{t}$ (i.e., types $t > \bar{t}$). If Candidate 1 instead deviates to $x'$, he will attract all types with indifference curves steeper than those of type $t'$ (i.e., types $t < t'$).

Let $F(\cdot)$ denote the cumulative distribution function of the median voter type $t_m(\omega)$. In the $(\bar{x}^0, \bar{x}^1)$ configuration, Candidate 1 attracts the votes of all types $t > \bar{t}$, so that his winning probability is $1 - F(\bar{t})$, while Candidate 0 wins with probability $F(\bar{t})$. If Candidate 1 plays his optimal outflanking deviation, Candidate 1 attracts the votes of all types $t < t'$, so that his winning probability is $F(t')$. Since $t' < \bar{t}$, $F(t') < F(\bar{t})$. Thus, Candidate 1’s winning probability with his optimal outflanking deviation is strictly less than his opponent’s winning probability in the $(\bar{x}^0, \bar{x}^1)$ configuration. An analogous argument holds for Candidate 0. Thus, a sufficient condition for $(\bar{a}^0, \bar{a}^1, \bar{t})$ to be an equilibrium is that both candidate’s winning probabilities are close to $1/2$. This is stated formally in the following proposition.

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16Note that $x'$ is Candidate 1’s optimal outflanking deviation, as the indifference curve of type $t'$ is tangent to his production possibility frontier. Candidate 1 cannot appeal to types with flatter indifference curves (as their indifference curve through $\bar{x}^0$ never touches his production possibility frontier.)
Proposition 2 Let \((\bar{a}^0, \bar{a}^1, \bar{t})\) denote the strategy configuration characterized in Proposition 1.

1. A deviation is always strictly detrimental for a candidate whose winning probability is at least \(1/2\).

2. There exists \(\varepsilon > 0\) such that, if \(F(\bar{t}) \in (0.5 - \varepsilon, 0.5 + \varepsilon)\), a deviation is always strictly detrimental for both candidates.

5.2 Welfare

We now turn to the welfare properties of the equilibrium. There is a general intuitive notion that policy convergence such as the one arising in the one-dimensional standard model is excessive over-convergence, effectively depriving voters of a real choice. This notion of essentially equivalent candidates is not true in the equilibrium of our model, where almost all voters have a strict preference for one of the two candidates. Nevertheless, from a social point of view, the candidates converge too much in equilibrium. If a social planner could force both candidates to put more emphasis on the policy area in which they are strong, then (with probability 1), a majority of the population would be better off.

Intuitively, the reason why a majority of the population would be better off if the candidates focused marginally more on their strong issue is the following. In equilibrium, both candidates choose, from their respective sets of available policies, the one that maximizes the utility of the cutoff voter \(\bar{t}\). If Candidate 0 wins, this means that a majority of voters cares relatively more about good 0 provision than the cutoff voter. The preferred budget share allocated to good 0 production for each member of this majority is larger than what is optimal for the cutoff voter. An analogous argument shows that, if Candidate 1 wins the election, a majority of voters would be better off with a lower \(a^1\) (i.e., with a stronger focus on Candidate 1’s strength in good 1 production).

Proposition 3 Suppose that \((a^0, a^1)\) is an equilibrium in which both candidates have a strictly positive probability of winning, and that \(t_m(\omega)\) has a strictly positive density. Then the following is true with probability 1.\(^{17}\)

1. If Candidate 0 wins, then there exists \(a_0' > a^0\) such that, ex-post, a majority of voters would strictly prefer \(a_0'\) to \(a^0\).

2. If Candidate 1 wins, then there exists \(a_1' < a^1\) such that, ex-post, a majority of voters would strictly prefer \(a_1'\) to \(a^1\).

\(^{17}\)The reason why the statements in the proposition are only true “with probability 1” (rather than “always”) is that, in principle, it is possible that the cutoff voter \(\bar{t}\) is also the realized median voter. In this case, a marginal change of policy would make a (bare) majority worse off. However, note that \(t_m(\omega) = \bar{t}\) occurs only with probability 0.
To better understand the reasons for the inefficiency, it is useful to refer to the definitions of ex-ante majority-efficiency and competition-efficiency in Krasa and Polborn (2006). Ex-ante majority-efficiency compares the voters’ utilities when the candidate is elected and implements his equilibrium platform \(a\) with the voters’ utilities if he instead implements some alternative platform \(a’\). Whether a majority of the electorate is better or worse off with \(a\) or \(a’\) depends on the state \(\omega\) that determines the voter preference distribution. A candidate’s platform \(a\) is ex-ante majority-efficient if there is no other platform \(a’\) that is more likely to make a majority of the electorate better off than worse off.

In contrast, competition-efficiency refers to the equilibrium pair of platforms, \((a^0, a^1)\) in comparison to some other pair of platforms \((\tilde{a}^0, \tilde{a}^1)\). Given the platforms, the state of the world \(\omega\) determines which candidate wins and which policy is implemented, and thus ultimately whether a majority of voters would prefer what they receive under \((a^0, a^1)\) or under \((\tilde{a}^0, \tilde{a}^1)\). The equilibrium \((a^0, a^1)\) is called competition-efficient if a majority of the electorate is more likely to be better off under \((a^0, a^1)\) than under \((\tilde{a}^0, \tilde{a}^1)\), for any other pair of platforms \((\tilde{a}^0, \tilde{a}^1)\).

In our model, the candidates’ equilibrium strategies are generally not ex-ante majority-efficient, because the cutoff voter is usually different from the “median median voter” (i.e., the median realization over \(\omega\) of the median voter). In the equilibrium of our model, there may be a very high probability that the median voter, and thus a majority of the electorate, would prefer (say) a higher emphasis on spending on good 0 than both candidates choose to provide in equilibrium. Yet, candidates would suffer a reduction in their winning probability if they catered more to the (likely) majority interests. We demonstrate this possibility by example in Section 5.4 below.

Now consider what happens if this effect is not present, because the cutoff voter happens to be equal to the median median voter. In this case, each candidate’s platform is ex-ante majority-efficient, because it is more likely that a majority of the electorate prefers the equilibrium platform to any other platform with higher or lower spending on good 0. However, the equilibrium pair of platforms \((a^0, a^1)\) is still not competition-efficient, because both candidates maximize the utility of the same type \(\bar{t}\). Uncertainty about the distribution of voter preferences implies that the realized median voter is almost never identical to \(\bar{t}\).

Specifically, let \((\bar{a}^0, \bar{a}^1) = (a^0 + \epsilon, a^1 - \epsilon)\) (with \(\epsilon > 0\) but sufficiently small, i.e., both candidates choose a platform that is a bit more “extreme” than their equilibrium platform in the sense that it is preferred by most of their supporters to their respective equilibrium platform, while all voters who prefer their respective opponent are worse off with the new platform in comparison to the equilibrium platform. Under the pair of platforms \((\bar{a}^0, \bar{a}^1)\), Candidate 0 wins if and only if low types are in the majority, i.e. if \(t_m(\omega) < \bar{t}\), and in these cases, the majority prefers the stronger emphasis on good 0 production in \(\bar{a}^0\) relative to \(a^0\). Conversely, Candidate 1 wins if and only if high types are in the majority, i.e. if \(t_m(\omega) > \bar{t}\), and in these cases, the majority prefers the stronger emphasis on good 1 production in \(\bar{a}^1\) relative to \(a^1\).

\[18\] This working paper version of Krasa and Polborn (2009c) contains more general results than the published version, in particular an analysis of the case with uncertainty about voter preferences which is relevant here.
The importance of this second effect depends on how uncertain the position of the median voter is for candidates, while the size of the first effect (due to the difference between median and cutoff voter) is completely independent of the specific uncertainty in the voter distribution.

The result that candidates’ platforms are “too moderate” with probability 1 differentiates our model from most standard one-dimensional models with policy divergence. Consider, for example, the citizen-candidate model of Osborne and Slivinski (1996). In their model, there exists (for large parameter sets) an equilibrium in which two candidates located symmetrically at opposite sides of the median voter run against each other, and each wins with probability 1/2. Independent of whether the right-wing or left-wing candidate wins the election, a majority of voters would like the winning candidate to implement a more moderate policy (i.e., a policy that is closer to the median). The same result applies in models where policy divergence is due to entry deterrence (Palfrey (1984), Callander (2005)). Likewise, in Calvert’s (1985) model in which two policy-motivated candidates are uncertain about the median voter’s preferred position and choose platforms to maximize their own expected utility from the implemented policy, divergence arises because each candidate chooses his position trading-off an increased probability of winning from moderating his platform, and a lower utility from the more moderate policy. If the election outcome is sufficiently close, then the realized median voter’s preferred position is between the two candidates’ positions, and consequently, a majority of the electorate would strictly prefer that the election winner adopts a more moderate position than promised during the campaign.\footnote{If the election result is lopsided in the Calvert (1985) model, then the realized median voter’s preferred position may be more extreme than the platform proposed by the winning candidate, so that a majority would prefer the implementation of a more extreme platform. However, this situation certainly does not arise with probability 1, as in our model.}

Bernhardt, Duggan, and Squintani (2009), who analyze a standard model with uncertainty about the position of the median also find that voters may benefit in expectation from platform divergence that results when parties are policy-motivated instead of office-motivated.\footnote{More generally, Krasa and Polborn (2006), Theorems 5 and 6 show that, in a class of models containing the standard model, the candidates’ equilibrium platforms are competition-efficient if and only if there is no uncertainty about the preferred position of the median voter.} Note, though, that the extent of the inefficiency in their model is limited if uncertainty about the location of the median is small. In contrast, the size of the inefficiency in our model remains generally bounded away from 0 even if the uncertainty about the position of the median goes to zero.

5.3 Comparative statics

We now consider what happens to the equilibrium policies when one of the candidates becomes more productive. Suppose, for concreteness, that Candidate 0 becomes more efficient in the production of good $i$. It is clear that this change increases the electoral support for Candidate 0, i.e. the cutoff voter moves to the right ($\bar{t}$ increases). Candidate 1’s productivity did not change, but we know that he chooses his equilibrium policy with the objective of appealing to the new cutoff voter, who is more interested in
good 1 relative to good 0 than the previous cutoff voter. Consequently, Candidate 1 lowers $a_1$ in order to increase his production of good 1. More generally, the candidate whose productivity did not increase is forced to focus more strongly on the production of the good in which he has an advantage.

For Candidate 0, there are two effects that possibly go in different directions. First, the same indirect “competition” effect discussed in the previous paragraph implies that, in order to appeal to the new cutoff voter, Candidate 0 has an incentive to increase his production of good 1. The direct “substitution” effect, in contrast, depends on which of the two production functions became more productive. If Candidate 0’s productivity in good $i$ production increased, then every voter type prefers a higher level of good $i$ production than before. Thus, if Candidate 0’s productivity in producing good 1 increased, then both the indirect and the direct effect go in the same direction, and Candidate 0 will choose a lower value of $a_0$ (i.e., more good 1 production) than before. In contrast, if Candidate 0’s productivity in producing good 0 increased, then the indirect and the direct effect go in opposite directions, and the sign of the total effect is, in general, unclear.

**Proposition 4**

1. Any increase of Candidate 0’s productivity induces Candidate 1 to increase his good 1 provision (and, correspondingly, to decrease his good 0 provision): $\frac{da_1}{dy_0} < 0$ and $\frac{da_0}{dy_1} < 0$.

2. An improvement of Candidate 0’s productivity in good 1 production induces Candidate 0 to provide more of good 1: $\frac{da_0}{dy_1} < 0$

3. An improvement of Candidate 0’s productivity in good 0 production may induce Candidate 0 to provide more or less of good 0.

Groseclose (2001) provides an influential theoretical model with policy-motivated candidates and differential additive valence and uncertainty about the median voter’s position. Without valence differences, the two candidates locate symmetrically around the expected median. When the valence of one of the candidates increases, his equilibrium position initially becomes more moderate before eventually (i.e., for sufficiently high valence advantage) becoming more extreme. In contrast, the disadvantaged candidate always becomes more extreme as his opponent’s valence increases. Thus, his model provides an explanation for the contradictory results in empirical studies of the “marginality hypothesis” that posits that weaker candidates “moderate” their policy position in order to increase their reelection probability.

Proposition 4 shows that our model provides an alternative theory for somewhat similar results, though based on different fundamental reasons than in Groseclose (2001), where divergence arises based on policy motivation. In our model the weaker candidate becomes more extreme (i.e., focuses more strongly on his strong good), while the effects for the candidate whose productivity increases are more subtle, as competition effect and substitution effect may go in opposite directions.

Another exogenous change that one can analyze is what happens when the budget increases. In classical microeconomic household theory, a household with a homogeneous utility function always
spends the same fraction of his income on each good, no matter how rich he is. The CES-utility function in our canonical example is homogeneous and, consequently, an exogenous increase of the budget would leave the budget fraction allocated to each good that is optimal for the cutoff voter unaffected. It is also easy to check that the type of the indifferent voter does not change when candidates leave their budget allocation unchanged. Thus, for homogeneous voter utility functions, a change in the government’s budget does not change the equilibrium budget allocations for the two public goods (relative to the total size of the budget). This would change for non-homogeneous voter utility functions, as in this case, an increase of the budget may affect the cutoff voter type, with corresponding changes in equilibrium platforms.

5.4 Application: Voter preferences with constant elasticity of substitution

In this section, we determine the equilibrium solution of the model for the case of utility functions with constant elasticity of substitution (CES utility functions), given by

\[ v(x_0, x_1, t) = \left( (1 - t)x_0^\rho + tx_1^\rho \right)^{1/\rho}, \]

where \( \rho \in (-\infty, 1] \). Our main interest in this section is how properties of the voters’ utility functions (in particular, the degree of substitutability between the public goods) influence whether candidates use the proposed budget allocation to strengthen their strong issue, or to partially compensate for their weakness.

To understand the role of \( \rho \), observe that the marginal rate of substitution (the slope of the indifference curve) is given by

\[ \frac{dx_1}{dx_0} = -\frac{(1 - t)x_0^{\rho - 1} \left( (1 - t)x_0^\rho + tx_1^\rho \right)^{\frac{\rho - 1}{\rho}}}{tx_1^{\rho - 1} \left( (1 - t)x_0^\rho + tx_1^\rho \right)^{\frac{\rho - 1}{\rho}}} = -\frac{(1 - t)}{t} \left( \frac{x_1}{x_0} \right)^{1-\rho}. \]

In the CES utility function given in (6), \( \frac{1}{1-\rho} \) is referred to as elasticity of substitution and measures the curvature of the voter’s indifference curve. If \( \rho = 1 \), then voters are only interested in a weighted sum of \( x_0 \) and \( x_1 \); the weights \( t \) and \( 1 - t \) differ between voters, but the slope of each voter’s indifference curve is constant at \(-\frac{(1-t)}{t}\) (as \( (x_1/x_0)^{0} = 1 \) for all \( x_0 \) and \( x_1 \)). The constant marginal rate of substitution implies, for example, for voter type \( t = 2/3 \), an increase of \( x_1 \) by one unit is always worth as much as an increase of \( x_0 \) by two units. In contrast, for \( \rho < 1 \), the voters’ marginal rate of substitution depends on \( x_0 \) and \( x_1 \) (as well as, of course, on \( t \)). For example, the case of \( \rho \to 0 \) corresponds to Cobb-Douglas preferences, and \( \rho \to -\infty \) corresponds to L-shaped “Leontief” indifference curves.\(^{21}\)

Given policy proposals \( a^0 \) and \( a^1 \), the voter who is indifferent between the two candidates is given by the value of \( t \) that solves

\[ \left( (1 - t)[\gamma_0^0 a^0 + t\gamma_1^0 (1 - a^0)]^{\rho} \right)^{1/\rho} = \left( (1 - t)[\gamma_0^1 a^1 + t\gamma_1^1 (1 - a^1)]^{\rho} \right)^{1/\rho}, \]

\(^{21}\)To see the latter, note that, for \( \rho \to -\infty \), \( \frac{\gamma_0}{x_0} \) is very large if \( x_1 > x_0 \), and is close to zero if \( x_1 < x_0 \).
which is
\[ t(a^0, a^1) = \frac{(\gamma^1_0 a^1)^\rho - (\gamma^0_0 a^0)^\rho}{(\gamma^1_0 a^1)^\rho - (\gamma^0_0 a^0)^\rho - (\gamma^1_1 (1 - a^1))^\rho + (\gamma^0_1 (1 - a^0))^\rho}. \tag{9} \]

Candidate 0’s objective is to increase \( t(a^0, a^1) \) as far as possible, because each voter \( t \leq t(a^0, a^1) \) votes for Candidate 0. Similarly, Candidate 1’s objective is to decrease \( t(a^0, a^1) \) as far as possible, because each voter \( t \geq t(a^0, a^1) \) votes for Candidate 1. As we show in the Appendix, the corresponding first-order conditions can be rearranged to yield
\[ \frac{1-a^0}{1-a^1} = \left[ \frac{\gamma^0_0 / \gamma^1_1}{\gamma^0_1 / \gamma^1_0} \right]^{\frac{2}{1-\rho}}. \tag{10} \]

Note that the term in square brackets is smaller than 1 (as \( \gamma^0_0 > \gamma^1_0 \), and \( \gamma^1_1 > \gamma^0_1 \)). Thus, if \( \rho \in (0, 1] \), the left-hand side of (10) is smaller than 1, which implies \( a^0 > a^1 \). Conversely, if \( \rho < 0 \), the left-hand side of (10) is greater than 1, which implies \( a^0 < a^1 \), and \( \rho = 0 \) is the boundary case where \( a^0 = a^1 \).\(^{22}\)

**Proposition 5** For the class of CES-utility functions given by (6), the following results hold.

1. If \( \rho \in (0, 1] \), then \( a^0 > a^1 \);
2. If \( \rho = 0 \), then \( a^0 = a^1 \);
3. If \( \rho < 0 \), then \( a^0 < a^1 \);

**Proof.** See Appendix. \( \blacksquare \)

Thus, in cases where the two public goods are relatively good substitutes, candidates choose platforms that further strengthen their respective strong point; that is, Candidate 0 chooses to put more money into the production of good 0 than Candidate 1, and vice versa for good 1. In contrast, in cases where the two public goods are relatively poor substitutes, candidates choose platforms in which they compensate for their weakness; that is, each candidate puts less money than his opponent into the production of the good in which he is strong; this allows the candidate to spend more money on his weak good, partially offsetting the advantage of his competitor there.

In order to analyze existence conditions for the equilibrium, it is useful to assume a symmetric ability distribution such that \( \gamma^0_0 = \gamma^1_1 = r \) and \( \gamma^0_1 = \gamma^1_0 = 1 - r \), where \( r \geq 0.5 \) measures the extent of (symmetric) specialization. If \( r \) is close to 1/2, a candidate’s advantage in his better field is very limited, while if \( r \) is high, each candidate is a specialist in his strong field and a rookie in the other field.

\(^{22}\)The CES utility function is not defined for \( \rho = 0 \), but it is well known that the logarithmic utility function (equivalent to a Cobb-Douglas utility function here) is a utility function with constant elasticity of substitution equal to 1 (= 1 - \( \rho \)).
We know from Proposition 1 that candidates choose positions that maximize the utility of the cutoff voter. Maximizing

\[
[(1 - t)(ra^0)^\rho + t((1 - r)(1 - a^0))^\rho]^{1/\rho}
\]

with respect to \(a^0\) for Candidate 0, and an analogous problem for Candidate 1, and substituting \(\bar{t} = 1/2\) (because of the symmetry of the problem, the cutoff voter must be located at 1/2) yields

\[
\bar{a}^0 = \frac{1}{1 + (\frac{1-r}{r})^{\frac{\rho}{1-\rho}}}, \quad \bar{a}^1 = \frac{(\frac{1-r}{r})^{\frac{\rho}{1-\rho}}}{1 + (\frac{1-r}{r})^{\frac{\rho}{1-\rho}}}
\]

(12)

The corresponding production levels are

\[
\bar{x}^0_0 = \bar{x}^1_1 = \frac{r}{1 + (\frac{1-r}{r})^{\frac{\rho}{1-\rho}}}, \quad \bar{x}^0_1 = \bar{x}^1_0 = \frac{(1 - r)(\frac{1-r}{r})^{\frac{\rho}{1-\rho}}}{1 + (\frac{1-r}{r})^{\frac{\rho}{1-\rho}}}.
\]

(13)

Figure 4 provides the cutoff individual as a function of Candidate 1’s choice of \(a^1\) (\(r = 0.55, \rho = 0.5\)).

Figure 4 provides the cutoff individual as a function of Candidate 1’s budget allocation \(a^1\), for \(\rho = 0.5\) and \(r = 0.55\) (i.e., very moderate specialization of candidates). If \(a^1 < 0.6318\), then Candidate 1 attracts all voters located above the cutoff in the left panel. Consequently, Candidate 1’s set of supporters is maximized (in this range) for \(a^1 = 0.45\), for which the cutoff is 0.5. This, of course, is just \(\bar{a}^1\) given in (12). Allocating slightly more money to good 0 production just loses moderate voters (i.e., the cutoff.
For $a^1 \in [0.6318, 0.6722]$, all voters prefer Candidate 0. The outflanking deviations start from $a^1 > 0.6722$. Candidate 1 now appeals to voter types below the cutoff, i.e., those voters who care primarily about good 0. Thus, it is optimal for him to maximize the cutoff in this range. This is achieved by setting $a^1 \approx 0.8195$, which generates a cutoff of slightly less than 0.298, so that Candidate 1’s winning probability with the optimal deviation is about $F(0.298)$. Thus, if $1 - F(0.5) > F(0.298)$ (i.e., if the probability that the median voter is a type larger than 0.5 is larger than the probability that the median voter type is below 0.298), then even the optimal deviation decreases Candidate 1’s winning probability.

For example, suppose that the location of the median voter is normally distributed around 0.4 with standard deviation $\sigma_t$. For any $\sigma_t$, the actions characterized by (12) are the unique equilibrium, because $1 - F(0.5) > F(0.298)$. From a welfare perspective, the equilibrium in which candidates maximize the utility of voter type 0.5 appears very inefficient: As $\sigma_t \to 0$, the median voter is almost certainly close to 0.4 and Candidate 0 wins the election with probability close to 1 (as he has a comparative advantage in the production of the good that the majority cares about more). However, he does so with a platform that, from a social point of view, caters too much to the interests of the (likely) minority that cares more about good 1.

### 6 Extensions

In this section, we analyze the robustness of the model with respect to three important assumptions of the basic model. First, candidates are exogenously assumed in the basic model to have differential productivities. Here, we want to analyze a setup in which, instead, parties choose their respective candidates from a set that contains both balanced and specialized potential candidates. Second, we assume in the basic model that the candidate’s tax rate is exogenously fixed at the same level for both candidates; here, we analyze what would happen when candidates can choose the tax rate as part of their platform (in addition to the budget allocation). Third, we present an important re-interpretation of the model.

#### 6.1 Party Choice of Candidates

A key ingredient of our model is that candidates have differentiated abilities. Since the equilibrium is much different when both candidates instead have the same abilities, it is crucial to analyze the incentives of the parties whom to nominate when there is a choice between several different potential candidates. In particular, we are interested in a setup in which the parties’ choice sets overlap, so that they could, in principle, nominate two candidates who have exactly the same capabilities. It is then meaningful to ask whether parties select candidates that coincide or differ from the opponent chosen by the other party.

The choice behavior of parties depends on their objectives. A party can be either office-motivated or

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23 The exact boundaries of this interval are $139/220$ and $121/180$. 

25
policy-motivated. While we believe that there are good arguments that parties representing their members are more policy-motivated than candidates, we initially focus on office-motivated parties, because (i) this is the harder case for divergence\textsuperscript{24} and (ii), it allows us to show clearly the difference between the standard model and our model.

As a benchmark case, consider the following nomination model in a standard one-dimensional Downsian framework with office motivated parties: Suppose that voters’ ideal points are distributed in [0, 1] and that the median median is located at 0.5 — recall that receiving the support of the median-median implies that the winning probability is at least 50%. Suppose furthermore that the liberal party can select a candidate $\theta_L \in [0, 0.5]$, while the conservative party can select a candidate $\theta_R \in [0.5, 1]$.\textsuperscript{25} Candidates are citizen-candidates in the sense that they cannot credibly commit during the election campaign to another policy than their most preferred one.

If the parties only care about winning, then it is optimal for them to choose identical candidates, i.e., $\theta_L = \theta_R = 0.5$. Differentiated candidates will only be chosen if parties care about policy. Suppose the typical liberal party member prefers a policy strictly to the left of 0.5 and the conservative party a policy to the right of 0.5, then $\theta_L < \theta_R$. However, parties are now trading off getting their party into office against getting their most preferred policy implemented. In other words, in the standard framework, satisfying the policy objectives of a party’s rank and file members and maximizing the winning probability of the party’s candidate are conflicting objectives.

Indeed, it would be problematic if a similar intuition applied in our model, because it would suggest that parties select candidates that are very similar, unless they have a very substantial policy motivation. However, we now show that, in contrast to the standard framework, parties have a strong incentive to chose differentially skilled candidates in our model.

We choose a setup that is completely symmetric to the one we just discussed for the Downsian model. We assume that party 0 is composed of individuals more keen on good 0 than the median median (i.e., $t < t_m$ for party 0 supporters), while party 1 consists of individuals that care more about good 1 ($t > t_m$). Each party must choose between a “balanced candidate” and another candidate, who is better in providing the good party members like, but worse in producing the other good. After candidates are nominated, they choose which combination of goods to propose from their budget set.

Suppose first that parties choose balanced candidates. The equilibrium in the following subgame is depicted in the top left panel of Figure 5. Both candidates’ production possibility frontiers coincide, and they choose the same policy $x_m$ that results in provision of $x_m$ of public goods that maximize the median voter’s utility. Each candidate wins with 50% probability.

Now suppose that party 0 nominates instead a candidate who is better in producing good 0 and worse in producing good 1. Assume, for the moment, that this candidate could still provide $x_m$. The

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\textsuperscript{24}It is well known that policy-motivation leads to divergence in the standard model, and the same would be true here as well.

\textsuperscript{25}The argument below remains valid even if there is a larger overlap in the parties’ feasible intervals.
resulting equilibrium is shown in the right top panel of figure 5. Note that $\bar{x}_0$, the equilibrium public good allocation by Candidate 0, is strictly preferred by the median type to $\bar{x}_1$ (Candidate 1’s equilibrium response), so that Candidate 0’s winning probability is now strictly larger than 50% because $t_m$ and even some types $t > t_m$ now support Candidate 0. Thus, even a purely office-motivated party prefers the specialized candidate. Note, though, that another beneficial aspect of differentiation from the perspective of party members is that they all prefer $\bar{x}_0$ to $x^m$: Party members identify more with the platform that is proposed by a specialized candidate. The lower left panel of Figure 5 shows that a symmetric argument applies to party 1.

Note that the new candidate’s production possibility frontier does not have to go through $x^m$ in order for the above effect to work. Consider, for example, the lower right panel of Figure 5. Given the solid production possibility frontier of Candidate 1, $\bar{x}_1$ is optimal, and the median voter is indifferent between the two candidates, and each of them wins with 50% probability. If the production possibility set is moved to the right (e.g., the dashed line), then Candidate 1’s winning probability is strictly larger than 50%, even though he may not be able to provide $x_m$. 

Figure 5: Endogenous Differentiation
In summary, the forces that determine the optimal candidate choice by parties in our model differ significantly from those present in the standard framework. In the standard model, choosing a “more extreme” candidate may please party members (if they are policy motivated), but the probability that the party’s nominee wins the election suffers. In contrast, choosing a more specialized candidate (who is better at producing the party’s preferred good even at the expense of being worse in producing the other good) has the potential of both pleasing party members and increasing the winning probability of the party’s candidate. Thus, the forces that induce parties to choose differentiated candidates in our model appear stronger than those that lead to policy differentiation in the standard framework, and so the assumption that candidates are, in fact, differentiated with respect to their productivities in different policy fields appear quite robust.

6.2 Endogenous taxation

In the basic model, we assume that both candidates raise the same taxes and thus face the same budget constraint. In this section, we consider what happens when the level of taxation is another choice variable for candidates.

A voter’s type is now determined by the voter’s income \( m \) in addition to the preference parameter \( t \). If the tax rate is \( \tau \), then private consumption is \( c = (1 - \tau)m \). The voter’s utility is given by

\[
\ln(c) + v(x_0, x_1, t),
\]

where \( v \) is homogeneous of degree \( k > 0 \) in \((x_0, x_1)\),\(^26\) which, for example, is the case for CES preferences. For simplicity, suppose that there is no uncertainty about the average income, which we denote by \( \bar{m} \). Candidates choose a platform consisting of a tax rate and a budget allocation, \((\tau^j, a^j)\).

We now show that the equilibrium budget allocations of the basic model, \( \bar{a}^j, j = 0, 1 \), remain an equilibrium allocation in the extended model with taxes. Voter \((t, m)\) is indifferent between the two candidates if

\[
\ln((1 - \tau^0)m) + v(x_0^0, x_1^1, t) = \ln((1 - \tau^1)m) + v(x_0^1, x_1^0, t).
\]

Since \( \ln((1 - \tau^j)m) = \ln(1 - \tau^j) + \ln(m) \) it follows that \( t \) is independent of \( m \). Hence there exists a cutoff voter \( \bar{t} \) as in the basic model, and in equilibrium each candidate must maximize \( \bar{t} \)'s utility.

If Candidate \( j \)'s proposed tax rate is \( \tau^j \), then his public good production is \( x_0^j = \gamma_0^ja^j\tau^j\bar{m} \) and \( x_1^j = \gamma_1^j(1 - a^j)\tau^j\bar{m} \). Thus, Candidate \( j \) solves

\[
\max_{a^j, \tau^j} \ln((1 - \tau^j)m) + v(\gamma_0^ja^j\tau^j\bar{m}, \gamma_1^j(1 - a^j)\tau^j\bar{m}, t),
\]

which, because \( v \) is homogeneous of degree \( k \), is equivalent to

\[
\max_{a^j, \tau^j} \ln((1 - \tau^j)m) + (\tau^j)^k \cdot v(\gamma_0^ja^j\bar{m}, \gamma_1^j(1 - a^j)\bar{m}, t),
\]

\(^26\)That is, \( v(\lambda x_0, \lambda x_1, t) = \lambda^k v(x_0, x_1, t) \).
Let \( \bar{a}^j, j = 0, 1 \) be the equilibrium budget allocation of the basic model and let \( \bar{t} \) be the corresponding cutoff voter. Then \( \bar{a}^j \) solves (16) for \( t = \bar{t} \) since the first summand in the objective, \( \ln((1 - \tau^j)m) \), does not depend on \( a^j \), and by definition \( \bar{a}^j \) solves \( \max_{a^j} -v(\gamma_0^j a^j \bar{m}, \gamma_1^j (1 - a^j) \bar{m}, t) \).

The optimal tax rate, \( \bar{\tau}^j \), for voter \( \bar{t} \) is the solution to the first order condition of (15) with respect to \( \tau^j \), given by

\[
-\frac{1}{1 - \tau^j} + k \cdot (\tau^j)^{k-1} v(x_0^j, x_1^j, \bar{t}) = 0. \tag{17}
\]

In the basic model voter \( \bar{t} \) is indifferent between the candidates’ proposals, i.e., \( v(x_0^0, x_1^0, \bar{t}) = v(x_1^0, x_1^1, \bar{t}) \). Hence (17) implies that both candidates propose the same tax rate, i.e., \( \bar{\tau}^0 = \bar{\tau}^1 \). Thus, \( \bar{\tau}^j, \bar{a}^j \), is an equilibrium of the extended model. Any deviation by Candidate \( j \) would lower voter \( \bar{t} \)'s utility from \( j \)'s policy. Voter \( \bar{t} \) would therefore strictly prefer the opposing candidate, and the set of voters supporting Candidate \( j \) would be strictly smaller. Thus, it is not optimal for candidate \( j \) to deviate from \((\bar{\tau}^j, \bar{a}^j)\).

If the distribution of the media voter \( m(\omega) \) has a strictly positive density around \( \bar{t} \), then \((\bar{\tau}^j, \bar{a}^j)\), \( j = 0, 1 \) is in fact the unique Nash equilibrium (mixed or pure). To see this, suppose there exists another pure strategy equilibrium \((\hat{\tau}^j, \hat{a}^j), j = 0, 1 \). Denote the cutoff voter by \( \hat{t} \). Then \( \hat{t} \neq \bar{t} \), else the above argument implies that the equilibrium must be the same, i.e., \((\bar{\tau}^j, \bar{a}^j) = (\hat{\tau}^j, \hat{a}^j)\). Suppose that \( \hat{t} < \bar{t} \) (the argument for \( \hat{t} > \bar{t} \) is analogous). Then Candidate 0 gets the support of all voters \( t < \hat{t} \), while in the original equilibrium also types \( t \) with \( \hat{t} \leq t < \bar{t} \) support him. If Candidate 0 chooses \((\hat{\tau}^0, \hat{a}^0)\) then he maximizes voter \( \hat{t} \)'s utility. In the original equilibrium both candidates maximized \( \bar{t} \)'s utility and \( \bar{t} \) was indifferent between them. Thus, the deviation ensures that Candidate 0 receives the support of at least all voters \( t < \hat{t} \), and of \( \bar{t} \) and some voters \( t > \bar{t} \) if \((\hat{\tau}^1, \hat{a}^1) \neq (\bar{\tau}^1, \bar{a}^1)\). As the median voter lies between \( \hat{t} \) and \( \bar{t} \) with strictly positive probability, Candidate 0’s deviation strictly increases the winning probability. Thus, \((\hat{\tau}^j, \hat{a}^j), j = 0, 1 \) cannot be an equilibrium. The argument can be extended along the lines of Theorem 2 in the Appendix to show that there is no equilibrium in mixed strategies, and hence \((\bar{\tau}^j, \bar{a}^j), j = 0, 1 \) is the unique Nash equilibrium.

It is somewhat surprising that tax rates are identical even if the candidates’ productivities are asymmetric. Suppose, for example, that \( \gamma_0^0 = 12, \gamma_1^0 = 10, \gamma_0^1 = 10, \gamma_1^1 = 11 \). In this case, Candidate 0 seems to be “on average more productive” than his opponent: Both candidates have a productivity of 10 in their worse good, but Candidate 0 has a productivity of 12 in his better good, which is better than Candidate 1’s productivity in his better good. Thus, it would seem at first glance that Candidate 0 should propose a higher tax rate in order to capitalize on his higher average productivity. Yet, this is not true in equilibrium.

Voter \( t = 1/2 \) cares equally about both goods. Hence if both candidates maximizes the utility of \( t = 1/2 \) then Candidate 0 would take advantage of being more productive by increasing production of public goods, and he would finance this spending by charging higher taxes than his opponent, Candidate 0. However, \( t = 1/2 \) is not the cutoff voter, as \( t = 1/2 \) is strictly better off with Candidate 0. Instead, the cutoff voter’s type is \( \bar{t} > 1/2 \) and cares more for good 1 than for good 0. Thus, Candidate 0’s production
advantage is not as important for \( \bar{t} \) as for type \( t = 1/2 \). At the same time, Candidate 1 is better at providing at good 1. At \( \bar{t} \), the relative advantages of both candidates balance each other exactly such that the benefit (or costs) of increasing taxes are identical for both candidates. As a consequence, both candidates propose the same tax rate.

6.3 Uncertainty and Disagreement about the production process of public goods

Finally, it is useful to point out that our model can be re-interpreted as one in which only one ultimate public good is provided, and all voters just want the highest quantity possible. However, there is disagreement among voters how the ultimate public good is provided from two intermediate goods.

Consider the following example. The ultimate public good that all voters care about is “national security.” The two main inputs that affect the level of national security are “international goodwill” and “military power.” International goodwill reduces the likelihood that other actors such as foreign states or ethnic or religious communities want to undertake aggressive actions that are detrimental to the interests of our country. Military power works both as a deterrent and increases our ability to deal with an aggressive move, should one occur. Both “international goodwill” and “military power” can be increased by spending money on, say, development aid or military hardware, respectively. However, it is also quite plausible that the identity of the winning candidate matters. For example, in the last presidential election Obama was generally thought to be able to provide more “international goodwill” than McCain. It is also plausible that, because of his military background, a majority of voters believed that McCain had a competence advantage in increasing “military power”.

This model is analytically equivalent to the two-goods setup that we analyze. In our formal model, citizens directly derive utility from two public goods, and a parameter measures how much they care about each good. A key role for the analysis is played by the voters’ indifference curves, i.e., all those combinations of the public goods that lead to the same utility for a voter. In the intermediate goods scenario, voters differ in how effective they believe that certain intermediate goods are at producing the ultimate good; thus, each voter’s indifference curves in this scenario are effectively the “isoquants” of the production process in which the voter believes.

During the campaign, candidates can make policy proposals that imply how much they would invest in the two intermediate goods. Voters who believe that military power matters most for national security will prefer the candidate whose platform offers more of it, and vice versa for those who believe that international goodwill is more important. Conversely, candidates have an incentive to offer a platform that emphasizes their strength (with respect to the intermediate good that they are better at producing).

We should note that game theorists sometimes find it problematic to assume that agents differ in their beliefs about how the world works (the “common prior assumption” in game theory). Yet, in practice, the phenomenon that actors genuinely disagree about complex causation mechanisms appears to be wide-
spread. Since the national security outcome is a very complex and longterm process, we would argue that it is quite plausible that voters have substantial and stable differences of opinion about how international goodwill and military power interact in generating national security: Even though they may genuinely be interested in the same ultimate outcome, some voters may believe that what matters is primarily hard military power, while others may believe that international goodwill matters substantially, too.

7 Conclusion

In this paper, we have developed a formal model of political competition between candidates with heterogeneous capabilities in different policy fields. Candidates are office-motivated and compete by proposing how to allocate government resources to different policy fields. The model has a unique equilibrium that differs substantially from the standard one-dimensional model. While candidates compete for the support of a moderate voter type, this cutoff voter differs from the expected median voter. Moreover, no voter type except the cutoff voter is indifferent between the candidates in equilibrium. The model predicts that candidates respond to changes in the preferences of voters in a very rigid way. We also analyze under which conditions candidates choose to strengthen the issues in which they have a competence advantage, and when they rather compensate for their weaknesses.

Finally, we show that when parties can choose the qualities of their nominee, they have an incentive to go for a candidate who is a specialist in the production of the good that party members care about more, rather than a balanced generalist. This is because parties know that their candidate will eventually choose his platform to appeal to a moderate cutoff voter, but the more specialized he is in the production of the good that party members care about most, the more he will provide of that good in equilibrium.

Our model opens up several avenues for future research. We have already discussed in Section 2 how our model can inform empirical studies. One interesting theoretical issue is the nature of political campaigning in our framework. In a standard Downsian model, trying to influence the distribution of voter ideal points has no benefit for a candidate who is interested only in winning, because candidates converge on the same platform anyway. In contrast, since candidates in the differential skills model have no opportunity to gain votes through pandering to marginal supporters of their opponent, an attractive option for a campaign may be to persuade voters that the issue in which the candidate has an advantage is “really important” (in the sense of trying to influence the \( t \) in voters’ utility functions). In this respect, it may be useful to combine our framework with the campaign model of Hammond and Humes (1993).\(^{27}\)

\(^{27}\)Hammond and Humes (1993) study issue-framing by candidates in a two-dimensional Euclidean model. In their model, voters are initially uninformed about candidates’ (exogenous) positions, and candidates can only make their position in one dimension known to voters, and they can choose which one they want to broadcast (that is, they can choose to frame “what the election is about”).
8 Appendix

**Theorem 1**  Suppose that utility \( v(x, t) \) is continuous in \( t \) and \( x \), strictly monotone, and strictly quasiconcave in \( x \), and satisfies the single crossing property\(^{28}\)

\[
\frac{\partial}{\partial t} \left[ \frac{\partial v(x_0, x_1, t)}{\partial x_0} \frac{\partial v(x_0, x_1, t)}{\partial x_1} \right] < 0. \tag{18}
\]

Assume that candidate \( j \) has a relative advantage in providing good \( j \), i.e., \( \gamma_j^j > \gamma_i^j \) for \( i \neq j \). Let \( \xi \) be a lower bound for the elasticity of substitution for all consumption bundles \((x_0, x_1)\) and all types \( t \). Suppose that

\[
\left( \frac{\gamma_0^1 \gamma_1^1}{(\gamma_1^j - \gamma_0^j)(\gamma_0^0 - \gamma_1^0)} \right)^{1/\xi} \leq \min \left\{ \frac{\gamma_1^j}{\gamma_1^0}, \frac{\gamma_0^j}{\gamma_0^0} \right\}. \tag{19}
\]

Then there exists a pure strategy Nash equilibrium with the following properties:

1. There exists a voter type \( \bar{t} \) who is indifferent between candidates 0 and 1; all types \( t < \bar{t} \) strictly prefer Candidate 0 and all types \( t > \bar{t} \) strictly prefer Candidate 1.

2. Both candidates’ equilibrium strategies maximize the utility of voter \( \bar{t} \).

3. The equilibrium strategies are independent of the distribution \( \mu \) of voter types.

4. Candidate 0 provides strictly more of public good 0 than Candidate 1, while Candidate 1 provides strictly more of public good 1 than Candidate 0.

The following Lemma is used in the proof of Theorem 1.

**Lemma 1**  Let \( x^0, x^1 \) be the amount of public goods offered by the two candidates. Let \( D = \{ t | v(x^j, t) \geq v(x^{-j}, t) \} \) be the set of types \( t \) that weakly prefer \( x^j \) to \( x^{-j} \). Then \( D \) is an interval. Moreover, if \( D \neq [0, 1] \), then \( v(x^j, t) = v(x^{-j}, t) \) only for the endpoint of the interval \( D \) that is strictly inside \([0, 1] \).

**Proof of Lemma 1.**  Suppose by way of contradiction that \( D \) is not an interval for some \( x^j, x^{-j} \). Note that we must have \( x^j \neq x^{-j} \), else \( D = [0, 1] \). Then there exist \( t < t' < t'' \) such that \( t, t'' \in D \) but \( t' \notin D \). Continuity of utility in \( t \) implies that there exists \( t_0 < t_1 \) such that \( v(x^j, t_0) = v(x^{-j}, t_0) \) and \( v(x^j, t_1) = v(x^{-j}, t_1) \). Thus, the indifference curves of voters \( t_0 \) and \( t_1 \) intersect twice, which is a contradiction to (18). Hence, \( D \) is an interval.

Moreover, if \( D \neq [0, 1] \), the preceding argument also implies that there cannot be two different types in \( D \) who are indifferent between \( x^0 \) and \( x^1 \). \[\blacksquare\]

**Proof of Theorem 1.** Let

\[ H^j(t) = \max_{a \in [0,1]} v(G^j_0(a), G^j_1(a), t) \]  

We first focus on what turns out to be the “interesting case” where no candidate can attract all of the voters, i.e., suppose that \( H^0(0) > H^1(0) \) and \( H^0(1) < H^1(1) \). Continuity of \( H^j, j = 0, 1 \) therefore implies that there exists \( \bar{t} \) such that \( H^0(\bar{t}) = H^1(\bar{t}) \). Let \( x^j, i = 0, 1 \), be the output of public goods and \( \bar{a}^j \) be the optimal allocation of the input, i.e.,

\[ H^j(\bar{t}) = v(x^j, \bar{t}), \quad x^j_0 = G^j_0(\bar{a}^j), \quad x^j_1 = G^j_1(\bar{a}^j) \]  

We now show that \( \bar{a}^j, i = 0, 1 \) is an equilibrium.

Suppose by way of contradiction, that Candidate 1 can improve by deviating to producing \( \hat{x}^1 \). Let \( D = \{ t \mid v(\hat{x}^1, t) \geq v(x^0, t) \} \). If \( 1 \in D \), then \( D \) is of the form \( [\bar{t}, 1] \), where \( \bar{t} \geq \hat{t} \). (Suppose otherwise; then, by Lemma 1, \( v(\hat{x}^1, \hat{t}) > v(x^0, \hat{t}) \), which contradicts (21), i.e., that \( x^1 \) maximizes the utility of type \( \bar{t} \).) Thus, a deviation such that \( 1 \in D \) cannot increase Candidate 1’s winning probability, as the set of types that vote for Candidate 1 is weakly smaller. Hence, the following claim completes the proof that Candidate 1 has no profitable deviation.

**Claim 1.** \( 1 \in D \).

Figure 6 illustrates the intuition for the proof. The left panel of figure 6 illustrates the relationship between type \( \bar{t} \)’s indifference curve and the equilibrium production levels \( \hat{x}^0 \) and \( \hat{x}^1 \) of both candidates. Clearly, the indifference curve must be tangent to the transformation frontier at both points. Suppose that \( \hat{x}^0 \) is to the right of \( \hat{x}^0 \) as depicted in the left panel. It is then immediate that type 0, whose dashed indifference curve is steeper than that of type \( \bar{t} \), is strictly better off with \( \hat{x}^0 \) than with any public good.

\[ \text{Figure 6: Illustration of the proof of claim 1 in Theorem 1} \]
bundle that Candidate 1 could offer. Hence type 0 would never vote for Candidate 1. Since $D$ must either contain type 0 or type 1 by Lemma 1, this implies that $1 \in D$. Thus, in order to conclude the proof we must exclude the scenario depicted in the right panel of figure 6, where $\tilde{x}^0$ is to the left of $\tilde{x}^0$. If the goods are sufficiently well substitutable, i.e., if (19) holds, then this limits the amount by which the MRS can change along the indifference curve (the limit on the change of the MRS can be related to a lower bound on the elasticity of substitution). In particular, suppose we move along the indifference curve of type $\tilde{t}$, starting from $\tilde{x}^0$ and ending at the intersection with the dashed line $\xi$. If the MRS at this intersection is still less than $\Delta$, then $\tilde{x}^1$ is above the indifference curve, as indicated in the right panel. This, however, means that voter $\tilde{t}$ is not indifferent between the candidates. Candidate 1 could find a policy, such as $\tilde{x}^1$, that would make $\tilde{t}$ strictly prefer him, which cannot be the case in equilibrium. We now proceed to the formal proof.

**Proof of Claim 1.** It is easy to check that candidate $j$'s transformation frontier is

$$TF^j = \left\{ (x^j_0, x^j_1) \in \mathbb{R}^2_+ \mid x^j_1 = \gamma^j_1 \frac{y^j_1}{y^j_0} x^j_0 \right\}. \quad (22)$$

Since $\tilde{x}^j$ satisfies (21) it follows that the marginal rate of substitution of voter $\tilde{t}$ must equal negative of the slope of the transformation frontier:

$$\text{MRS}_{\tilde{t}}(\tilde{x}^j) = \frac{\gamma^j_1}{\gamma^j_0}. \quad (23)$$

The maximum amount of good 0 that Candidate 1 can provide is $\gamma^1_0$. Let $\tilde{x}^0 \in TF^0$ be

$$\tilde{x}^0_0 = \gamma^1_0, \quad \tilde{x}^0_1 = \gamma^1_0 \left[ 1 - \frac{\gamma^0_0}{\gamma^1_0} \right]. \quad (24)$$

Similarly, the maximum amount of good 1 that Candidate 0 can provide is $\gamma^0_1$. Let $\tilde{x}^1 \in TF^1$ be

$$\tilde{x}^1_0 = \gamma^0_1 \left[ 1 - \frac{\gamma^1_0}{\gamma^1_1} \right], \quad \tilde{x}^1_1 = \gamma^0_1. \quad (25)$$

For $0 \in D$, we now show that $\tilde{x}^0_0 < \tilde{x}^0_0$ must hold. To see this, note that no point on the transformation frontier of Candidate 1 is strictly preferred to $\tilde{x}^0$ by voter $\tilde{t}$. The single crossing property (18) therefore implies that $v(x^1,0) < v(\tilde{x}^0,0)$ for any point $x^1$ with $x^1_0 \leq \tilde{x}^0_0$. Thus, a necessary condition for the deviation to attract type 0 is that $\tilde{x}^0_0 < \tilde{x}^0_0$.

Let $L = \{ \alpha \tilde{x}^0 + (1 - \alpha)\tilde{x}^1 | 0 < \alpha < 1 \}$ be the open line segment connecting $\tilde{x}^0$ and $\tilde{x}^1$, so that

$$\Delta = \frac{\tilde{x}^1_1 - \tilde{x}^0_1}{\tilde{x}^0_1 - \tilde{x}^0_0} \quad (26)$$

is the (negative of the) slope of this line segment.
We next show that
\[ \text{MRS}_I(\bar{x}^1) \geq \Delta \]  
(27)

Suppose by way of contradiction that \( \text{MRS}_I(\bar{x}^1) < \Delta \). Then quasiconcavity of utility implies that
\[ v(\bar{x}^1, \bar{r}) > v(x, \bar{r}) \text{ for all } x \in L. \]  
(28)

Recall that if \( 0 \in D \) then \( x_0^0 < x_0^1 \) must hold. Further, \( x_0^1 < x_0^0 \) since candidate 0 is better at providing good 0. Thus, there exists \( x \in L \) with \( x \geq \bar{x}^0 \). Monotonicity of preferences implies that \( v(x^0, t) < v(\bar{x}^1, t) \leq v(\bar{x}^1, \bar{r}) \), where the last inequality follows from (28). Thus \( \bar{r} \) is not indifferent between the candidates, and therefore not the cutoff voter, a contradiction.

Equations (24), (25) and (26) imply
\[ \frac{\Delta}{\text{MRS}_I(x^0)} = \frac{\gamma_1^0}{\gamma_1}. \]  
(29)

Further (24) and (25) yield
\[ \frac{x_1^1}{x_0^1} = \frac{\gamma_0^0 \gamma_1}{(\gamma_0 - \gamma_1)(\gamma_0^1 - \gamma_0^0)}. \]  
(30)

Let \((x_1/x_0)(\text{MRS})\) be the good ratio \( x_1 / x_0 \) on \( \bar{I} \) as a function of the MRS. Since \( \bar{\xi} \) is a lower bound for the elasticity of substitution we get
\[ \frac{(x_1/x_0)(\text{MRS})}{(x_1/x_0)(\text{MRS})} \geq \frac{\xi}{\text{MRS}}. \]  
(31)

Integrating both sides of (31) from \( \text{MRS}_I(x^0) \) to \( \Delta \) and taking the exponential yields
\[ \left( \frac{(x_1/x_0)(\Delta)}{(x_1/x_0)(\text{MRS}_I(x^0))} \right)^{1/\bar{\xi}} \geq \frac{\Delta}{\text{MRS}_I(x^0)}. \]  
(32)

By definition \((x_1/x_0)(\text{MRS}_I(x^0)) = \frac{x_0^0}{x_0^1}, i.e., the good ratio at which the MRS of type \( \bar{I} \) is \( \text{MRS}_I(x^0) \) must be \( x_0^0 / x_0^1 \). We have shown that \( x_0^0 < x_0^1 \) if \( 0 \in D \). Thus, \((x_1/x_0)(\text{MRS}_I(x^0)) > (x_1/x_0)(\text{MRS}_I(x^0)) \). Further, as indicated in the right panel of figure 6, \((x_1/x_0)(\Delta) \geq x_0^1 / x_0^0 \). In particular, by construction, voter \( \bar{I} \) is indifferent between the candidates. Thus, \( \bar{x}^1 \) cannot be above indifference curve \( \bar{I} \). In order for this to be the case, the slope of \( \bar{I} \) at good ratio \( x_1^1 / x_0^0 \) must be at least \( \Delta \). Hence (32) implies
\[ \left( \frac{x_1^1}{x_0^0} \right)^{1/\bar{\xi}} \geq \frac{\Delta}{\text{MRS}_I(x^0)}. \]  
(33)

Substituting (29) and (30) into (33) contradicts (19). Thus, \( 1 \in D \).

The proof that a deviation by Candidate 0 is not optimal it similar, except that we must replace (29) by
\[ \frac{\text{MRS}_I(x^1)}{\Delta} = \frac{\gamma_0}{\gamma_1}. \]  
(34)
As \( \text{MRS}_0(x^0) < \text{MRS}_0(x^1) \), strict quasiconcavity implies that \( x_0^0 > x_1^0 \) and \( x_1^0 > x_1^1 \).

Finally note that the distribution of types does not affect the equilibrium. This proves the first statement.

*The case where \( H(0,c^0) \leq H(0,c^1) \) or \( H(1,c^0) \geq H(1,c^1) \).*

Consider the first of the two scenarios as the other case is similar. Let \( x^1 \) be the consumption bundle provided by Candidate 1 that maximizes type 0’s utility. Then \( v(x^1,0) \geq v(x,0) \) for any \( x \in \text{TF}^0 \). The single crossing property (18) immediately implies that \( v(x^1,t) > v(x,t) \) for any \( x \in \text{TF}^0 \) and for any \( t > 0 \) and hence all citizens \( t > 0 \) vote for Candidate 1 independently of Candidate 0’s strategy. Thus, \((x^0, x^1)\) is a Nash equilibrium, where \( x^0 \) is the consumption bundle that maximizes type 0’s utility on \( \text{TF}^0 \). Clearly, \( x_0^0 > x_1^0 \) and \( x_1^0 > x_1^1 \). ■

The next result shows that the equilibrium characterized in Theorem 1 is unique and strict, provided that there is sufficient uncertainty about the position of the median voter type.

**Theorem 2** Suppose that the conditions of Theorem 1 hold and that the distribution of the median voter \( t_m(\omega) \) has a strictly positive density on \([0, 1]\). Then, one of the following is true:

1. The equilibrium is strict and it is the unique Nash equilibrium (pure or mixed).

2. One of the candidate wins with probability 1 and receives 100% of the votes in almost all states \( \omega \in \Omega \).

**Proof of Theorem 2.** *Proof of Part 2.* Let \((x^0, x^1)\) be the allocation of public goods offered by the candidates in a pure strategy equilibrium. By Lemma 1, \( D^0 = \{ t|v(x^0,t) \geq v(x^1,t) \} \) and \( D^1 = \{ t|v(x^1,t) \geq v(x^0,t) \} \) are intervals.

First, suppose that \( D^0 = D^1 = [0, 1] \). Clearly, each candidate’s winning probability is 0.5. Given the single crossing property (18) this implies \( x^0 = x^1 \). Let \( t_m(\omega) \) be the realization of the median voter type, and let \( \hat{t} \) be the median of the distribution of \( t_m(\omega) \). Since \( \gamma_j > \gamma_j^* \) the transformation frontiers have different slopes. Thus, for at least one candidate \( \text{MRS}_i(x^j) \) does not equal the slope of the candidate’s transformation frontier. As a consequence, there exists a bundle of public goods \( \hat{x}^j \) for Candidate \( i \) such that \( v(\hat{x}^j, \hat{t}) > v(x^j, \hat{t}) = v(x^j, \hat{t}) \). Thus, Lemma 1 implies that \( \hat{t} \) is in the interior of \( \hat{D} = \{ t|v(\hat{x}^j,t) \geq v(x^j,t) \} \). Given that \( \hat{D} \) contains the median of the median voters in its interior, and given that the distribution of types has strictly positive density, the winning probability for Candidate \( j \) is strictly increased, a contradiction to the assumption that \( x^0 = x^1 \) is a Nash equilibrium. Hence, \( D^0 \) and \( D^1 \) cannot both be equal to \([0, 1]\).

Next, suppose that \( D^j \) consists of only a single, point, i.e., \( D^j = \{ 0 \} \) or \( D^j = \{ 1 \} \). Continuity of preferences then implies that no citizen in \( D^j \) has a strict preferences for Candidate \( i \), and all of them will
therefore abstain. Finally, since $t_m(\omega) = 0$ or $t_m(\omega) = 1$ with probability 0, this implies that the other candidate will receive a strictly positive number of votes and therefore win 100% of all votes cast.

Thus, let $D^I \neq [0, 1]$ for $i = 0, 1$. Further, by continuity of $v$ there exists exactly one type $t^*$ for which $v(x^0, t^*) = v(x^1, t^*)$. Suppose by way of contradiction that $H^0(t^*) \neq v(x^0, t^*)$, where $H^0$ is defined in (20). If $0 \in D^0$ then Lemma 1 implies $v(x^0, 0) > v(x^1, 0)$. Hence, there exists some $x^0$ such that $v(x^0, 0) > v(x^0, t^*) > v(x^0, t^*) = v(x^1, t^*)$. Thus, $t^*$ is in the interior of $D^0 = \{t|v(x^0, t) \geq v(x^1, t)\}$. Since $0 \in D^0$ and $D^0$ is an interval it follows that $D^0$ is a strict superset of $D^0$. Since the distribution of types has strictly positive density, this implies that the winning probability for Candidate 0 strictly increases, a contradiction. The proof where $1 \in D^0$ or agent 1 deviates is similar. Thus, the cutoff voter $t^*$ at any equilibrium must satisfy $H^0(t^*) = v(x^0, t^*)$.

We now show that there exists exactly one $t$ that solves $H^0(t) = H^1(t)$. Suppose by way of contradiction that there exist $t < t'$ such that $H^0(t) = H^1(t)$ and $H^0(t') = H^1(t')$. Then the indifference curves of type $i$'s and that of type $t'$ must be tangent both to $T^0$ and $T^1$. This, however, is only possible if the indifference curves intersect at at least two points, contradicting the single crossing property (18).

Given that a unique $t$ solves $H^0(t) = H^1(t)$, the Nash equilibrium is unique among all pure strategy Nash equilibria. Now suppose that there exists a mixed strategy equilibrium. Without loss of generality suppose that Candidate 1 mixes. By selecting $x^0$ Candidate 0 can ensure that at least all types $t < \bar{t}$ vote for him. However, since Candidate 1 mixes, the candidate chooses $x^1$ with probability less than 1. In such a case, there exists $\hat{\bar{t}} > \bar{t}$ such that all citizens $t < \hat{\bar{t}}$ vote for Candidate 0, which strictly increases Candidate 0’s winning probability as $t_m(\omega)$ has a strictly positive density. Thus, Candidate 0’s winning probability in the mixed strategy equilibrium must be strictly larger than that in the pure strategy equilibrium. Similarly, it follows that Candidate 1’s winning probability in the mixed strategy equilibrium must be at least as large as in the pure strategy equilibrium, a contradiction since the winning probabilities must add up to 1.

Finally, the Nash equilibrium is strict since preferences are strictly quasiconcave and therefore the solution to maximization problem (20) is unique. As a consequence, any deviation by Candidate 1 from $x^1$ to $\tilde{x}^1$ implies that $v_i(x^{-j}) > v_i(\tilde{x}^i)$. Hence, Candidate $i$ loses type $\tilde{t}$. Since the distribution of types has a strictly positive density, this implies that Candidate $i$’s winning probability strictly decreases.

**Proof of Proposition 3.** Denote the cutoff voter by $\bar{t}$. Suppose Candidate 0 wins. Then the median voter in state $\omega$ must be to the left of $\bar{t}$, i.e., $t_m(\omega) < \bar{t}$. Consider the optimal budget allocation by Candidate 0 for a voter of type $t$. The optimization problem

\[
\max_{a^0} v(\gamma_0^0, \gamma_1^0(1 - a^0), t))
\]

has the first-order condition

\[
\frac{\partial v}{\partial x_0} \gamma_0^0 - \frac{\partial v}{\partial x_1} \gamma_1^0 = 0
\]

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which is equivalent to
\[ \frac{\partial v(x_0, x_1, t)}{\partial x_0} \gamma_0^0 - \frac{\partial v(x_0, x_1, t)}{\partial x_1} \gamma_0^1 = 0. \]  
(36)

By (18), \( \frac{\partial}{\partial t} \left( \frac{\partial v(x_0, x_1, t)}{\partial x_0} \right) \) < 0. Since we know from Theorem 1 that, in equilibrium, (36) holds for \( t = \bar{t} \), it follows that (36) is positive for all \( t < \bar{t} \). This implies that all types \( t < \bar{t} \) have an optimal level of \( a \) that is greater than \( \bar{a}^0 \). The argument if Candidate 1 wins is analogous. ■

**Proof of Proposition 4.** Consider the optimal budget allocation by Candidate 1 for a type \( t \) voter:

\[ \max v(\gamma_0^1 a^1, \gamma_1^1 (1 - a^1), t), \]  
(37)

The first order condition is
\[ \frac{\partial v(x_0, x_1, t)}{\partial x_0} = \gamma_1^1 \gamma_0^1. \]  
(38)

By (18), the left-hand side of (38) is decreasing in \( t \). Moreover, since \( v \) is concave in both arguments, it follows that the left-hand side of (38) is decreasing in \( a^1 \). Thus, a voter with a higher type \( t \) has a lower preferred value of \( a^1 \). Since an increase in \( \gamma_0^0 \) or \( \gamma_1^0 \) moves the equilibrium cutoff voter to the right (i.e., increases \( \bar{t} \)), and we know from Proposition 1 that Candidate 1 chooses \( a^1 \) to maximize the utility of the new cutoff voter, this proves the first part of the theorem.

The equivalent condition to (38) for Candidate 0 is (36) in the proof of Proposition 3. Totally differentiating (36) with respect to \( \gamma_0^1 \) and \( a \) yields
\[ \frac{\partial}{\partial a} \left( \frac{\partial u}{\partial x_0} \right) da + \frac{\partial}{\partial t} \left( \frac{\partial u}{\partial x_1} \right) dt - \frac{1}{\gamma_0^1} \gamma_1^1 dy_1^0 = 0. \]  
(39)

The first term is negative (by the second-order condition of maximization). Furthermore, as argued above, \( \frac{\partial}{\partial a} \left( \frac{\partial u}{\partial x_0} \right) < 0 \) and \( \frac{\partial}{\partial y_1^0} > 0 \), so that the term in square brackets is negative. Thus, \( da^0 / dy_1^0 < 0 \), as claimed.

Going through the same steps as above for \( \gamma_0^0 \) yields
\[ \frac{da^0}{dy_0^0} = -\frac{\frac{\partial}{\partial a} \left( \frac{\partial u}{\partial x_1} \right) da + \frac{\partial}{\partial t} \left( \frac{\partial u}{\partial x_1} \right) dt - \frac{\gamma_0^0}{\gamma_1^1}}{\frac{\partial}{\partial a} \left( \frac{\partial u}{\partial x_1} \right)} \]  
(40)

The first term in the numerator is the product of a negative and a positive number, while the second term is positive. Consequently, the sign of the numerator, and thus of \( da^0 / dy_0^0 \) is ambiguous. ■
Proof of Proposition 5. Differentiating (9) with respect to $a^0$ and $a^1$ and canceling the respective denominators and $\rho$ yields

$$\begin{align*}
(\gamma_1^0)^\rho(1-a^0)^{\rho-1}[(\gamma_0^0 a^0)^\rho - (\gamma_0^1 a^1)^\rho] - (\gamma_0^0)^\rho(a^0)^{\rho-1}[(\gamma_1^1(1-a^1))^\rho - (\gamma_1^0(1-a^0))^\rho] &= 0 \quad (41) \\
(\gamma_1^1)^\rho(1-a^1)^{\rho-1}[(\gamma_0^0 a^0)^\rho - (\gamma_0^1 a^1)^\rho] - (\gamma_0^1)^\rho(a^1)^{\rho-1}[(\gamma_1^1(1-a^1))^\rho - (\gamma_1^0(1-a^0))^\rho] &= 0 \quad (42)
\end{align*}$$

Rearranging gives

$$\left(\frac{\gamma_1^0}{\gamma_0^0}\right)^\rho \left(1 - a^0\right)^{\rho-1} = \left(\frac{\gamma_1^1}{\gamma_0^1}\right)^\rho \left(1 - a^1\right)^{\rho-1}$$

Rearranging gives equation (10) in the text. The remaining steps of the argument are in the main text. ■
References


