Integrative Negotiation: An Economic Perspective*

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Abstract

We expand the canonical bargaining framework from the economics literature to incorporate the process of integrative negotiation, a phase in negotiation that has received significant attention outside of economics. Integrative negotiation, the first stage of our model, consists of collaborative attempts of the negotiators to express their priorities and interests, and to jointly acquire information to increase surplus. The second stage corresponds to a classical bargaining problem. We show that there is complex interplay between integrative negotiation and classical bargaining, and that findings that do not incorporate integrative negotiation into the bargaining problem can be misleading.

Keywords: Negotiation, Bargaining, Learning.
JEL Codes: C78, D74, D83

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1 Introduction

There is a long-standing literature in economics on the "bargaining problem," which, as defined by Jevons (1879, p. 124), is the process of a "buyer ascertaining the lowest price at which the seller is willing to part with his object, without disclosing if possible the highest price which he, the buyer, is willing to give." Proposed solutions range from the cooperative game theoretic approach of Nash (1950) to the non-cooperative description of Ståhl (1972) and Rubinstein (1982). In this body of literature, bargaining is characterized as the back-and-forth haggling between negotiating parties. In psychology and management, this process is often referred to as *distributive negotiation* (Steinel and Harinck, 2020).

The starting point for this paper is that an exclusive focus on the "bargaining problem"—i.e., distributive negotiation—overlooks important aspects of real-world negotiations. At least since the work of Mary Parker Follett (1940), researchers have recognized that negotiations are not merely one-dimensional processes in which each party simply tries to best the other. Consider the scenario posed by Follett in a lecture in 1925. Two sisters each needed an orange, but only had one available. After negotiating, they decided to split the orange in half, a classical solution to the "divide the pie" problem. However, they later learned that one of them only needed the peel and the other the juice, meaning their negotiation process failed to recognize these salient details. Using the terminology of Walton and McKersie (1965), the sisters did not engage in *integrative negotiation*, a process in which the negotiated outcome "reconciles (i.e., integrates) the parties' interests and thus provides high benefit to both of them" (Pruitt, 2013, p. 137). The literature in management distinguishes between integrative and distributive negotiation as two distinct modes, analyzing the impact of each on the negotiation outcome (Brett and Thompson, 2016).

The economics literature largely ignores integrative negotiation, while the management literature recognizes its importance but treats it as separate from the distributive process. The contribution of this paper is to meld integrative negotiation into the classic bargaining problem of Nash (1950), and to understand the interplay between integrative and distributive negotiation in determining negotiated settlements.

Our model involves two players $i = 1, 2$, who negotiate between two possible solutions: a status quo (e.g., splitting an orange in half), and an "unknown" alternative. Using the standard Harsanyi approach, we adopt a Bayesian framework to model the alternative solution: the players are symmetrically uninformed and have a prior assessment about the alternative's effectiveness in replacing the status quo. The negotiation process consists of two phases: integrative negotiation and distributive negotiation. In the integrative phase, the players mutually decide to spend resources on gathering more information about the alternative solution. This process is modeled as public information acquisition, where a player's *negotiation ability* is defined as the inverse of the cost of acquiring information. After integrative negotiation, the players enter the distributive phase, where the focus shifts to dividing the surplus. The literature on negotiation emphasizes the role of bargaining power in distributive negotiation, defined as a person's ability to secure a larger share of the surplus (Kim et al., 2005). We capture this aspect by employing the generalized Nash bargaining solution, with each player's *bargaining power* (q_i) defined according to Kalai (1977).

We show that the set of feasible negotiation outcomes is primarily determined by each player's *negotiation affinity*, defined as the product of negotiation ability and bargaining power. When the players' negotiation affinities are identical, their interests in the integrative phase are perfectly aligned, resulting in a unique outcome that Pareto dominates all others. However, as negotiation affinities begin to differ, conflicts of interest arise during the integrative phase. Specifically, if one player's negotiation affinity is too low, integrative negotiation may completely break down. It is important to note that negotiation affinity consists of two parameters: one influencing integrative negotiation and the other governing distributive negotiation. This highlights the interplay between the two phases—analysis of one cannot be fully understood without considering the other.

We next show that changing a player's bargaining power, q_i, has a non-monotone impact on payoffs. While increasing q_i directly benefits player i by enlarging their share during distributive negotiation, it simultaneously discourages the other player from engaging in integrative negotiation, reducing the available total surplus. If q_i grows too large, this negative effect can dominate the benefits of a larger share. Further, changes in a player's negotiation ability also can have a non-monotonic effect on the other player's payoff, because disparity in negotiation skills would make it difficult for two negotiators to agree upon the way they conduct integrative negotiation. In particular, a player's payoff decreases if the other player's negotiation ability becomes too high. Finally, we show that increasing one player's status quo payoff may not always decrease their willingness to engage in integrative negotiation. In fact, the breakdown of integrative negotiation can be triggered by the opponent, whose status quo payoff remains unchanged.

We then analyze two extensions of our baseline model. First, we introduce the possibility of monetary transfers in the integrative phase, to induce reluctant players to spend more time investigating the alternative solution. In practice, such arrangements are quite rare, with a notable exception of payments for individuals to participate in timeshares sales meeting. Whether monetary transfers are allowed is assumed to be an institutional feature of the environment that both parties would have to agree on ex-ante, and must therefore be Pareto improving. Our results show that side payments are only Pareto improving when there is a large disparity in the players' negotiation affinities. In this case, the payments are made by the player with the higher negotiation affinity.

In the second extension, we assume that monetary payments are not permitted throughout the negotiation process, effectively removing the distributive negotiation phase. Compared to the baseline model, this setup has the advantage of ensuring that differences in bargaining power do not affect integrative negotiation. However, this approach also introduces a potential cost: the alternative solution may not be chosen when it would be socially optimal. We demonstrate that the benefits can outweigh the costs, and hence the absence of distributive negotiation may improve the outcome.

Literature Review: The observation in Schelling (1960) that individuals interacting with others often face "mixed-motives," involving both conflict and cooperation, is particularly relevant to negotiations. In social psychology, integrative negotiation is typically viewed as the cooperative component, whereas the distributive phase is associated with conflict (Deutsch, 1973; Rubin et al., 1994). However, we argue that this distinction is overly simplistic, as divergence of interest can also emerge during integrative negotiation.

The key parameter for the distributive phase is a player's bargaining power. While we use the cooperative game-theoretic approach of Kalai (1977), bargaining has also been explored in non-cooperative models. For example, starting with Baron and Ferejohn (1987) and Banks and Duggan (2000) the literature on legislative bargaining models bargaining power by the likelihood of being able to make a proposal in distributive negotiation. Another aspect of bargaining power is modeled in Bowen et al. (2022) and Hwang and Krasa (2023) based on the notion of personal power from Weber (1922). Some of the economics literature also connects bargaining power with patience (Binmore et al., 1986; Cramton, 1991).

The main parameter that characterizes the integrative phase of the negotiation is negotiation ability. Although this concept is not well-established in the economics literature, there is a significant literature in management and psychology to determine individual characteristics that facilitate integrative negotiation. Thompson (1990) discusses the impact of cognitive aspects on a player's ability to engage in integrative negotiation. First experiments on this link were done by Pruitt and Lewis (1975) and are further discussed in Section 4.2. Beyond cognitive factors, we also argue that opportunity costs of time affect negotiation ability.

We model integrative negotiation as a process of discovering a solution that could improve upon the status quo, which is most effectively represented as a symmetric learning process. Alternatively, one may stretch the interpretation of the model to a scenario where players make physical investments to increase the overall surplus; however, unlike in Grossman and Hart (1986), these investments are the subject of integrative negotiation, making them observable and contractible. Nevertheless, this interpretation does not adequately capture many environments of integrative negotiation. For instance, in Follett's story of the two sisters, this approach would mean that they invest in technology that increases the yield from the juice or peel of their respective orange halves. Such pre-bargaining investment would not lead them closer to the true integrative solution, as the essence of integrative negotiation lies in understanding their distinct needs, not in physical investments.

The key feature of our model is that players can determine how much information and what type of information is acquired during the negotiation. Thus, while our information acquisition is endogenous, there is a large literature where exogenous shocks or signals may change the bargaining environment. Buisseret and Bernhardt (2017) consider a bargaining problem where the identity of the negotiators may change, e.g., due to a parliamentary election. In Basak and Deb (2020), two parties bargain in an environment where public opinion stochastically changes over time. While information in our model is acquired about the total size of the pie, it is equivalent to learning about the surplus by resolving uncertainty about outside options. In this sense our paper provides a different perspective to a large literature starting with Epple and Riordan (1987) on an endogenous status quo. In these models the status quo for the next period is the negotiated settlement from the previous period (c.f., Kalandrakis (2004); Duggan and Kalandrakis (2012); Bowen et al. (2014); Jeon and Hwang (2022), and the survey by Eraslan et al. (2022)).

The existing literature on bargaining with incomplete information (Fudenberg et al., 1985; Abreu and Gul, 2000; Deneckere and Liang, 2006) typically focuses on the effect of private information. More recently, Jackson et al. (2023) consider a multidimensional model with two-sided incomplete information, in which the (net) surplus is constant.[1] In contrast, we focus on symmetric information because the resulting model matches more closely the insights from the literature on integrative negotiation from Psychology. In particular, Pruitt (2013) points out that there are three types of behaviors observed during integrative negotiations: Explicit information exchange, implicit information exchange, and heuristic trial and error. Brett and Thompson (2016) argue that implicit information exchange—which would be caused by private information in a formal model—requires a distributive approach and does not usually facilitate integrative negotiation. In contrast, modeling players as being symmetrically uninformed and jointly acquiring information represents explicit information exchange, which is most commonly observed in practice.

Finally, in a review of the history of social psychology studies on conflict and negotiation, Pruitt (2012, pp. 446-47) identifies several key questions for future research. One of these questions—"under what conditions do parties go into negotiation?"—is directly addressed by our model: We show that negotiation affinity is the key determinant for a successful negotiation.

2 Model

2.1 Model Description

Two players, $i = 1, 2$ engage in a negotiation involving two possible solutions: a status quo solution and an alternative solution. The status quo solution yields known payoffs $w_i > 0$ for each player $i = 1, 2$. The alternative solution can be either effective or ineffective, described by states H and L, which provide gross surpluses of θ_H and θ_L, respectively. An effective alternative solution improves upon the status quo, meaning that the net surplus is positive: $\theta_H - w_1 - w_2 > 0$. If the alternative solution is ineffective, it generates a negative net surplus: $\theta_L - w_1 - w_2 < 0$. Prior to negotiation, both players are symmetrically uninformed

[1] See also Bac and Raff (1996), Chakraborty and Harbaugh (2003), and Acharya and Ortner (2013) for models of bargaining with multiple issues.

about the state and share the same prior belief, $p_0 \in (0, 1)$, that the alternative solution is effective. The players are risk-neutral and aim to maximize their respective expected payoffs.

The negotiation process consists of two phases:

Integrative negotiation: We model integrative negotiation as a joint and public learning about the effectiveness of the alternative solution. Public learning is formally described by a signal structure σ, which consists of a random variable (S, θ) on a probability space $(\Omega\times\{L, H\}, \mathcal{F}, P)$ such that $P(\Omega\times\{H\}) = p_0$. Let Σ denote the set of all signal structures. Both players observe the realization $S \in \Omega$. The cost of information acquisition is $C_i(\sigma)$ for player i, and its functional form is specified below. To reflect the cooperative nature of integrative negotiation, we assume that the players jointly choose a Pareto efficient and individually rational signal structure.

Distributive negotiation: In this phase, the players determine how to divide the surplus based on the information obtained during the integrative phase. In the baseline model, we assume that transfers between players are allowed during the distributive phase. Let $p \in [0, 1]$ denote the players' common posterior belief following the integrative phase. Then, any payoff vector (u_1, u_2) satisfying $u_1 + u_2 \leq \max\{w_1 + w_2, p\theta_H + (1 - p)\theta_L\}$ is feasible. The division of surplus follows the generalized Nash bargaining solution (Kalai, 1977), where the status quo payoffs (w_1, w_2) as the disagreement points, and player i's *bargaining power* is given by $q_i \in [0, 1]$, where $q_1 + q_2 = 1$.

The costs of information acquisition in the integrative phase depend on the amount of information that is acquired. In particular, let $h\colon [0, 1] \to \mathbb{R}$ be twice continuously differentiable with $h'' < 0$. Further, h satisfies Inada conditions at $p = 0$ and $p = 1$, i.e., $h'(0) = \infty$ and $h'(1) = -\infty$.[2] Then, the cost of information acquisition for each player $i = 1, 2$ is

$$C_i(\sigma) = k_i\Big(h(p_0) - E_\sigma[h(p)]\Big), \tag{1}$$

where $k_i > 0$ is a constant, and $E_\sigma[\cdot]$ is the expectation over the posteriors generated by σ. We define $a_i = 1/k_i$ as player i's *negotiation ability*, where a higher ability implies that the player needs to spend less resources in integrative negotiations.

Note that $C_i(\sigma) = 0$ if σ is independent of θ, i.e., if no information about θ is revealed. The costs $C_i(\sigma)$ are interpreted as player i's costs of integrative negotiation. There are two sources for the costs. First, the cost of time it takes to absorb that information, and second a possible direct cost of paying third parties to acquire information. If both of these costs are proportional to the amount of information that is transmitted,

[2] This simplifies the analysis because we always get interior solutions in our model. All results extend if we do not assume Inada conditions, which, for example, is the case if h is quadratic.

then $h(p)$ would be a multiple of Shannon entropy, e.g., $h(p) = -p\log(p) - (1-p)\log(1-p)$.[3] We use this specification for h in all numerical examples below.

There are two possible cases in the distributive phase depending on the value of posterior p. First, p is sufficiently low such that the players will take the status quo option. Second, p is sufficiently high for the alternative solution to be effective. The cutoff value p^* between these two cases is given by

$$p^*\theta_H + (1-p^*)\theta_L = w_1 + w_2 \quad \Longleftrightarrow \quad p^* = \frac{w_1 + w_2 - \theta_L}{\theta_H - \theta_L}. \tag{2}$$

Then, Kalai (1977) implies that player i's payoff from distributive negotiation is

$$v_i(p) = w_i + q_i(\theta_H - \theta_L)\max\{p - p^*, 0\}. \tag{3}$$

The ex-ante expected payoff of individual i is given by $E_\sigma[v_i(p)] - C_i(\sigma)$. Finally, let $\underline{u} = (\underline{u}_1, \underline{u}_2)$, where $\underline{u}_i = v_i(p_0)$, denote the vector of the players' reservation payoffs. Note that player i would obtain $\underline{u}_i$ if there is no integrative negotiation.

2.2 Discussion of Model Assumptions

2.2.1 Integrative Negotiation

Note that we do not adopt a strategic modeling approach to integrative negotiation; rather, our model is based on an extension of a Nash bargaining solution. By doing so, we bypass the need for modeling specific negotiation protocol, and instead focus on the full range of potential outcomes that can emerge from integrative negotiation. We believe this approach provides a more effective starting point for understanding integrative negotiations. In the remainder of the paper, we characterize the set of all Pareto efficient and individually rational outcomes and explore how this set is influenced by the model's primitives.

We assume that the parties jointly, rather than privately, learn about the alternative solution. In other words, the integrative phase does not involve persuasion or manipulation of the other party. The literature on integrative negotiation emphasizes aspects of creating value and "enlarging the pie" (Kong et al., 2014). Our model highlights this cooperative aspect of negotiation. For the same reason, we do not consider private information. Nevertheless, as we will show, the parties' interests are not necessarily aligned in the integrative phase, leading them to "engage in constructive conflict … to develop integrative agreements (Pruitt, 2013, p.194)."

In most integrative negotiations we consider, the status quo solution serves as the default option, meaning that the players would select the status quo in the absence of integrative negotiation. For instance, in the Follett's story of two sisters, the status quo solution of splitting the orange would be adopted by default.

[3] Note that $h(p)$ is the Shannon entropy multiplied by $\log(2)$.

In our model, this corresponds to the case where $p_0 < p^*$. However, our model can also accommodate scenarios where the alternative is the default option, i.e., $p_0 > p^*$. For example, imagine a seller and a prospective buyer for a product, where the status quo is "no purchase" and the alternative is"purchase", and the players are uncertain about the match value in case of purchase. In this case, while the players may be optimistic enough for the purchase to be the default option, they could still engage in integrative negotiation to learn more about the value.

Our model assumes a strict separation between integrative and distributive negotiations. It is natural for integrative negotiation to precede distributive negotiation, as the core elements of a potential agreement need to be determined before price or other distributive factors are discussed. The empirical literature on negotiation shows that once price negotiations begin, it is rare for parties to return to integrative negotiations. For example, Putnam (1983) points out that due to individuals' tendencies to reciprocate, it becomes difficult to reenter the cooperative phase of integrative negotiation once distributive topics have been introduced (see also Brett and Thompson (2016)).[4]

2.2.2 Distributive Negotiation

The objective of a player in distributive negotiation is to secure a larger share of the pie. It is understood that the ability to do so is related to the player's "power," which Weber (1922) defines as the ability to assert one's will even against opposition. However, power in negotiation involves multiple dimensions.

In our model, two key variables capture power in distributive negotiation: bargaining power (q_i) and status quo payoff (w_i). When the status quo represents a breakdown in negotiation, w_i can be interpreted as the breakdown payoffs or the "Best Alternative To the Negotiated Agreement" (BATNA). Fisher et al. (2011) emphasize BATNA as a key element in a negotiator's ability to achieve a higher payoff by applying pressure on the opponent, for example, through the threat of leaving the negotiation. However, as Galinsky et al. (2017) note, power in negotiation extends beyond BATNA and includes factors such as a negotiator's status or social capital. Similarly, Kim et al. (2005) identify coercive power and legitimate power (i.e., the extent to which a player's position is supported by lawful authorities) as additional dimensions of power. In our model, these other aspects of power are reflected by bargaining power q_i. In Section 4, we demonstrate that our two parameters of power—q_i and w_i—have distinct implications for the negotiation process.

[4]There is also literature analyzing the contrast between the integrative and distributive modes of negotiations, treating them as distinct processes (c.f., De Dreu et al. 1998). In Section 5.2, we extend our model to incorporate situations where only integrative negotiation exists, and provide a more subtle comparison between the two modes of negotiation.

2.3 Example

After the fall of Sadam Hussein, numerous conflicts arose between farmers and international oil companies about land usage.[5] Fisher et al. (2011, pp. 5–6) describe one such case, which nearly escalated into an armed conflict between farmers and representatives of an oil company. A government official intervened and found out that an alternative solution was possible due to differing temporal valuations for land use. Table 1 provides a stylized description.

Farmers (F)	Eviction	Farming
θ_L	0	0
θ_H	0	100

Oil Company (C)	Eviction	Farming
θ_L	400	0
θ_H	400	500

Table 1: Benefits for farmers and the oil company from land usage.

At the start of the negotiation, the farmers (player F) and the oil company (player C) are uncertain about how the oil company's land usage would affect their respective operations. We assume there are two equally likely states, θ_L and θ_H. There are two possible solutions: eviction (status quo solution) or allowing farming in the next few years (alternative solution). In the eviction scenario, the farmers are forced off the land, resulting in a payoff of zero. The oil company, although able to operate, incurs costs due to delays and conflict, leading to a payoff of 400.

Alternatively, if the farmers are allowed to continue farming, there are two possibilities. In state θ_L, the oil company's operations interfere with farming, leading to zero payoffs for both players. However, if the state is θ_H, the two operations do not interfere. As noted by Fisher et al. (2011), *only after integrative negotiation* did it become apparent that allowing farming was beneficial for both parties: the farmers needed the land in the initial period because they had already planted crops, while the oil company planned to utilize the land later, after completing their exploratory phase. In this case, we assume that the farmers and the company achieve highest possible payoffs of 100 and 500, respectively.

Since states are equally likely, the total payoff from eviction is 400, while the expected total payoff from allowing farming is $0.5 \times 600 = 300$. Therefore, in the absence of further investigation, eviction would be chosen. Allowing farming increases the social surplus if and only if the posterior p of θ_H exceeds $p^* = (400 - 0)/(600 - 0) = 2/3$.

Note that integrative negotiation is only beneficial if there is positive probability that the negotiators

[5] See, for example, https://www.reuters.com/article/idUSTRE74S0RP/ or https://www.nytimes.com/2009/09/06/world/middleeast/06iraqoil.html.

receive a signal optimistic enough to push their posterior above p^*. The key question is how the players trade off more accurate information—i.e., further integrative negotiation—against the associated costs. Also, it is unclear whether both players would agree on what information to acquire, or if disagreements could lead to a breakdown of integrative negotiation. In the following section, we analyze the outcome of integrative negotiation before returning to this example.

3 Determinants of the Negotiated Outcomes

In this section, we characterize the set of payoffs that the players can obtain. A vector of payoffs $(u_1, u_2) \in \mathbb{R}^2$ is *feasible* if there exists a signal structure $\sigma \in \Sigma$ such that

$$u_i = E_\sigma[v_i(p)] - C_i(\sigma), \tag{4}$$

for $i = 1, 2$. Further, we say that (u_1, u_2) is *individually rational* if $u_i \geq \underline{u}_i$ for all $i = 1, 2$. Let U be the set of all feasible payoffs.

Lemma 1 *The set U of feasible payoffs is convex and compact.*

Compactness follows from standard topology arguments. To understand why U is convex, note that we can define convex combinations of signal structures σ and σ' by defining signal structure that selects σ with some probability λ, and σ' with the probability $1 - \lambda$. It is easy to verify the the cost of the new signal structure is the linear combination of the costs of σ and σ's respectively. Convexity of U then follows because the cost function is convex in signals.

Lemma 1 and the separating hyperplane theorem imply that, for any extreme point $u \in \text{bd}(U)$, there exists $\vec{\beta} = (\beta_1, \beta_2) \in \mathbb{R}^2$ such that u is the solution of

$$\max_{\sigma \in \Sigma} \beta_1 \Big(E_\sigma \left[v_1(p) \right] - C_1(\sigma) \Big) + \beta_2 \Big(E_\sigma \left[v_2(p) \right] - C_2(\sigma) \Big). \tag{5}$$

Conversely, for arbitrary $\vec{\beta}$, the solution of Problem 5 is on the boundary of U.

To solve Problem 5, it is more convenient to rewrite the problem by maximizing over the posterior distributions induced by σ, rather than directly over the set of random variables σ themselves. Gentzkow and Kamenica (2014) demonstrate the equivalence between these approaches, and their argument readily extends to our setting. With a slight abuse of notation, we denote the probability distributions over posterior beliefs, p, also by σ. Further, we now restrict attention to points on the Pareto frontier, where $\beta_i \geq 0$ for both $i = 1, 2$. We provide a general analysis of the entire set of feasible payoff vectors in Appendix B.

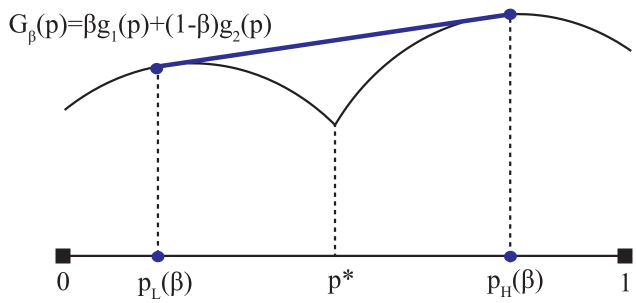


Figure 1: Illustration of the tangency condition (7).

Normalizing the weights, we can set $\beta_1 = \beta$ and $\beta_2 = 1 - \beta$, where $0 \leq \beta \leq 1$. Further, let $g_i(p) = v_i(p) + k_i h(p)$ for $i = 1, 2$. Then optimizing over posterior distributions instead of signal structures, Problem 5 can be rewritten as

$$\max_{\sigma} \beta E_\sigma[g_1(p)] + (1-\beta) E_\sigma[g_2(p)] \ \text{ s.t. } \ E_\sigma[p] = p_0. \tag{6}$$

Let $G_\beta(p) = \beta g_1(p) + (1-\beta) g_2(p)$ denote the objective of Problem 6. It is straightforward to observe that G_β is piecewise concave with a kink at p^*. Thus, the solution to Problem 6 has either a degenerate or a binary structure. To identify the signal structure, we construct the tangency line for the graph of G_β as shown in Figure 1. The tangency points, $p_L(\beta)$ and $p_H(\beta)$, are given by

$$G'_\beta(p_L(\beta)) = \frac{G_\beta(p_H(\beta)) - G_\beta(p_L(\beta))}{p_H(\beta) - p_L(\beta)} = G'_\beta(p_H(\beta)). \tag{7}$$

If $p_0 \in (p_L(\beta), p_H(\beta))$, then the optimal signal structure $\sigma^*(\beta)$ is binary, inducing posteriors $p_L(\beta)$ and $p_H(\beta)$ with probabilities $(p_H(\beta) - p_0)/(p_H(\beta) - p_L(\beta))$ and $(p_0 - p_L(\beta))/(p_H(\beta) - p_L(\beta))$, respectively. If the prior lies outside of the interval $(p_L(\beta), p_H(\beta))$, then any non-degenerate signal structure strictly decreases the objective function, and thus $\sigma^*(\beta)$ is degenerate. The following proposition summarizes the result.

Proposition 1 *Let $p_L(\beta)$ and $p_H(\beta)$ be the solutions to (7). Problem 6 has a unique solution $\sigma^*(\beta)$ for every $\beta \in [0, 1]$, which is binary if and only if $p_0 \in (p_L(\beta), p_H(\beta))$ and degenerate otherwise. If the signal is binary, then the realizations are $p_L(\beta)$ and $p_H(\beta)$, respectively.*

Proposition 1 provides the signal structure $\sigma^*(\beta)$ for each $\beta \in [0, 1]$. Then the Pareto frontier of the set of feasible payoffs, denoted by $\text{PF}(U)$, is given by

$$\text{PF}(U) = \left\{ (u_1^*(\beta), u_2^*(\beta)) \,\middle|\, \beta \in [0, 1] \right\},$$

where $u_i^*(\beta) = E_{\sigma^*(\beta)}[v_i(p)] - C_i(\sigma^*(\beta))$ is player i's expected utility given signal structure $\sigma^*(\beta)$.

Analyzing the Pareto frontier of U allows us to understand how and under what circumstances integrative negotiation occurs. Recall that $a_i = 1/k_i$ represents player i's negotiation ability during the integrative phase, and q_i is player i's bargaining power in the distributive phase. Notably, the qualitative properties of the Pareto frontier do not depend solely on either parameter. In particular, the outcome of integrative negotiation is determined by the *product* of a player's negotiation ability and bargaining power. Define player i's *negotiation affinity* α_i as:

$$\text{negotiation affinity } (\alpha_i) = \text{negotiation ability } (a_i) \times \text{bargaining power } (q_i). \tag{8}$$

First, we consider the case where the players have the same negotiation affinity.

Proposition 2 *Suppose that both players have the same negotiation affinity α_i. Then there is a unique utility allocation $u^* = (u_1^*, u_2^*)$ that is Pareto efficient.*

Proposition 2 implies that both players' interests during integrative negotiation are perfectly aligned when they have the same negotiation affinity. Further, because the Pareto frontier consists of a single point, u^*, it also serves as the predicted outcome of the negotiation. To prove Proposition 2, observe that by applying (3) and renormalizing the formula, we can rewrite the objective function of Problem 6 as

$$E_\sigma\Big[\alpha(\beta)(\theta_H - \theta_L)\max\{p - p^*, 0\} + h(p)\Big], \text{ where } \alpha(\beta) = \frac{\beta q_1 + (1-\beta) q_2}{\beta k_1 + (1-\beta) k_2}. \tag{9}$$

Note that if $\alpha_1 = \alpha_2$, then $\alpha(\beta)$ is a constant function of β. Therefore, the solution $\sigma^*(\beta)$ of Problem 6 is independent of β, leading to uniqueness of the Pareto efficient outcome.

If players have different negotiation affinities, their preferences during integrative negotiation are no longer aligned. To characterize this case, note that Problem 6 becomes player 1's (resp. player 2's) individual optimization problem if $\beta = 1$ (resp. $\beta = 0$). Define $I_1 = (p_L(1), p_H(1))$ and $I_2 = (p_L(0), p_H(0))$. Then Proposition 1 implies that player i's individually optimal signal structure is degenerate if and only if $p_0 \notin I_i$.

The next lemma describes how the players' incentives for integrative negotiation differ when they have different negotiation affinities.

Lemma 2 *Suppose that $\alpha_i > \alpha_j$. Then the following must hold:*

1. *As β puts more weight on player i,[6] $p_L(\beta)$ strictly decreases and $p_H(\beta)$ strictly increases;*

2. *I_j is a strict subset of I_i;*

3. *If σ is strictly individually rational for player j, then it is also strictly individually rational for player i.*

[6]We say that β puts more weight on player 1 (resp. player 2) if β increases (resp. decreases).

The main implication of Lemma 2 is that there is a monotonic relationship on the optimal signal structure for each β. Intuitively, the player with a greater negotiation affinity generally prefers to acquire more precise information. In our model, this preference is reflected by the inclusion relationship between I_i and I_j (Item 2). Further, as β puts more weight on the player with a higher α_i, the corresponding interval $(p_L(\beta), p_H(\beta))$ monotonically widens (Item 1). The intuition for the proof of Item 1 is again based on the normalized objective function in (9). Note that the first term of the objective—which captures the value of information—is proportional to $\alpha(\beta)$ while the second term is independent of $\alpha(\beta)$. Thus, the optimal signal structure $\sigma^*(\beta)$ becomes more precise as $\alpha(\beta)$ increases. Since $\alpha(\beta)$ monotonically increases (resp. decreases) in β if $\alpha_1 > \alpha_2$ (resp. $\alpha_1 < \alpha_2$), we obtain the desired result. The last item of Lemma 2 states that a monotonic relationship also exists in each player's set of individually rational signal structures.

Lemma 2 enables us to characterize the Pareto frontier of the feasible set, which can be one of the three qualitatively different cases depending on the prior belief p_0.

Proposition 3 *Suppose that $\alpha_i > \alpha_j$, so that I_j is a strict subset of I_i. Then the Pareto frontier of the set of feasible payoffs U is characterized as follows:*[7]

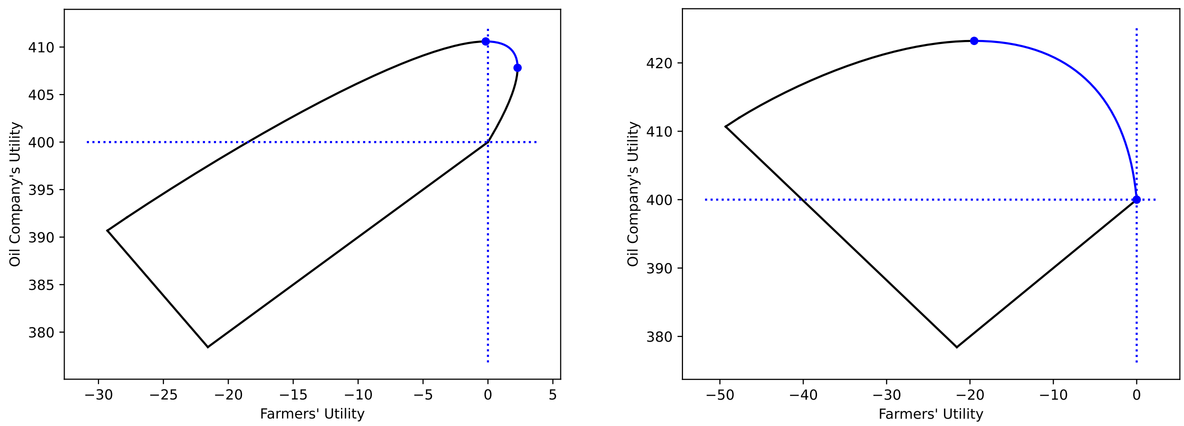
Case 1 *If $p_0 \notin I_i \cup I_j$, then* $\mathrm{PF}(U) = \{\underline{u}\}$.

Case 2 *If $p_0 \in I_i$ and $p_0 \notin I_j$, then* $\mathrm{PF}(U)$ *is a strictly concave curve connecting $\underline{u}$ and a point u^a, where $u^a_i > \underline{u}_i$ and $u^a_j < \underline{u}_j$. For any point on* $\mathrm{PF}(U)\backslash\{\underline{u}\}$*, player j's payoff is strictly below $\underline{u}_j$.*

Case 3 *If $p_0 \in I_i \cap I_j$, then* $\mathrm{PF}(U)$ *is a strictly concave curve connecting points u^b and u^c, where $u^b > \underline{u}$ and $u^c_i > \underline{u}_i$.*

For Case 1, if $p_0 \notin I_i \cup I_j$, it is clear that neither player prefers to obtain any informative signals, and thus Pareto frontier consists only of the reservation payoff vector $\underline{u}$. Second, consider Case 2, where $p_0 \in I_i$ but $p_0 \notin I_j$. Item 1 of Lemma 2 implies that the two endpoints of the Pareto frontier are obtained for the values $\beta = 0$ and $\beta = 1$, i.e., payoffs under each player's individually optimal signal structure. Because player j's optimal signal structure is degenerate, one endpoint of $\mathrm{PF}(U)$ is the reservation payoff vector. Further, player j's expected payoff at all other points on the Pareto frontier must be strictly below $\underline{u}_j$. On the other hand, $p_0 \in I_i$ implies that player i is strictly better off than his reservation payoff under the other endpoint of $\mathrm{PF}(U)$. Finally, consider Case 3 in which $p_0 \in I_i \cap I_j$. The previous argument for Case 2 implies that each player's highest possible payoff is strictly greater than their respective reservation payoff. Now assume, without loss of generality, that the two endpoint u^b and u^c of $\mathrm{PF}(U)$ satisfy $u^b_j > \underline{u}_j$ and $u^c_i > \underline{u}_i$. Then Item 3 of Lemma 2 implies that $u^b_i > \underline{u}_i$, showing the desired result.

[7] Appendix B provides a full characterization of U, including payoffs that are not Pareto efficient.



(a) $(q_F, q_C) = (0.4, 0.6)$ (b) $(q_F, q_C) = (0.2, 0.8)$

Figure 2: The set of feasible payoffs and the Pareto frontier for the example in Section 2.3. The left and right panels describe Case 3 and Case 2 of Proposition 3, respectively.

Before we revisit our example of Section 2.3 and discuss Cases 2 and 3 of Proposition 3 in more detail, we state two corollaries of our characterization result. First, we identify the necessary and sufficient condition for which the players engage in a meaningful integrative negotiation. We say that there exists *non-trivial integrative negotiation* if there exists a feasible and individually rational signal structure σ such that at least one player is strictly better off under σ compared to no information acquisition.

Corollary 1 *The occurrence of integrative negotiation is determined by the player with the lower negotiation affinity. Specifically, non-trivial integrative negotiation exists if and only if* $p_0 \in I_i \cap I_j$.

The next corollary states that if the players have different negotiation affinities, there is disagreement about integrative negotiation as long as at least one player prefers to acquire non-trivial information. The result is a direct implication of Cases 2 and 3 of Proposition 3.

Corollary 2 *Suppose* $\alpha_i \neq \alpha_j$ *and* $p_0 \in I_i \cup I_j$. *Then, disagreement arises during integrative negotiation: for any signal structure* σ, *there exists an alternative structure* $\sigma' \neq \sigma$ *that one of the players strictly prefers.*

Corollary 2 implies that if negotiation affinities differ, integrative negotiation cannot be an entirely cooperative process. Therefore, the players' bargaining power must also matter in shaping the outcome of integrative negotiation.

Example Revisited: Consider again the example of Section 2.3. As the New York Times article referenced in footnote 5 highlights, there was a significant power imbalance between the farmers and the oil company.

The Iraqi government sided with the oil companies, and dissent was seen as dangerous, given the historical context where opposing the government "carried the risk of death." In light of this, we explore two cases where the oil company has greater bargaining power: $(q_F, q_C) = (0.4, 0.6)$ and $(q_F, q_C) = (0.2, 0.8)$, where the subscripts F and C represent the farmers and the oil company, respectively. We assume both players have equal negotiation ability $a_i = 0.01$ $(i = F, C)$. Figure 2 depicts the feasible payoff sets in each case, with the Pareto frontiers indicated by blue lines bordered by blue dots. The reservation payoffs $(\underline{u}_F, \underline{u}_C) = (0, 400)$ are marked by the dotted lines.

We calculate intervals I_C and I_F in each scenario of Figure 2 to identify which case of Proposition 3 applies. In Figure 2a, where $q_C = 0.6$, solving Problem 6 for $\beta = 0$ and $\beta = 1$ yields $I_C = (p_L(0), p_H(0)) \simeq (0.282, 0.935)$ and $I_F = (p_L(1), p_H(1)) \simeq (0.394, 0.878)$, respectively. Since $p_0 = 0.5 \in I_C \cap I_F$, Case 3 of Proposition 3 applies. This implies that there exist Pareto efficient payoffs that make both parties strictly better off than the status quo, indicating that non-trivial integrative negotiation is likely to occur. Now consider Figure 2b, where q_C is increased to 0.8. In this case, we find that $I_C \simeq (0.195, 0.967)$ and $I_F \simeq (0.528, 0.788)$. Since p_0 lies in I_C but not in I_F, Case 2 of Proposition 3 applies. Thus, the only Pareto efficient and individually rational outcome is the reservation payoff vector $\underline{u}$, implying that non-trivial integrative negotiation cannot occur. As we will see in Section 4.1, if one player has sufficiently high bargaining power, the parties may fail to engage in integrative negotiation.

In this example, we argue that the involvement of a government official as an intermediary likely made the significant difference. Within the framework of our model, this likely had two key impacts. First, the official may have shifted the power balance between the players, resulting in more equal bargaining powers, q_C and q_F. Second, the presence of a skilled intermediary would likely reduce negotiation costs, or equivalently, increase negotiation ability a_i. In what follows, we further explore how these and other model primitives influence the negotiated outcome.

4 Comparative Statics

In this section, we investigate how the outcomes from integrative negotiation depends on the model parameters such as players' bargaining power (q_i), negotiation ability (a_i), and the status quo payoffs (w_i).

Recall from the previous section that whenever $\alpha_i \neq \alpha_j$ and a non-trivial integrative negotiation occurs, there exist multiple Pareto efficient outcomes. In other words, there is a disagreement during integrative negotiation about what type of information should be acquired. In this case, the player with the higher bargaining power will be able to move the integrative negotiation process in their favor. Given this observation, we assume that the outcome is determined by the generalized Nash bargaining solution where the feasible payoff set is U and the disagreement payoff vector is $\underline{u}$. Formally, define the Nash solution $\hat{u}$ for integrative negotiation as[8]

$$\hat{u} = \arg\max_{u \in U} (u_1 - \underline{u}_1)^{q_1} (u_2 - \underline{u}_2)^{q_2}. \tag{10}$$

Henceforth, we assume that any conflict in the integrative part of the negotiation is resolved by the generalized Nash bargaining solution (10).

4.1 Bargaining Power

We begin by examining how bargaining power (q_i) influences the outcome of integrative negotiation. As the first result, we characterize the range of q_i's for which non-trivial integrative negotiation occurs.

Proposition 4 *There exists* $\underline{q}, \overline{q} \in [0, 1]$ *such that non-trivial integrative negotiation occurs if and only if* $q_1 \in (\underline{q}, \overline{q})$.

The intuition for Proposition 4 is clear: If bargaining power is sufficiently asymmetric (i.e., q_1 close to either zero or one), one of the players obtains a vanishingly small share of potential surplus. Therefore, such a player would not have an incentive to participate in integrative negotiation. This result shares a superficial similarity with the feature in Grossman and Hart (1986) that initial investments are inefficiently low when firms are not integrated. The similarity is superficial because our negotiated outcome is always Pareto efficient in our setting as the extent of information acquisition is contractible—the only reason why no integrative negotiation occurs is that it would violate individual rationality for the player with the weaker negotiation affinity.

To understand the effect of bargaining power, observe that there are two distinct impacts of increased bargaining power on a player's payoff, $\hat{u}_i$. The more straightforward impact is the *larger-share effect*, where a player secures a larger share of the pie as their bargaining power increases. This effect is clearly monotonic in q_i. However, there is also the *integrative negotiation effect*: as discussed in the previous section, players engage more in integrative negotiation when their negotiation affinities are more balanced. The interplay of these two forces implies that a player's payoff, $\hat{u}_i$, may be non-monotonic in their bargaining power:

Proposition 5 *A player's payoff is non-monotone in their bargaining power for priors* $p_0 \in (p_L(0.5), p^*)$.

1. *If* $p_0 \in (p_L(0.5), p_H(0.5))$, *then for* q_i *close to the points where* $\alpha_1 = \alpha_2$, $\hat{u}_i$ *is strictly increasing in* q_i.
2. *If* $p_0 \le p^*$, *then* $\hat{u}_i$ *converges to* $\underline{u}_i = w_i$ *as* $q_i \to 1$.

[8] Note that we use the bargaining power q_i that is identical to that of distributive phase. In principle, one could assume that a player's bargaining power in integrative negotiation, say $q_{i,I}$, differs from that of distributive negotiation, $q_{i,D}$. Keeping these parameters separate does not fundamentally change our results. More importantly, we would argue that bargaining power is a player's feature that should affect the entire negotiation process, which means that $q_{i,I}$ and $q_{i,D}$ should be similar.

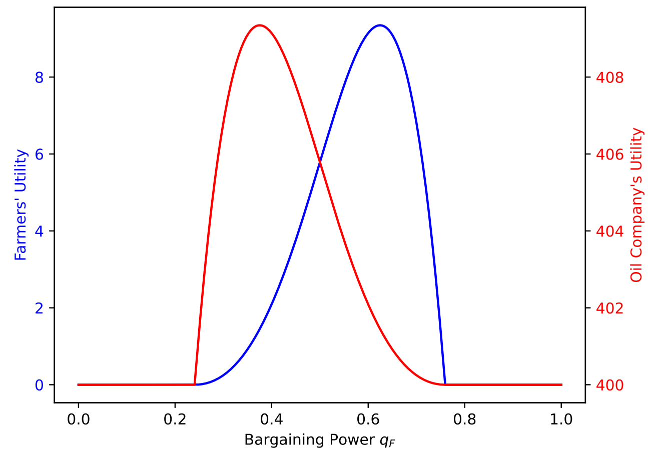


Figure 3: The payoff of the farmers (blue line) and the oil company (red line) in the example of Section 2.3 as a function of the farmers' bargaining power, q_F.

Continuing the example from Section 2.3, Figure 3 illustrates the effects of bargaining power on the payoffs of the farmers (blue line) and the oil company (red line). When the farmers' bargaining power q_F is very low, as indicated by Proposition 4, integrative negotiation does not occur and the status quo solution is chosen, leaving the farmers with a payoff of zero. As q_F increase towards 0.5, the negotiation affinities α_i converge to each other (recall that the players have the same negotiation ability $a_F = a_C = 0.01$), which enhances integrative negotiation. Simultaneously, increasing q_F also raises the farmers' share of the pie. With both effects reenforcing each other, the farmers' payoff increases rapidly. As q_F is pushed above 0.5, the negotiation affinities become more unequal, reducing the amount of integrative negotiation effect and, consequently, decreasing the surplus. However, the larger-share effect dominates initially, and the farmers' payoff continues to increase, albeit at a slower rate. Once the players' negotiation affinities diverge significantly, the farmers' payoffs begin to decline. Finally, as q_F becomes close to one, Proposition 4 again implies that the farmers' payoff drops back to zero. The graph for the oil company's payoff is symmetric.

The following corollary is straightforward from Proposition 5.

Corollary 3 *If the prior satisfies $p_0 \in (p_L(0.5), p^*)$ then the optimal level of bargaining power for player i is strictly less than 1.*

In summary, if one considers solely the competitive aspect of a negotiation, i.e., the distributive part, it is well known that having more bargaining power is better for a player, because it leads to a larger share of the pie. In contrast, if we consider the entire negotiation process, the effectiveness of integrative negotiation is determined by players' negotiation affinities, which are the product of their negotiation abilities and bargaining powers. Because integrative negotiation is a cooperative process it works best when the players' negotiation affinities are equal. Thus, negotiators encounters a basic tradeoff: They want to generate as much surplus as they can, which is achieved by having similar negotiation affinities, but at the same time they would like to have more bargaining power so that they can claim a larger share of the pie.

4.2 Negotiation Ability

We now investigate the impact of a player's negotiation ability a_i on the final outcome. Negotiation ability is determined by the direct and opportunity costs associated with engaging in integrative negotiation. These costs can be influenced by the negotiators' cognitive abilities, which in turn affect the time and effort spent on negotiating.

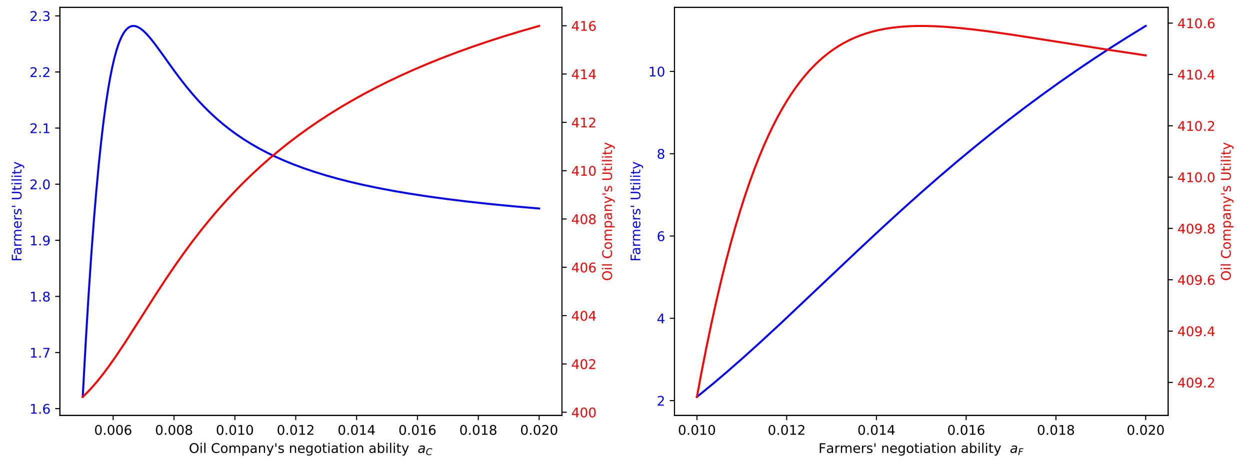
In one of the first papers on this topic, Pruitt and Lewis (1975) conducted an experiment in which they classified negotiators into two groups: high-complexity and low-complexity individuals. They compare pairs of two high-complexity subjects to pairs containing two low-complexity subjects. In the experiment most relevant to our paper,[9] a negotiation could only succeed if the parties engage in non-trivial integrative negotiation. To place their findings in the context of our model, we observe that in their experiment 67% of high-complexity pairs succeeded vs. only 46% of low complexity pairs.[10] We compute the associated p-value of 0.16, which is statistically insignificant almost certainly because their experiment only considered 25 pairs of subjects, so that it was too under-powered. Consistent with their result, our model predicts that increasing negotiation ability increases the likelihood of integrative negotiation, as higher negotiation abilities only expand the feasible set U. Additionally, Thompson et al. (1996) demonstrate in an experiment that teams outperform individual negotiators in generating joint surplus. They hypothesize that teams are more willing to engage in information exchange, which corresponds to a higher negotiation ability in our model. Their results are also consistent with our predictions.

Pruitt and Lewis (1975) do not pair high and low complexity individuals. It turns out that the effect of such pairing is subtle: a player's negotiation ability has a non-monotonic effect on *the other player's* payoff.

Proposition 6 *Player i's payoff in the Nash solution $\hat{u}_i$ is non-monotonic in a_j, and is maximized at $a_j = \alpha_i/q_j$. Further, if $p_0 < p^*$, then $\hat{u}_i$ converges to w_i as $a_j \to 0$.*

[9] Consider the description of Experiment 2 on pp. 628–631 and the data summarized in Table 4 on page 630. We focus on their high-limit case, which requires each participant to achieve a payoff of 2,300. This payoff can only be obtained by engaging in integrative negotiation, as their status quo option (termed the "horizontal option") only yields a total payoff of 4,000 for both players combined.

[10] Pruitt and Lewis (1975) focused on the average surplus generated of teams that negotiated successfully, which is marginally lower for the high-complexity pair, but the more appropriate metric for our model is the number of pairs that negotiated successfully.



(a) payoffs as functions of a_C (b) payoffs as functions of a_F

Figure 4: Payoffs of the farmers (blue line) and the oil company (red line) from the Nash solution $\hat{u}$ for the example of Section 2.3 as functions of negotiation abilities, with bargaining power of $(q_F, q_C) = (0.4, 0.6)$.

When player j's negotiation ability is low, then any increase in a_j benefits player i, because more information will be acquired. More surprisingly, if player j's negotiation ability is too high then player i becomes worse off. One key reason is that negotiation affinities become more unequal as a_j is increased, resulting in a bigger difference between the signal structures preferred by each player. The Pareto frontier shifts in the direction of player j, resulting in a bigger payoff for player j and smaller one for player i.

The non-monotonic effect of a_j on $\hat{u}_i$ is illustrated in Figure 4, using the numbers from the example in Section 2.3. As the proposition indicates player i's payoff is maximized if the negotiation abilities satisfy $a_j = \alpha_i/q_j$. This is the case for the farmers when $a_C = 0.0067$ (left panel), and for the oil company when $a_F = 0.015$ (right panel). In these cases, negotiation affinities α_i are the same. Consequently, there is no conflict of interest in integrative negotiation, and the signal structure on the Pareto frontier coincides with individually optimal one. For any other values of a_j, a distortion from conflict of interest may occur, making player i weakly worse off. Further, if player j's negotiation ability is too low, then integrative negotiation becomes prohibitively costly, and the status quo is chosen, because $p_0 < p^*$.

There is an interesting parallel to the non-monotonicity result of Proposition 6 with the experiment in Nunnari et al. (2024). In their paper subjects first perform an intelligence test, and then are paired with subjects with various cognitive abilities. Subjects are first informed about the opponents ability and then negotiate on whether to play a prisoner's dilemma game or harmony game, i.e., a game in which the dominant strategy leads to the Pareto efficient outcome. They show that pairs of low ability and pairs of high ability are more likely to agree to the harmony game.

4.3 Status Quo Payoff

We now consider the effect of changes in the status quo payoffs, w_i. These payoffs may be either the result of a negotiated share of the status quo solution or an exogenous "breakdown payoff" when the parties cannot agree on the alternative solution. The former case applies to the orange example by Follett (1940), where she implicitly assumes that the sisters have equal bargaining power, and hence the status quo is to split the orange into two halves. In contrast, in the case of the Iraqi farmers and the oil company in Section 2.3, w_i corresponds to a breakdown payoff rather than a negotiated settlement. In this latter case, w_i corresponds to the "Best Alternative To the Negotiated Agreement" (BATNA) (Fisher et al., 2011).

As discussed in Section 2.2, w_i and q_i model different aspects of a negotiator's "power." However, we now show that they affect the negotiation outcome in substantially different ways.

Proposition 7 *Consider two pairs of status quo payoffs w_i and $\tilde{w}_i$ $(i = 1, 2)$ with $w_1 + w_2 = \tilde{w}_1 + \tilde{w}_2$. Then for any $\beta \in [0, 1]$, the optimal signal structure $\sigma^*(\beta)$ is the same under either status quo payoffs. Moreover, let U and $\tilde{U}$ be the corresponding sets of feasible payoffs, then $U - (w_1, w_2) = \tilde{U} - (\tilde{w}_1, \tilde{w}_2)$.*

Proposition 7 shows that players' behavior during integrative negotiation remains unchanged as long as the sum of the status quo payoffs stays the same. In other words, if one player's BATNA is increased and the other player's BATNA is decreased by the same amount, the integrative negotiation process and the total surplus remain unaffected. This is because integrative negotiation is focused on generating surplus, which depends on the sum of the status quo payoffs, not on their individual levels. The individual status quo payoffs only impact distributive negotiation.

Formally, note that p^*, as defined in (2), does not change if the sum of the status quo payoffs remains constant. Therefore, the objective function of Problem (6) shifts only vertically by $\beta(\hat{w}_1 - w_1) + (1-\beta)(\hat{w}_2 - w_2)$. As a result, the optimization problem yields the same signal structure in both cases, leading to only a parallel shift of the Pareto frontier, as indicated in the proposition. The result in Appendix B confirms that this holds true for all other points in the feasible set as well.

If the sum of the status quo payoffs changes, it impacts the outcome of integrative negotiation, as stated in the next proposition.

Proposition 8 *Suppose that the sum of the status quo payoffs $w_1 + w_2$ is increasing. Then for any $\beta \in [0, 1]$,*

1. *the interval of priors for which integrative negotiation occurs shifts to the right, i.e., both $p_L(\beta)$ and $p_H(\beta)$ strictly increase; and*

2. *the length of the interval, $p_H(\beta) - p_L(\beta)$, strictly increases if and only if $h''(p_H(\beta)) > h''(p_L(\beta))$.*

To understand the first statement of Proposition 8, note from (2) that p^* strictly increases in $w_1 + w_2$. Integrative negotiation is generally more beneficial if p_0 is closer to p^*, as the implemented solution depends on whether the posterior is greater or less than p^*. Therefore, the interval of integrative negotiation $(p_L(\beta), p_H(\beta))$ follows the change in p^* and shifts to the right. Depending on whether p_0 is below or above p^*, this change can either enhance or reduce the extent of integrative negotiation. The second statement of Proposition 8 implies that the length of the interval $p_H(\beta) - p_L(\beta)$ is single-peaked in $w_1 + w_2$ if $h''(p)$ is single-peaked. For example, this is the case when $h(p)$ is the entropy function, for which $h''(p)$ is maximized at 0.5. This reflects the fact that a prior of $p_0 = 0.5$ contains the least amount of information about the alternative solution's effectiveness. Hence, the benefit of learning is maximized at that point, leading to the widest interval of integrative negotiation.

Propositions 7 and 8 indicate that changes in status quo payoffs affects the negotiated outcome differently from changes in negotiation affinity. To see this, consider the optimal signal structure for player 1, which is the solution of Problem 6 for $\beta = 1$. If we change player 2's negotiation affinity α_2, then player 1's most preferred signal structure remains unchanged. In contrast, changing player 2's status quo payoff w_2 alters the signal structure that player 1 desires. As discussed above, the change in the sum of the status-quo payoffs affects the net surplus, which in turn affects the player's incentive to engage in integrative negotiation.

We illustrate the results in Proposition 8 using the example of Section 2.3. Consider an alternative scenario where the oil company incurs lower costs from evicting the farmers, so that the oil company's status quo payoff increases from 400 to 460, as shown in Table 2—all other payoffs remain the same.

Farmers

	Eviction	Farming
θ_L	0	0
θ_H	0	100

Oil Company

	Eviction	Farming
θ_L	460	0
θ_H	460	500

Table 2: Payoffs for the farmers and the oil company when eviction costs are lower.

Proposition 8 implies that the interval of priors for which integrative negotiation occurs shifts to the right when comparing the current example to the original one in Table 1. Suppose again that $(q_F, q_C) = (0.4, 0.6)$. Then, I_F moves from $(0.394, 0.878)$ to $(0.528, 0.925)$, and I_C from $(0.282, 0.935)$ to $(0.416, 0.963)$. Because $p_0 = 0.5$ is no longer within the farmers' interval, Corollary 1 implies that integrative negotiation fails under the new payoffs. Notably, the breakdown of negotiations is caused by the farmers and not by the oil company, even though the farmers' payoffs are unchanged. This is due to the fact that the payoffs of both players impact p^*, which in this case reduces the farmers' incentive to engage in integrative negotiation.

5 Transfers

In this section, we investigate the role of monetary transfers in negotiation. First, we examine a model where the players can make monetary transfers during integrative negotiation. Second, we consider the case of "purely integrative negotiation," where no monetary transfers are allowed throughout the entire negotiation process, thereby eliminating the distributive phase.

5.1 Side Payments During Integrative Negotiation

Payments between negotiators are standard in distributive negotiations. However, their role in integrative negotiation has not been well investigated. One such example occurs in timeshare sales, where prospective buyers are offered free or discounted vacations in exchange for attending a sales presentation.[11] On the other hand, such payments are very uncommon in most other negotiations: for instance, it is unlikely that the sisters in Follett's example would make side payments during integrative negotiation. The objective of this section is to examine the conditions under which such payments are permitted in integrative negotiation.

To address this question, we extend the baseline model to incorporate side payments during integrative negotiation. The timeline of the extended model is as follows. First, players jointly decide whether side payments are permissible during integrative negotiation. If both players agree to allow side payments, they determine an signal structure σ *and* a net payment $m \in \mathbb{R}$ from player 1 to player 2 during integrative negotiation. If at least one player rejects side payments, the model reverts to the baseline version, where only σ is determined. As in the baseline model, distributive negotiation follows integrative negotiation.

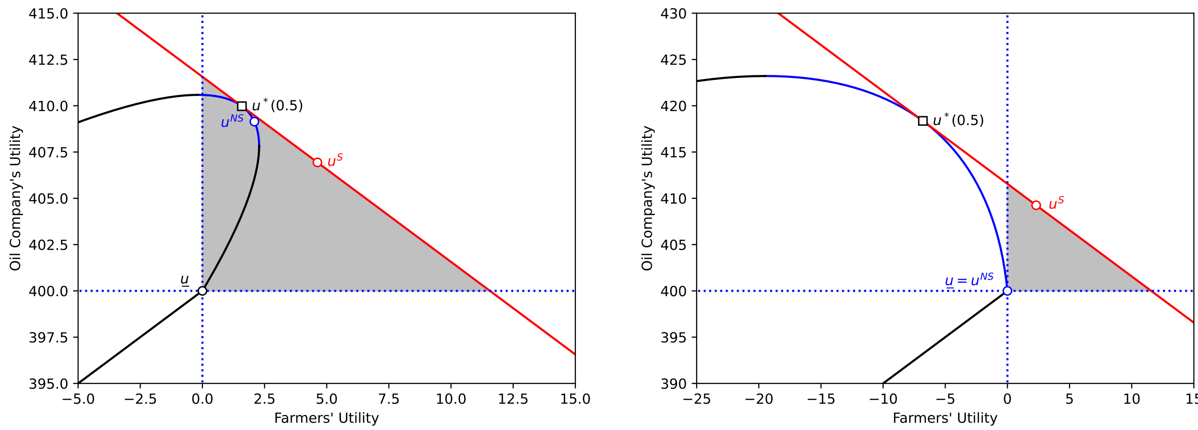
Let u^S (resp. u^{NS}) be the players' payoffs when side payments are permitted (resp. not permitted). It is clear that the players agree to allow side payments during integrative negotiation if and only if $u^S \geq u^{NS}$, i.e. allowing side payments induces a (weak) Pareto improvement. We continue to assume that any conflict in integrative negotiation is resolved by the generalized Nash bargaining solution. This immediately implies that u^{NS} equals $\hat{u}$ as defined in (10).

To determine u^S, note that the side payment m during integrative negotiation effectively serves as a utility transfer between the two players, resulting in a Pareto frontier that is a straight line with slope -1. The new Pareto frontier is tangent to the feasible set U from the baseline model. Formally, this extended set of feasible payoffs, $\tilde{U}$, is defined by

$$\tilde{U} = \{(\tilde{u}_1, \tilde{u}_2) : \tilde{u}_1 + \tilde{u}_2 \leq \max_{u \in U}(u_1 + u_2)\}.$$

We define u^S as a solution of a modified version of (10), where the feasible set U is replaced with $\tilde{U}$. Note that with side payments, the optimal signal structure is always $\sigma^*(0.5)$, as solving Problem 6 with $\beta = 0.5$

[11] For a discussion of the timeshare market see Wang and Krishna (2006).



(a) $(q_F, q_C) = (0.4, 0.6)$ (b) $(q_F, q_C) = (0.2, 0.8)$

Figure 5: The set of feasible and individually rational payoffs with side payments for the example in Section 2.3.

yields the maximum utilitarian payoff. As discussed in the proof of Proposition 3, the Pareto frontier of U is strictly convex, and thus all other points on PF(U) result in a strictly lower utilitarian payoff.

Figure 5 adds side payments during integrative negotiation to the example in Section 2.3. The outcomes of the baseline model are denoted by u^{NS}. If we allow for side payments, the set of feasible and individually rational outcomes is depicted by the shaded sets. In Figure 5a, the negotiated outcome u^S makes the oil company worse off compared to u^{NS}, and the oil company would therefore not agree to side payments. In contrast, in Figure 5b, the resulting outcome u^S is a strict Pareto improvement over u^{NS}, because integrative negotiation only occurs with side-payments. It turns out that the crucial difference between the two scenarios is that the point $u^*(0.5)$ which anchors the feasible set is strictly above the autarky point.

Proposition 9 *Suppose that $u_i^*(0.5) > \underline{u}_i$ for all $i = 1, 2$. Then allowing for side payments is not Pareto improving: the player with the higher negotiation affinity is strictly worse off under side payment.*

As the player with the higher negotiation affinity, the oil company has to make a net-payment to the farmers. This net-payments dominates the benefit from the farmer's increased willingness to engage in integrative negotiation, as long as negotiation affinities are not too asymmetric or the potential surplus for an individual player is not too small. To understand why, observe from Figure 5a that the Pareto frontier without side payments is skewed towards the oil company. This means that the oil company secures most of the bargaining surplus, despite not having significantly more bargaining power than the farmers. With side payments, the farmers receive 40% of the maximized surplus, given $(q_F, q_C) = (0.4, 0.6)$. The distance between u^S and $u^*(0.5)$ represents the payments m made by the oil company to the farmers. Although side payments increase the surplus, the gain is relatively small compared to the size of m, as shown in the gap between u^{NS} and the red line.

The reason side payments are beneficial in Figure 5b is that without them, no integrative negotiation occurs, and the players obtain their reservation payoffs. As long as there is sufficient surplus from integrative negotiation, i.e., $\sum_{i=1}^{2}\left(u_i^*(0.5) - \underline{u}_i\right)$ is strictly positive, the set of feasible and individually rational payoffs is non-trivial, making side payments Pareto improving. In the timeshare sales example, visiting a timeshare property incurs costs for prospective buyers, along with the opportunity cost of spending vacation time listening to a sales pitch. In our model, this corresponds to the prospective buyer having a low negotiation ability. Thus, a payment from the seller to the buyer can incentivize the buyer to engage in negotiations and attend the presentation.

5.2 Purely Integrative Negotiation

In the Follett's example from the introduction, no distributive negotiation occurred, meaning that no money or utility transfers took place. Thompson (1991) argues that many real-world negotiations follow a similar pattern. Additionally, in legislative negotiations, utility is often not directly transferable; instead, multiple discrete issues are combined into a bill during an integrative negotiation process. For example, the border security bill of February 2024 negotiated by a bipartisan group of US senators[12] paired aid to Ukraine with increase border security measures to appeal to both liberals and conservatives, as direct payoffs to these groups were not feasible. In this section, we modify our model to incorporate the case of *purely integrative negotiation*, where there is no distributive phase.

We continue to assume that there is a status quo solution with payoff of w_i and an alternative solution with two possible states, H and L. Departing from the baseline model, we now assume that player i derives individual payoff $\theta_{i,H}$ and $\theta_{i,L}$ from the alternative solution in each state. For the alternative to be effective only in state H, we assume $\theta_{i,H} > w_i$ for all $i = 1, 2$, and $\theta_{i,L} < w_i$ for some i. After the integrative phase, the players decide which solution to adopt. The alternative solution is selected only if both players prefer it. Importantly, we assume purely integrative negotiation: no monetary transfers are allowed, and each player receives the realized individual payoff.

Unlike the baseline model, each player has a different threshold belief p_i^*, where they are indifferent between the status quo and the alternative solution. Player i's expected payoff from the alternative solution is $p\theta_{i,H} + (1-p)\theta_{i,L}$, which player i prefers if $p \geq p_i^* := (w_i - \theta_{i,L})/(\theta_{i,H} - \theta_{i,L})$. The alternative solution is only adopted if both players prefer it, i.e., if $p \geq \max\{p_1^*, p_2^*\}$.

[12]See, for example https://www.reuters.com/world/us/us-senate-unveils-118-billion-bipartisan-bill-tighten-border-security-aid-2024-02-04/

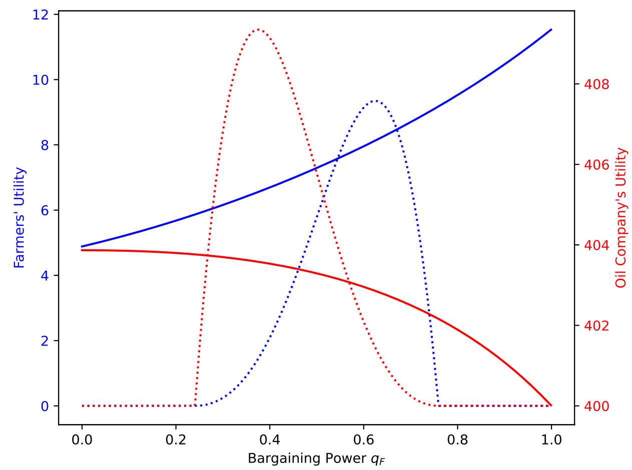


Figure 6: Payoffs of the farmers (blue) and the oil company (red) in the example of Section 2.3 as functions of the farmers' bargaining power (q_F), under purely integrative negotiation (solid lines) and the baseline model (dotted lines).

This decision rule reveals the inherent cost of not having distributive negotiation. Recall that p^* is the corresponding threshold in the baseline model. It is immediate that $p^* < \max\{p_1^*, p_2^*\}$ whenever $p_1^* \neq p_2^*$. Because p^* is the socially efficient threshold, the alternative solution is under-used in purely integrative negotiation whenever $p^* < p < \max\{p_1^*, p_2^*\}$. The intuition is straightforward: as in many economic environments, banning transfers introduces friction in utilizing information, leading to inefficient outcomes in purely integrative negotiations when information is fixed.

However, there is also a potential benefit from purely integrative negotiation. Without a distributive phase, the players' incentives during integrative negotiation are no longer influenced by their bargaining power. As a result, the Pareto frontier is independent of bargaining power. This observation suggests a potential benefit of banning transfers: it could encourage more information acquisition during integrative negotiation, particularly when bargaining power is very asymmetric.

Figure 6 illustrates this argument by depicting a case where the players are better off under purely integrative negotiation. The figure reproduces Figure 3 from the example in Section 2.3. The dashed lines represent the players' payoffs in the baseline model as functions of the farmers' bargaining power (q_F), while the solid lines represent their payoffs under purely integrative negotiation, with the individual payoffs $u_{i,H}$ and $u_{I,L}$ derived from Table 1. First, note that payoffs under purely integrative negotiation are not constant in q_F; even though the Pareto frontier is independent of q_F, bargaining power still affects the negotiated outcome in the integrative phase. More importantly, observe that when q_F is close to either 0 or 1, the outcome under purely integrative negotiation Pareto dominates that in the baseline model. As discussed in

Section 4.1, integrative negotiation fails when the players' bargaining powers are sufficiently asymmetric. In this case, not allowing transfers acts as a commitment device, providing both players with sufficient incentives to engage in integrative negotiation.

6 Conclusion

Integrative negotiation is defined as process in which players jointly determine ways how to increase the surplus. In contrast, distributive negotiation determines how a fixed surplus is distributed. The literature on negotiation in psychology and management focus on the former while the economics literature emphasizes the latter. This paper explores the interaction between integrative and distributive negotiations, and shows that they are closely connected. Our findings suggest that insights derived from models that focus solely on either distributive or integrative negotiation can be misleading.

For example, while an increase in a player's bargaining power is always beneficial in a purely distributive negotiation model, this advantage may not hold when integrative negotiation is also considered. Additionally, we formally introduce the concept of negotiation ability, which measures a player's propensity towards engaging in integrative negotiations. We define negotiation affinity as the product of negotiation ability and bargaining power, showing that it is a prime determinant of the outcome of integrative negotiation. Furthermore, we show that surplus is maximized when both players have equal negotiation affinity.

We extend our baseline model to investigate under what circumstance one player may induce the other player to engage in integrative negotiation in exchange for financial payments. Our main results show that unless negotiation affinities are very unequal, such side payments will not occur. We also modify our model to investigate cases in which distributive negotiation cannot occur or is severely limited because monetary payments are not feasible.

Our results indicate that studying the interplay between integrative and distributive negotiation is important to understand how negotiated settlements are obtained. Our approach can be used more generally to analyze outcomes in many non-market environments.

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Appendix A Proofs

Proof of Lemma 1. Let $u, u' \in U$ and $\sigma, \sigma' \in \Sigma$ such that $u_i = E_\sigma[v_i(p)] - C_i(\sigma)$ and $u'_i = E_{\sigma'}[v_i(p)] - C_i(\sigma')$. Let $0 \le \lambda \le 1$, and define σ_λ as the following signal structure: With probability λ signal structure σ is chosen, and with probability $1 - \lambda$ signal structure σ'. Let u_i^λ be the payoffs induced by σ_λ. Then

$$\begin{aligned} u_i^\lambda &= E_{\sigma_\lambda}[v_i(p)] - k_i\Big(h(p_0) - E_{\sigma_\lambda}[h(p)]\Big) \\ &= \lambda \left(E_\sigma[v_i(p)] - k_i\Big(h(p_0) - E_\sigma[h(p)]\Big)\right) + (1-\lambda)\left(E_{\sigma'}[v_i(p)] - k_i\Big(h(p_0) - E_{\sigma'}[h(p)]\Big)\right) \\ &= \lambda u_i + (1-\lambda) u'_i. \end{aligned} \tag{11}$$

Thus, the line connecting u and u' can be generated by the signal structure σ_λ. Thus, U is convex.

To prove that U is closed, let u^n, $n \in \mathbb{N}$ be a sequence in U that converges to u. We must show that $u \in U$. For each u^n there exists a signal structure $\sigma^n \in \Sigma$. The distributions on posteriors generated by signal structures are probability measures on $[0, 1]$. The Banach-Alaoglu theorem implies that the set of all probabilities on $[0, 1]$ is is compact in the weak* topology. With a slight abuse of notation denote by σ the distribution over posteriors on $[0, 1]$. It is immediate that the set of posteriors is closed in the weak* topology, and hence compact, because Σ is a subset of the set of all probability distributions on $[0, 1]$. Thus, there exists a subsequence σ^{n_k}, $k \in \mathbb{N}$ that converges to some σ^* in Σ. By the definition of the weak* topology it follows that $\lim_{k\to\infty} u_i^{n_k} = \lim_{k\to\infty} E_{\sigma^{n_k}}[v_i(p)] - C_i(\sigma^{n_k}) = E_{\sigma^*}[v_i(p)] - C_i(\sigma*) = u_i$ for $i = B, S$. Hence $u \in U$.

Finally, because both the gains of trade and the information cost are bounded, U is bounded. ■

Proof of Proposition 1. Suppose σ is not binary. Then define a new signal with two posteriors $p_L = E_\sigma[p | p < p^*]$ and $p_H = E_\sigma[p | p \ge p^*]$, that occur with the appropriate probabilities. The rest follows from strict concavity of h.

Next, suppose that σ and σ' both solve Problem 5. Let σ_α be the signal structure that uses σ with probability α and σ' with probability $1 - \alpha$. Then σ_α satisfies the constraint and the objective has the same value, i.e., σ_α solves the optimization problem. However, the first part of the argument implies that optimal signals are at most binary, a contradiction. ■

Proof of Proposition 2. See text. ■

Proof of Lemma 2. *Item 1.* Define $W(p) = (\theta_H - \theta_L)(p - p^*)$, and let $\alpha(\beta)$ be given by (9). Note that $\alpha(\beta)$ is strictly increasing in β if $\alpha_1 > \alpha_2$ and strictly decreasing if the inequality is reversed. Using the above notations, the tangency conditions (7) can be rewritten as

$$\alpha(\beta)W(p_H(\beta)) + h(p_H(\beta)) - h(p_L(\beta)) = h'(p_L(\beta))(p_H(\beta) - p_L(\beta)); \tag{12}$$

$$\alpha(\beta)W(p_H(\beta)) + h(p_H(\beta)) - h(p_L(\beta)) = \Big(\alpha(\beta)W'(p_H(\beta)) + h'(p_H(\beta))\Big)(p_H(\beta) - p_L(\beta)). \tag{13}$$

Taking the derivatives of (12) and (13) with respect to β yields

$$\begin{aligned}
\alpha'(\beta)W(p_H(\beta))+&\Big(\alpha(\beta)W'(p_H(\beta)) + h'(p_H(\beta))\Big)p'_H(\beta) - h'(p_L(\beta))p'_L(\beta)\\
&= h''(p_L(\beta))p_L(\beta)(p_H(\beta) - p_L(\beta)) + h'(p_L(\beta))(p'_H(\beta) - p'_L(\beta)),\\
\alpha'(\beta)W(p_H(\beta))+&\Big(\alpha(\beta)W'(p_H(\beta)) + h'(p_H(\beta))\Big)p'_H(\beta) - h'(p_L(\beta))p'_L(\beta)\\
&= \Big(\alpha'(\beta)W'(p_H(\beta)) + h''(p_H(\beta))p'_H(\beta)\Big)(p_H(\beta) - p_L(\beta))\\
&\quad + \Big(\alpha(\beta)W'(p_H(\beta)) + h'(p_H(\beta))\Big)(p'_H(\beta) - p'_L(\beta)).
\end{aligned}$$

Plugging in $\alpha(\beta)W'(p_H(\beta)) + h'(p_H(\beta)) = h'(p_L(\beta))$ (which is implied from $G'_\beta(p_L(\beta)) = G'_\beta(p_H(\beta))$) and simplifying yield

$$\begin{aligned}
\alpha'(\beta)W(p_H(\beta)) =& h''(p_L(\beta))p'_L(\beta)(p_H(\beta) - p_L(\beta)),\\
\alpha'(\beta)W(p_H(\beta)) =& \Big(\alpha'(\beta)W'(p_H(\beta)) + h''(p_H(\beta))p'_H(\beta)\Big)(p_H(\beta) - p_L(\beta)).
\end{aligned}$$

Solving for $p'_L(\beta)$ and $p'_H(\beta)$, we obtain

$$p'_L(\beta) = \alpha'(\beta)\overbrace{\frac{W(p_H(\beta))}{h''(p_L(\beta))(p_H(\beta) - p_L(\beta))}}^{<0}, \tag{14}$$

$$p'_H(\beta) = \alpha'(\beta)\frac{W(p_H(\beta)) - W'(p_H(\beta))(p_H(\beta) - p_L(\beta))}{h''(p_H(\beta))(p_H(\beta) - p_L(\beta))} = \alpha'(\beta)\overbrace{\frac{\alpha(\beta)(\theta_H - \theta_L)}{h''(p_H(\beta))}}^{<0}\overbrace{\frac{p_L(\beta) - p^*}{p_H(\beta) - p_L(\beta)}}^{<0}. \tag{15}$$

Therefore, $\mathrm{sgn}(p'_L(\beta)) = -\mathrm{sgn}(\alpha'(\beta))$ and $\mathrm{sgn}(p'_H(\beta)) = \mathrm{sgn}(\alpha'(\beta))$. Since it is clear from the definition of $\alpha(\beta)$ that $\mathrm{sgn}(\alpha'(\beta)) = \mathrm{sgn}(\alpha_1 - \alpha_2)$ for all $\beta \in [0, 1]$, we obtain the desired result.

Item 2. This immediately follows from Item 1.

Item 3. Suppose that σ is individually rational for player j, i.e., $E_\sigma[v_j(p)] - C_j(\sigma) \geq v_j(p_0)$. Using (1) and (3), the inequality can be rewritten as

$$w_j + E_\sigma[q_j(\theta_H - \theta_L)\max\{p - p^*, 0\}] - k_j\Big(h(p_0) - E_\sigma[h(p)]\Big) \geq w_i + q_j(\theta_H - \theta_L)\max\{p_0 - p^*, 0\}.$$

Simplifying and rearranging yield

$$\alpha_j(\theta_H - \theta_L)\Big(E_\sigma[\max\{p - p^*, 0\}] - \max\{p_0 - p^*, 0\}\Big) \geq h(p_0) - E_\sigma[h(p)].$$

Since the function $z(p) = \max\{p - p^*, 0\}$ is convex, Jensen's inequality implies that the formula in the parenthesis of the left-hand side is nonnegative. Then since $\alpha_i \geq \alpha_j$, it follows that

$$\alpha_i(\theta_H - \theta_L)\Big(E_\sigma[\max\{p - p^*, 0\}] - \max\{p_0 - p^*, 0\}\Big) \geq h(p_0) - E_\sigma[h(p)],$$

which implies $E_\sigma[v_i(p)] - C_i(\sigma) \geq v_i(p_0)$. ■

Proof of Proposition 3. To complete the proof for Proposition 3, it remains to show that the Pareto frontier is described by a strictly concave function. To see this, suppose that σ and σ' are two signal structures that generate points u and u' on the Pareto frontier. By Proposition 1 both σ and σ' are either non-degenerate or binary, which have non-overlapping support. Then as in Lemma 1 we can construct a convex combination σ_λ of σ and σ' by choosing σ with probability λ and σ' with probability $1-\lambda$. As mentioned, costs are linear in λ, and hence by varying λ between 0 and 1, we generate the straight line connecting u and u'. However, if $0 < \lambda < 1$ then signal σ_λ generates a posterior that is not binary, hence, the line connecting u and u' is not on the Pareto frontier. ■

Proof of Proposition 4. Recall from Corollary 1 that non-trivial integrative negotiation occurs if and only if $p_0 \in I_1 \cap I_2$. Also, from the proof of Lemma 2 (especially equations (14) and (15)), it is clear that I_1 expands and I_2 shrinks in q_1. Finally, it is straightforward from the definition of $g_i(p)$ that player i's optimal information interval vanishes as $\alpha_i \to 0$. Therefore, by setting

$$\underline{q} = \inf\{p_0 : p_0 \in I_1\}, \qquad \overline{q} = \sup\{p_0 : p_0 \in I_2\},$$

we obtain our desired result. ■

Proof of Proposition 5. We focus on player $i = 1$ without loss. For any q_1, p_H, and $p_L \in (0,1)$ such that $p_L \leq p_0 \leq p_H$, let

$$\tilde{v}_1(p_H, p_L, q_1) := r_H v_1(p_H) + (1-r_H)v_1(p_L) - k_1\Big(h(p_0) - r_H h(p_H) - (1-r_H)h(p_L)\Big)$$

denote player 1's payoff from employing a signal that indues $p = p_H$ and p_L respectively with probabilities r_H and $1 - r_H$, where $r_H = (p_0 - p_L)/(p_H - p_L)$ is pinned down by the constraint $E_\sigma[p] = p_0$. For any fixed $p \in [0,1]$, $v_1(p)$ weakly increases in q_1. Thus,

$$\frac{\partial \tilde{v}_1(p_H, p_L, q_1)}{\partial q_1} \geq 0 \quad \forall p_H, p_L, q_1, \tag{16}$$

where the inequality holds strictly whenever $p_H > p^*$ and $r_H > 0$.

The first statement in Proposition 5 follows from a slight modification of the envelope theorem. Fix $q_1 = q^*$ such that $\alpha_1 = \alpha_2$. Lemma 2 implies that the solution of Problem 6 are independent of β when $\alpha_1 = \alpha_2$. Further, if $p_0 \in (p_L(0.5), p_H(0.5))$ then the signal structure is non-degenerate. Thus, the first order conditions of Problem 6 are satisfied for both $\beta = 0$ and $\beta = 1$. In particular,

$$\frac{\partial \tilde{v}_1(p_H(0), p_L(0), q^*)}{\partial p_L} = \frac{\partial \tilde{v}_1(p_H(0), p_L(0), q^*)}{\partial p_H} = 0, \tag{17}$$

where $p_H(0)$ and $p_L(0)$ denote two posterior beliefs in the support of player 2's optimal signal $\sigma^*(0)$ (player 2's optimal signal). Let $U_1(q_1) := \tilde{v}_1(p_H(0), p_L(0), q_1)$ be player 1's payoff from $\sigma^*(0)$. Then,

$$U_1'(q^*) = \frac{\partial p_H(0)}{\partial q_1}\frac{\partial \tilde{v}_1}{\partial p_H} + \frac{\partial p_L(0)}{\partial q_1}\frac{\partial \tilde{v}_1}{\partial p_L} + \frac{\partial \tilde{v}_1}{\partial q_1},$$

where the right-hand side is evaluated at $(p_H, p_L, q_1) = (p_H(0), p_L(0), q^*)$. $U_1'(q^*) > 0$ by (16) and (17). Hence, with σ being fixed at player 2's optimal $\sigma^*(0)$, player 1 is strictly better off as q_1 increases. In the bargaining solution (10), player 1 must be at least as well off than $U_1(q_1)$ because $U_1(q_1) \leq u_1$ for any Pareto efficient and individually rational $(u_1, u_2) \in U$. Therefore, increasing q_1 makes player 1 strictly better off.

The second statement is immediate. In particular, $p_L(0) \to p^*$ as $q_1 \to 1$. Hence, $p_0 < p_L(0)$ if q_1 is sufficiently large, i.e., there is no integrative negotiation, and the player 1's payoff is w_1. ■

Proof of Proposition 6. See text. ■

Proof of Proposition 7. See text. ■

Proof of Proposition 8. Taking the derivative of the equations in (7) with respect to w_i yields

$$\begin{aligned}
&\frac{\partial G_\beta(p_L)}{\partial p}(p_H' - p_L') + \frac{\partial^2 G_\beta(p_L)}{\partial p^2} p_L'(p_H - p_L) \\
&\qquad = \frac{\partial G_\beta(p_H)}{\partial p} p_H' - \frac{\partial G_\beta(p_L)}{\partial p} p_L' - \beta_i \\
&\qquad = \frac{\partial G_\beta(p_H)}{\partial p}(p_H' - p_L') + \frac{\partial^2 G_\beta(p_H)}{\partial p^2} p_H'(p_H - p_L),
\end{aligned} \tag{18}$$

where p_L' and p_H' are the derivatives of p_L and p_H with respect to w_i. Observe that (7) implies $\frac{\partial G_\beta(p_L)}{\partial p} = \frac{\partial G_\beta(p_H)}{\partial p}$. Substituting this into (18) and solving for p_L' and p_H' yield

$$p_L' = -\frac{\beta_i}{(p_H - p_L)\frac{\partial^2 G_\beta(p_L)}{\partial p^2}}; \qquad p_H' = -\frac{\beta_i}{(p_H - p_L)\frac{\partial^2 G_\beta(p_H)}{\partial p^2}}.$$

Thus, p_L and p_H are strictly increasing in w_i. Further, the change in the length of the interval of integrative negotiation is given by

$$p_H' - p_L' = \frac{\beta_i}{p_H - p_L}\left(\frac{1}{\frac{\partial^2 G_\beta(p_L)}{\partial p^2}} - \frac{1}{\frac{\partial^2 G_\beta(p_H)}{\partial p^2}}\right). \tag{19}$$

Thus, the interval length is strictly increasing if $\frac{\partial^2 G_\beta(p_H)}{\partial p^2} > \frac{\partial^2 G_\beta(p_L)}{\partial p^2}$, and strictly decreasing otherwise. ■

Proof of Proposition 9. Recall that u^{NS} as the Nash bargaining solution without side payment as defined in (10). Then the analysis in Section 3 implies that whenever $u^{NS} > \underline{u}$, there exists $\beta^{NS} \in [0, 1]$ such that $u^{NS} = u^*(\beta^{NS})$ where $u^*(\beta^{NS})$ is the solution for (6) with $\beta = \beta^{NS}$. Since the boundary of U must be tangent to a Nash indifference curve at $u = u^{NS}$, the following first-order condition must be satisfied:

$$-\frac{q_1}{q_2}\frac{u_2^{NS} - \underline{u}_2}{u_1^{NS} - \underline{u}_1} = -\frac{\beta^{NS}}{1 - \beta^{NS}}. \tag{20}$$

Next, consider the case that side payments are available prior to the integrative phase. Note that the maximum attainable social payoff $\max_{u \in U} u_1 + u_2$ is equal to $u_1^*(0.5) + u_2^*(0.5)$, the solution for (6) with

$\beta = 1/2$. Therefore, the Nash bargaining solution with side payments, denoted by $u^S = (u_1^S, u_2^S)$, solves

$$\max_{(u_1,u_2)} (u_1 - \underline{u}_1)^{q_1}(u_2 - \underline{u}_2)^{q_2} \quad \text{s.t.} \quad u_1 + u_2 \leq u_1^*(0.5) + u_2^*(0.5).$$

If $u^S > \underline{u}$, the Nash indifference curve must be tanget to the line $u_1 + u_2 = u_1^*(0.5) + u_2^*(0.5)$ at $u = u^S$. Thus, the following first-order condition must hold:

$$-\frac{q_1}{q_2}\frac{u_2^S - \underline{u}_2}{u_1^S - \underline{u}_1} = -1. \tag{21}$$

The first-order condition (20) implies that

$$\frac{\beta^{NS}}{1-\beta^{NS}} = \frac{q_1}{q_2}\frac{u_2^{NS} - \underline{u}_2}{u_1^{NS} - \underline{u}_1} = \frac{M(\sigma^*(\beta^{NS})) - \frac{1}{\alpha_2}C(\sigma^*(\beta^{NS}))}{M(\sigma^*(\beta^{NS})) - \frac{1}{\alpha_1}C(\sigma^*(\beta^{NS}))},$$

where $M(\sigma) = (\theta_H - \theta_L)(E_\sigma[\max\{p - p^*, 0\}] - \max\{p_0 - p^*, 0\})$ and $C(\sigma) = h(p_0) - E_\sigma[h(p)]$. Therefore, $\beta^{NS} < 1/2$ if $\alpha_1 > \alpha_2$, $\beta^{NS} = 1/2$ if $\alpha_1 = \alpha_2$, and $\beta^{NS} > 1/2$ if $\alpha_1 < \alpha_2$.

If $\alpha_1 = \alpha_2$, then $\beta^{NS} = 1/2$, and comparing the two first-order conditions (20) and (21) implies that $u^{NS} = u^S$, and thus allowing side payment does not change the outcome.

Next, consider the case with $\alpha_1 \neq \alpha_2$. Assume without loss of generality that $\alpha_1 < \alpha_2$. Since $\beta^{NS} > 1/2$, it must be that $u_1^{NS} > u_1^*(0.5)$ and $u_2^{NS} < u_2^*(0.5)$. In this case, we finish our proof by showing that player 2 is strictly worse off with side payments, i.e., $u_2^S < u_2^{NS}$.

Suppose to the contrary that $u_2^S \geq u_2^{NS}$. Then it must be the case that the slope of the Nash indifference curve at $(u_1, u_2) = (u_1^*(0.5) + u_2^*(0.5) - u_2^{NS}, u_2^{NS})$ is weakly greater than -1, i.e.,

$$-\frac{q_1}{q_2}\frac{u_2^{NS} - \underline{u}_2}{u_1^*(0.5) + u_2^*(0.5) - u_2^{NS} - \underline{u}_1} \geq -1.$$

Plugging in (20) yields

$$-\frac{\beta^{NS}}{1-\beta^{NS}}\frac{u_1^{NS} - \underline{u}_1}{u_1^*(0.5) + u_2^*(0.5) - u_2^{NS} - \underline{u}_1} \geq -1,$$

which can be simplified to

$$\beta^{NS}(u_1^{NS} - \underline{u}_1) + (1-\beta^{NS})(u_2^{NS} - \underline{u}_2) \leq (1-\beta^{NS})\left(u_1^*(0.5) + u_2^*(0.5) - \underline{u}_1 - \underline{u}_2\right).$$

Since $1 - \beta^{NS} < 1/2 < \beta^{NS}$ and $u_1^*(0.5) > \underline{u}_1$,

$$\beta^{NS}(u_1^{NS} - \underline{u}_1) + (1-\beta^{NS})(u_2^{NS} - \underline{u}_2) < \beta^{NS}(u_1^*(0.5) - \underline{u}_1) + (1-\beta^{NS})(u_2^*(0.5) - \underline{u}_2).$$

However, this means that $u^*(0.5)$ strictly increases the objective of Problem 5 when $(\beta_1, \beta_2) = (\beta^{NS}, 1-\beta^{NS})$, a contradiction to $u^{NS} = u^*(\beta^{NS})$ being a solution of the optimization problem. ■

Appendix B Full Characterization of the Set of Feasible Payoffs

In this section, we provide a full characterization of the set of feasible payoff vectors U by deriving the solution of Problem 5 for a general vector $\vec{\beta} \in \mathbb{R}^2 \setminus \{0\}$.

Let $q(\vec{\beta}) = \beta_1 q_1 + \beta_2 q_2$, $k(\vec{\beta}) = \beta_1 k_1 + \beta_2 k_2$, and $\alpha(\vec{\beta}) = q(\vec{\beta})/k(\vec{\beta})$. Then Problem 5 is equivalent to

$$\max_{\sigma \in \Sigma} E_\sigma[q(\vec{\beta})(\theta_H - \theta_L) \max\{p - p^*, 0\} + k(\vec{\beta}) h(p)]. \tag{22}$$

Our analysis is divided into the following three cases: (1) $k(\vec{\beta}) > 0$, (2) $k(\vec{\beta}) < 0$, and (3) $k(\vec{\beta}) = 0$.

Case 1: *$k(\vec{\beta}) > 0$.* Dividing the objective function by $k(\vec{\beta})$, (22) becomes

$$\max_{\sigma \in \Sigma} E_\sigma[\alpha(\vec{\beta})(\theta_H - \theta_L) \max\{p - p^*, 0\} + h(p)], \tag{23}$$

Using a polar coordinate, we can rewrite $\vec{\beta} = (\beta_1, \beta_2) = (r \cos\varphi, r \sin\varphi)$ for $r > 0$ and $\varphi \in [0, 2\pi]$. The next lemma establishes monotonicity of $\alpha(\vec{\beta})$ in the rotation of $\vec{\beta}$.

Lemma 3 *Suppose that $k(\vec{\beta}) > 0$. Then $\alpha(\vec{\beta})$ increases (resp. decreases) in φ if $\alpha_1 < \alpha_2$ (resp. $\alpha_1 > \alpha_2$).*

Proof. Using a polar coordinate $\vec{\beta} = (r \cos\varphi, r \sin\varphi)$, for $r > 0$ and $\varphi \in [0, 2\pi]$, we can rewrite $\alpha(\vec{\beta})$ as

$$\alpha(\vec{\beta}) = \frac{q_1 \cos\varphi + q_2 \sin\varphi}{k_1 \cos\varphi + k_2 \sin\varphi}.$$

Taking a derivative with respect to φ yields

$$\frac{\partial \alpha(\vec{\beta})}{\partial \varphi} = \frac{k_1 q_2 - k_2 q_1}{(k_1 \cos\varphi + k_2 \sin\varphi)^2} = k_1 k_2 \frac{\alpha_2 - \alpha_1}{(k_1 \cos\varphi + k_2 \sin\varphi)^2},$$

showing the desired result. ■

Let $g(p; \alpha) = \alpha(\theta_H - \theta_L) \max\{p - p^*, 0\} + h(p)$ be the objective function in (23). Note that $g(p; \alpha)$ is piecewise concave with a kink at $p = p^*$. Therefore, our analysis of characterizing Pareto frontier (Problem 6) extends to this case. Specifically, we continue to utilize the "concavification" method as in Kamenica and Gentzkow (2011), and thus Proposition 1 still holds. Also, it is straightforward that as $\alpha(\vec{\beta})$ increases, the (relative) value of information increases, which leads to a wider interval of learning $(p_L(\alpha(\vec{\beta})), p_H(\alpha(\vec{\beta})))$. The next lemma summarizes this result.

Lemma 4 *Suppose that $k(\vec{\beta}) > 0$. Then there exist functions $p_L : \mathbb{R} \to (0, p^*]$ and $p_H : \mathbb{R} \to [p^*, 1)$ such that the optimal signal structure is binary if $p_0 \in (p_L(\alpha(\vec{\beta})), p_H(\alpha(\vec{\beta})))$, and degenerate otherwise. Further, p_L and p_H satisfy the following properties:*

1. *For* $\alpha \le 0$, $p_L(\alpha) = p_H(\alpha) = p^*$.

2. *For* $\alpha > 0$, $p_L(\cdot)$ *strictly decreases and* $p_H(\cdot)$ *strictly increases in* α.

3. $\lim_{\alpha\to\infty} p_L(\alpha) = 0$, *and* $\lim_{\alpha\to\infty} p_H(\alpha) = 1$.

Proof. Item 1 is straightforward since $g(p;\alpha)$ is globally concave for $\alpha \le 0$, and thus the concavification of $g(p;\alpha)$ is the same as itself. The proof of Item 2 is done by slightly modifying the proof of Lemma 2. the tangency conditions (7) can be rewritten as

$$\alpha W(p_H(\alpha)) + h(p_H(\alpha)) - h(p_L(\alpha)) = h'(p_L(\alpha))(p_H(\alpha) - p_L(\alpha)),$$
$$\alpha W(p_H(\alpha)) + h(p_H(\alpha)) - h(p_L(\alpha)) = \Big(\alpha W'(p_H(\alpha)) + h'(p_H(\alpha))\Big)(p_H(\alpha) - p_L(\alpha)).$$

Following the same procedure as that of the proof of Lemma 2, we take the derivatives of above equations with respect to α and solve the system of equations to obtain $p_L'(\alpha)$ and $p_H'(\alpha)$, given by

$$p_L'(\alpha) = \frac{W(p_H(\alpha))}{h''(p_L(\alpha))(p_H(\alpha) - p_L(\alpha))} < 0,$$
$$p_H'(\alpha) = \frac{\alpha(\theta_H - \theta_L)}{h''(p_H(\alpha))}\frac{p_L(\alpha) - p^*}{p_H(\alpha) - p_L(\alpha)} > 0.$$

Item 3 is a straightforward implication of the Inada condition on $h(p)$. ■

Lemmas 3 and 4 enable us to characterize the optimal signal $\sigma^*(\vec{\beta})$ for any $\vec{\beta}$ that satisfies $k(\vec{\beta}) > 0$. Given a prior belief $p_0 \in (0,1)$, define

$$\alpha^* = \inf\{\alpha : p_0 \in (p_L(\alpha), p_H(\alpha))\}$$

Lemma 4 implies that $\alpha^* \in [0,\infty)$. Then $\sigma^*(\vec{\beta})$ is either degenerate or binary; more specifically,

$$\mathrm{Supp}(\sigma^*(\vec{\beta})) = \begin{cases} \{p_0\} & \text{if } \alpha(\vec{\beta}) \le \alpha^*, \\ \{p_L(\alpha), p_H(\alpha)\} & \text{if } \alpha(\vec{\beta}) > \alpha^*. \end{cases}$$

Case 2: $k(\vec{\beta}) < 0$. In this case, by dividing the objective function by by $k(\vec{\beta})$, (22) is equivalent to

$$\min_{\sigma\in\Sigma} E_\sigma[\alpha(\vec{\beta})(\theta_H - \theta_L)\max\{p - p^*, 0\} + h(p)], \tag{24}$$

which is a minimization problem of the expectation of a piecewise concave function. Let σ_f be a full information signal (i.e., $\mathrm{Supp}(\sigma_f) = \{0,1\}$), and let σ_w be the most costly signal that yields zero information value, i.e.,

$$\mathrm{Supp}(\sigma_w) = \begin{cases} \{0, p^*\} & \text{if } p_0 < p^*, \\ \{p^*\} & \text{if } p_0 = p^*, \\ \{p^*, 1\} & \text{if } p_0 > p^*. \end{cases}$$

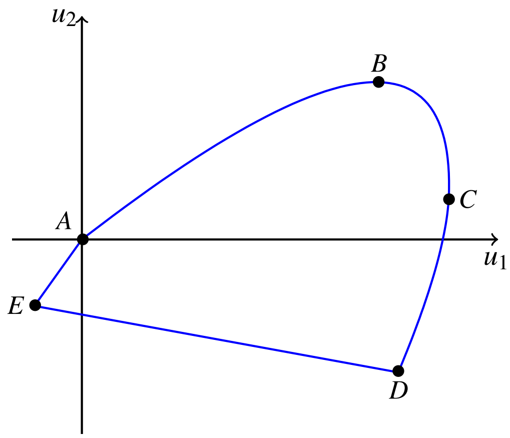

Figure 7: An illustration of the feasible set U for the case where $\alpha_1 > \alpha_2$. Points $A-E$ denote the following: $A = \underline{u}$; $B = u(\sigma^*(0,1))$; $C = u(\sigma^*(1,0))$; $D = u(\sigma_f)$; $E = u(\sigma_w)$;

From the concavification argument, it is clear that the solution to (24) must be either σ_f or σ_w. Then it remains to analyze when each signal structure is optimal. Under σ_f, the objective function becomes

$$p_0\alpha(\vec{\beta})(\theta_H - \theta_L)(1-p^*) + p_0 h(1) + (1-p_0)h(0),$$

and under σ_w, the objective function is $\frac{p_0}{p^*}h(p^*) + \frac{p^*-p_0}{p^*}h(0)$ for $p_0 < p^*$ and $\frac{p_0-p^*}{1-p^*}\alpha(\vec{\beta})(\theta_H - \theta_L)(1-p^*) + \frac{p_0-p^*}{1-p^*}h(1) + \frac{1-p_0}{1-p^*}h(p^*)$ for $p_0 > p^*$. Comparing the two cases yields that

$$\sigma^*(\vec{\beta}) = \begin{cases} \sigma_f & \text{if } \alpha(\vec{\beta}) > \hat{\alpha}, \\ \sigma_w & \text{if } \alpha(\vec{\beta}) < \hat{\alpha}, \end{cases}$$

where

$$\hat{\alpha} = \frac{h(p^*) - (p^* h(1) + (1-p^*)h(0))}{(\theta_H - \theta_L)p^*(1-p^*)} > 0.$$

If $\alpha(\vec{\beta}) = \hat{\alpha}$, any convex combination of σ_f and σ_w is optimal.

Case 3: $k(\vec{\beta}) = 0$**.** In this case, it is straightforward that the optimal signal $\sigma^*(\vec{\beta})$ is σ_f if $q(\vec{\beta}) > 0$ and is degenerate at p_0 if $q(\vec{\beta}) < 0$.

The above analysis completely characterizes boundary of U. For each $\vec{\beta}$, let $u(\sigma) = (u_1(\sigma), u_2(\sigma))$ be a vector of players' payoffs under σ. Then U is the convex hull of $u(\sigma^*(\vec{\beta}))$ for all $\vec{\beta} \in \mathbb{R}^2 \backslash \{0\}$.

Figure 7 illustrates U for the case where $\alpha_1 > \alpha_2$. A few points are worth noting. First, if $k(\vec{\beta}) > 0$, Lemma 3 implies that $\alpha(\vec{\beta})$ decreases in φ. In this case, as φ decreases (i.e., $\vec{\beta}$ rotates clockwise), $\alpha(\vec{\beta})$ increases, and the players' payoffs follow a strictly convex curve from point A (payoffs under no information) to point D (full information). Second, the Pareto frontier of U is the line between point B (seller's optimum) and C (buyer's optimum). Lastly, when $k(\vec{\beta}) < 0$, the payoffs under $\sigma^*(\vec{\beta})$ are either point D (payoffs under σ_f), point E (payoffs under σ_w), or convex combination of the two.