

# The Simple Analytics of Informed Finance\*

Dan Bernhardt and Stefan Krasa  
Department of Economics  
University of Illinois  
Champaign IL 61820  
<http://www.staff.uiuc.edu/~skrasa>  
danber@uiuc.edu

July 15, 2004

## Abstract

The paper derives two analytical consequences of informed finance: Equity leads to under-financing, while debt leads to over-financing. We show that our model can explain key qualitative and quantitative features of informed venture capital finance in the United States. Using only three model parameters we match: (1) the venture capitalist's equity share; (2) the venture capitalist's expected return and (3) its standard deviation; (4) the probability that a project receives funding; and (5) the probability the venture capitalist loses money on an investment. We then use the model to make predictions about features that cannot be readily identified from the data, such as the magnitude of under-financing with equity, and the surplus from using equity instead of debt.

---

\*Dan Bernhardt gratefully acknowledges financial support from NSF grant SES-0317700 . Stefan Krasa gratefully acknowledges financial support from NSF grant SES-0318394. We are grateful to Per Stromberg for helping us uncover the relevant empirical regularities. We thank John Cochrane for giving us access to his data and appreciate the guidance that he provided us. We also thank Burton Hollifield, George Pennacchi, Jano Zabojsnik and seminar participants at the University of Illinois. We are responsible for all errors.

# 1 Introduction

Our paper derives the simple analytical consequences of two well-established facts about venture capital finance. First, venture capitalists are better judges than entrepreneurs of a project's economic viability (see Garmaise [12], and references therein). Second, entrepreneurial projects require significant sweat equity from key personnel.<sup>1</sup>

We derive the qualitative and quantitative implications of these two facts. We first suppose that the financier is a venture capitalist who provides the capital in exchange for an equity share of the project. We show that venture-capital finance leads to under-financing. That is, a venture capitalist who is compensated with equity will fail to fund some projects that he expects to have a positive NPV. Here, a project has a positive NPV if, post-investigation, project payoffs are expected to cover the opportunity cost of the capital investment plus the entrepreneur's sweat equity.

This raises a basic question: If equity leads to under-financing, why not use debt instead? We show that debt may be unattractive for the opposite reason: debt leads to over-financing. That is, an informed lender will fund some projects whose payoffs he expects will cover his opportunity cost of capital, but will not fully compensate the entrepreneur and key personnel for their sweat equity.

We then show that these simple analytics can explain the key qualitative *and* quantitative features of venture capital finance in the United States. We also offer insights into why informed equity finance is primarily used in the United States, but debt is the main instrument of informed finance in Asia and Europe.

In our model, an entrepreneur has a project that requires external capital. The entrepreneur understands that an informed financier serves two key roles. First, the financier provides the required capital. Second, the financier investigates and filters entrepreneurial projects.<sup>2</sup> Following an investigation, the financier will fund a project that seems promising and discard a project whose prospects he deems poor. The entrepreneur recognizes that while he may have a potentially valuable invention, he may not be good judge of its implementability or market value. This reflects that in practice, most entrepreneurs are associated with only a few projects. In contrast, venture capitalists have extensive industry experience. Their extreme specialized knowledge permits venture capitalists to distinguish winners from losers (Fenn, Liang and Prowse [10]). Indeed, venture capitalists scrutinize serious projects intensively (Fried and Hisrich [11], Garmaise [12], Kaplan and Stromberg [15]), and reject about 90 percent of those that they investigate seriously.

Consider what happens if equity finance is pursued. At a minimum, for a venture capitalist to provide

---

<sup>1</sup>Sweat equity consists largely of the significant under-compensated time input that key personnel must put into a new project. The personnel are compensated for their efforts if the project succeeds, e.g., via stock and stock options.

<sup>2</sup>Kaplan and Stromberg [15] document empirically the significant time and effort spent by venture capitalists to evaluate and screen investment opportunities.

funding, the venture capitalist's share of the project's expected payoff conditional on his investigation must cover the venture capitalist's opportunity cost of capital. The entrepreneur understands this at the stage when he negotiates prospective equity terms. We show that equity leads to significant under-financing of good projects. Indeed, in order to increase his chances of funding, the entrepreneur willingly cedes an equity share that generates a significant excess return for the venture capitalist. That is, the equity share more than covers the opportunity cost of the venture capitalist's capital investment plus his investigation costs.

To understand the source of under-financing, recognize that the venture capitalist expects the payoffs from the marginal project that he funds to just cover his opportunity cost of capital. Ignoring investigation costs, the venture capitalist will therefore make money on all better projects. If the investigation costs are not too high and the proposed equity terms do not give the venture capitalist an excess return, then it necessarily follows that the marginal project funded must be close to the median project funded. The marginal project must therefore be very good, implying that many positive NPV projects remain unfunded. Recognizing this, the entrepreneur will negotiate equity terms that trade off between raising the probability of getting funded against ceding too much on very good projects.

We then calibrate our model to investigate its quantitative implications. The data pin down closely all but the the mean and variance of ex-ante project payoffs and the share of total project investment that is sweat equity. A back of the envelope calculation indicates that sweat equity is in the range of 50 percent of total value of the investment. We show that with the two remaining free parameters the equilibrium of our model does a remarkable job of matching five key variables characterizing venture capital finance: the venture capitalist's equity share of a funded project, the mean and standard deviation of the venture capitalist's return, the probability a project is funded, and the probability the venture capitalist loses money on his investment. For example, we predict the venture capitalist's equity share to be 43.7% versus 43-44% in the data (Kaplan and Stromberg [16]) and generate excess returns of 22% versus the 24% that Ljungqvist and Richardson [18] find. None of our predicted values differs substantially from their empirical counterparts. Importantly, the model's predictions are extremely robust to large variations in the values of all key parameters, and not just to variations in the mean and variance of ex-ante project payoffs.

Because our quantitative model robustly matches the key empirical features of venture capital finance that can be documented, our model can be used to glean insights into quantitative predictions about parameters that cannot be directly observed, and for which no data are available. In particular, we predict that with venture capital, only 38% of all positive NPV projects are funded. That is, we predict that venture capital gives rise to significant under-financing. We also get a measure of the percentage of ex-ante project surplus (i.e., accounting for the costs of investigating both funded and unfunded projects) that accrues to the venture capitalist. For reasonable investigation costs, this percentage ranges between 15 and 20 percent. This is

quite a large number, as it is pure rent in excess of venture capitalist's cost of capital and investigation costs. Interestingly enough, our model predicts that the venture capitalist's filtering raises the mean project return significantly, but the standard deviation of payoffs is only 25 percent less than that of unscreened projects. This high standard deviation and our model estimate that venture capitalists lose money on 33 percent of funded projects highlight the significant unresolved risk following a venture capitalist's investigation.

Finally, we provide general conditions under which informed debt gives rise to over-financing. To understand what drives this over-financing, first suppose that a lender's investigation *perfectly* reveals the project's payoff. Because debt gives the lender first priority, the lender funds a project if and only if the project payoff exceeds his opportunity cost of capital. That is, the lender does not internalize the fact that project payoffs may only partially compensate key personnel for their sweat equity. As a consequence, some projects that have a negative NPV after accounting for sweat equity are funded. Now suppose that the lender's investigation only resolves some uncertainty. This causes the lender to be more cautious in his lending decisions, mitigating, but not eliminating, the over-financing. To minimize over-financing, an entrepreneur will negotiate the lowest interest rate that is acceptable to the lender. Our quantitative model is calibrated to United States data, and predicts that over-financing with informed debt would be substantial—we predict that about 31 percent of all debt-funded projects would have a negative NPV. According to our model, debt would give rise to over three times as many projects being funded as equity.

For reasonable levels of investigation costs, we predict that for the United States equity finance increases total surplus relative to debt by between 8 and 12 percent. Thus, our model can reconcile why it is in the United States that equity is the most preferred form of informed finance.

Qualitatively, our model suggests that the mean of the ex-ante distribution of project payoffs reflects a negative NPV—most projects must be bad, and many of these would be financed by debt—but there is also a fat upper tail of very good projects that would be financed with equity. Still, our theoretical analysis provides insights into those settings for which debt becomes more attractive. Qualitatively, if the distribution of project payoffs places little probability weight on slightly negative NPV projects, then debt may dominate. Also, projects that require less sweat equity from the entrepreneur may be good targets for debt finance. Further, if there is more unresolved uncertainty in the economy—say because the economy is less-developed than that in the United States—then again debt may be more attractive. For example, the empirical findings of Hege, Palomino and Schwienbacher [14] suggest that in the US venture capitalists are better at screening projects than European Venture capitalists. Also, poorer property rights enforcement in emerging markets, Asia and Europe documented by Laporta et al. [17] and Beim and Calomiris [3] gives debt a further edge over equity. Combining these factors with our model can explain why informed debt dominates informed equity in these countries, and can explain the magnitude of underperforming loans in those countries.

## 1.1 Related Literature

There is an enormous literature that documents the apparently excessive returns that venture capitalists make (see, e.g., Cochrane [8] or Ljungqvist and Richardson [18]). Accepted wisdom is that these returns are hard to reconcile theoretically in a setting where venture capitalists do not have monopoly power. Ljungqvist and Richardson [18] suggest that this excess return is a premium for the illiquidity of the investment. Here, we show that even though an entrepreneur *could* negotiate competitive equity terms that just cover the venture capitalist's opportunity cost of capital plus investigation costs, the entrepreneur will *choose* to give the venture capitalist a share that is so much greater that we almost match the empirically-observed excess return.

Other models of entrepreneurial finance in which a venture capitalist can become better informed than the entrepreneur include Biais and Perotti [6], Ambec and Poitevin [2], Ueda [22], Bernhardt and Krasa [5] and Garmaise [12]). In the first three papers, the venture capitalists are better able to assess and implement a project, but the entrepreneur has to worry about stealing/free-riding on his idea/information. In contrast, in Bernhardt and Krasa, it is the informed financier (the expert) who is concerned about free-riding by uninformed potential investors.

There is also a large theoretical literature that introduces incomplete contracts and moral hazard of various forms to explain the choice of control right assignments and the division of cash flows. Representative papers include Aghion and Bolton [1], Dewatripont and Tirole [9], Hart and Moore [13], Repullo and Suarez [21], Bergemann and Hege [4], and Winton and Yerramilli [23]. Our paper shares the feature that the interests of the entrepreneur may not be aligned with the financier's, but in contrast to this literature, we do not introduce moral hazard, and control rights play no role. The problem with introducing moral hazard is that it can neither be measured nor calibrated readily. The purpose of this paper is to derive the qualitative and quantitative effects of informed debt and equity finance, the contracts that closely mirror those used in practice. Accordingly, we develop a sparsely-specified model with minimal free parameters and focus on pure debt and equity contracts, rather than specifying all primitive frictions and characterizing the fine details of the optimal contract.<sup>3</sup>

## 2 The Model

Consider a potential entrepreneur with a project. The project requires  $w$  units of the entrepreneur's sweat equity and  $1 - w$  units of external finance to generate output  $X + Y$ , where  $X$  and  $Y$  are independent random variables. Without loss of generality we normalize  $Y$  to have mean zero. We denote realizations of  $X$  and  $Y$

---

<sup>3</sup>In the context of our simple model, an optimal contract would be a franchise contract in which the financier pays the entrepreneur a fixed wage and is a residual claimant. This contract does not work in practice because it does not provide the entrepreneur the incentives to behave responsibly.

by  $x$  and  $y$ , respectively. We assume that  $X$  is distributed according to the density  $f(x)$  and  $Y$  is distributed according to the density  $g(y)$ . The associated cdfs are  $F(x)$  and  $G(y)$ . We assume that both  $f(x)$  and  $g(y)$  are strictly positive on their (possibly unbounded) supports. At a cost of  $c > 0$ , a financier can investigate the project and learn the realization  $x$  of  $X$ , thereby reducing the uncertainty about the project's payoff to  $x + Y$ . We assume that there are realizations  $x_1$  and  $x_2$  with  $f(x_1), f(x_2) > 0$  such that  $x_1 > 1 + r_i > x_2$ , where  $r_i$  is the risk-adjusted required expected rate of return for the venture capitalist's funds. This condition ensures that post-investigation some, but not all, projects have positive NPVs. This is a necessary condition for investigation to have value. The opportunity cost to the financier of investing  $(1 - w)$  in the project is  $(1 + r_i)(1 - w)$ .

If the project is not funded then the entrepreneur's payoff is normalized to zero. If the project is funded, the entrepreneur incurs an opportunity cost of  $(1 + r_i)w$  for providing sweat equity.<sup>4</sup> If the entrepreneur pursues his project then he can finance it with either debt or equity. If a debt contract with interest rate  $r$  is used, the financier is a lender who receives  $\min\{x + y, (1 + r)(1 - w)\}$ , while the entrepreneur receives  $x + y - \min\{x + y, (1 + r)(1 - w)\}$ . If, instead, an equity contract with share  $k$  is used, the financier is a venture capitalist who receives  $k(x + y)$ , while the entrepreneur receives  $(1 - k)(x + y)$ .

If an entrepreneur pursues funding, he first chooses whether to seek debt or equity finance. With debt finance, the entrepreneur proposes the interest rate  $r$ ; and with equity finance, the entrepreneur proposes the equity share  $k$  that the financier would receive. If informed finance is pursued, the terms must provide the financier an ex ante payoff of at least  $\bar{u} \geq 0$  net of information acquisition costs. We introduce  $\bar{u}$  to capture the possibility that the financier has some market power and can extract rents. When  $\bar{u} = 0$ , financial markets are perfectly competitive—the entrepreneur can play off potential investors against each other—in which case only the financier's opportunity cost of capital and investigation costs need be covered.

If the project is financed then these proposed terms or “term sheets” determine the funding conditions. This assumption captures standard industry practice (see Kaplan and Stromberg [15]). The term sheet is a summary of the terms and conditions that will apply if the venture capitalist and entrepreneur consummate their agreement. Obviously, funding is contingent on a positive project evaluation by the venture capitalist following his thorough investigation.

### **Timing of Decisions.**

**t=1** If the entrepreneur seeks finance then he proposes either a debt contract with interest rate  $r$  or an equity contract with share  $k$ .

---

<sup>4</sup>For simplicity, we assume that the entrepreneur and financier share a common discount factor. Qualitatively, none of our findings depend on this assumption.

**t=2** If finance is sought, the financier decides whether or not to investigate the project.

**t=3** If the financier investigates the project at  $t = 2$ , at the cost  $c$  he learns the realization  $x$  of  $X$ . The financier decides whether to accept or reject the funding terms that the entrepreneur proposed.

**t=4** If the project is funded, payoffs are realized and payments made according to the contract set at date one. If the project is not funded, the entrepreneur's payoff is zero, and the financier's payoff is  $-c$ .

### 3 Equity Finance

Suppose that the entrepreneur proposes equity terms that makes it worthwhile for the venture capitalist to investigate. Then, given a proposed share  $k$ , the venture capitalist will extend finance if and only if his expected payoff after learning  $x$  exceeds his opportunity cost of providing funds, i.e.,

$$E[k(x + Y)] \geq (1 + r_i)(1 - w).$$

Thus, funding will be extended if and only if  $x \geq x_E$ , where

$$x_E = \frac{(1 + r_i)(1 - w)}{k}. \quad (1)$$

The entrepreneur understands that the equity terms affect  $x_E$ , and hence the set of projects that the venture capitalist is willing to fund. The entrepreneur also understands that the venture capitalist will only investigate if it is in the venture capitalist's interest to do so. Therefore, if the entrepreneur seeks informed equity finance, then the proposed equity share will solve the following optimization problem.

#### Problem 1

$$\max_{k \in [0,1]} P(\{X \geq x_E\}) \left( E[(1 - k)(X + Y) | X \geq x_E] - w(1 + r_i) \right).$$

subject to

$$P(\{X \geq x_E\}) E[k(X + Y) - (1 + r_i)(1 - w) | X \geq x_E] - c \geq \bar{u}; \quad (2)$$

$$P(\{X \geq x_E\}) E[k(X + Y) - (1 + r_i)(1 - w) | X \geq x_E] - c \geq E[k(X + Y) - (1 + r_i)(1 - w)]. \quad (3)$$

The objective is the entrepreneur's ex-ante expected payoff given share  $k$ . Funding is offered with probability  $P(\{X \geq x_E\})$ , and  $E[(1 - k)(X + Y) | X \geq x_E] - w(1 + r_i)$  is the entrepreneur's expected payoff from a funded project. Constraint (2) says that the venture capitalist's equity share  $k$  must provide him a net expected payoff of at least  $\bar{u}$ . Constraint (3) says that it is in the venture capitalist's interest to investigate the project: the left-hand-side is his payoff from investigating, while the right-hand-side is his payoff from funding a project without an investigation.

We now show that under innocuous conditions, equity finance gives rise to under-financing. That is, the venture capitalist will not fund some projects that have a positive marginal social value. To increase the chance of funding, the entrepreneur willingly proposes to give the venture capitalist an equity share that generates a strictly positive excess return over and above the venture capitalist's required return.

**Proposition 1** *If  $c$  and  $\bar{u}$  are not too large then constraints (2) and (3) of Problem 1 do not bind and there is under-financing i.e.,  $x_E > 1 + r_i$ .*

The proof details the precise bounds on  $c$  and  $c + \bar{u}$  for the constraints to be slack, so that the venture capitalist's ex-ante expected profit strictly exceeds  $\bar{u}$ . Because the entrepreneur proposes the contract terms, one might expect that the entrepreneur would select a share that gives the venture capitalist only the minimum required return,  $\bar{u}$ . Proposition 1 shows that this is not the case.

To understand the intuition, recognize that from the point of view of the venture capitalist, the equity contract is a call option with strike price  $x_E$  that he buys at a price equal to his investigation cost,  $c$ . This is because the venture capitalist provides funding if and only if  $x \geq x_E$ . The payoff of the option is always strictly positive. If  $c$  and  $\bar{u}$  are small, then for the venture capitalist not to receive a profit in excess of  $\bar{u}$ , the strike price  $x_E$  must be close to the maximal project realization, which means that the option is almost never exercised. Rather than have a tiny chance of funding, the entrepreneur willingly cedes a larger share to the venture capitalist, raising  $k$  above the venture capitalist's break-even point. This lowers the strike price  $x_E$ , which increases the entrepreneur's probability of being funded. However, increasing  $k$  also transfers more surplus to the venture capitalist, particularly for outstanding projects. The optimal share trades off between these two factors.

To understand why under-investment occurs, note that it is socially efficient to fund any project with an expected return of at least  $(1 + r_i)$ . But as long as the constraints do not bind, the entrepreneur never gives up all of the surplus so that his return strictly exceeds  $w(1 + r_i)$ . The financier only funds a project if he expects a payoff that covers the opportunity cost of his funds  $(1 - w)(1 + r_i)$ . Adding, it follows that the expected payoff on the marginal project strictly exceeds  $(1 + r_i)$ , which implies that there is under-financing.

Indeed, under-financing typically results *even* if  $c + \bar{u}$  is large enough for the constraints to bind. In particular, the financier breaks even on the marginal product gross of costs  $c$ , while the minimum-payment constraint 2 that determines  $k$  reflects an average over all projects better than the marginal project. As long as  $c + \bar{u}$  is not inordinately large, the entrepreneur's equity share  $1 - k$  will exceed his sweat equity investment  $w$ . As a result, the marginal project will have a strictly positive NPV.

Further, our calibration analysis will reveal that for relevant parameterizations both constraints of Problem 1 are slack.



## 4 Debt Finance

Now suppose that the entrepreneur proposes debt finance terms that make it worthwhile for a lender to investigate the project. As with informed equity, the lender will offer funding after learning realization  $x$  if and only if he covers his opportunity cost of funds,  $(1 + r_i)(1 - w)$ . Formally, a lender will extend funding if and only if  $x \geq x_D$ , where  $x_D$  solves

$$E[\min\{x_D + Y, (1 + r)(1 - w)\}] = (1 + r_i)(1 - w). \quad (4)$$

The interest rate,  $r$ , offered by the entrepreneur therefore solves the following optimization problem:

### Problem 2

$$\max_r P(\{X \geq x_D\}) \left( E\left[ X + Y - \min\{X + Y, (1 + r)(1 - w)\} \mid X \geq x_D \right] - w(1 + r_i) \right)$$

subject to

$$P(\{X \geq x_D\}) E\left[ \min\{X + Y, (1 + r)(1 - w)\} - (1 + r_i)(1 - w) \mid X \geq x_D \right] - c \geq \bar{u}; \quad (5)$$

$$\begin{aligned} P(\{X \geq x_D\}) E\left[ \min\{X + Y, (1 + r)(1 - w)\} - (1 + r_i)(1 - w) \mid X \geq x_D \right] - c \\ \geq E\left[ \min\{X + Y, (1 + r)(1 - w)\} - (1 + r_i)(1 - w) \right]. \end{aligned} \quad (6)$$

The objective is the entrepreneur's ex-ante expected payoff from an interest rate  $r$ . The interest rate has two effects on the entrepreneur's payoff. The interest rate directly affects the amount of money,  $\min\{x + Y, (1 + r)(1 - w)\}$ , that the entrepreneur must repay. Further, the interest rate indirectly affects payoffs through its impact on the marginal-project funded,  $x_D$ , and hence on the funding probability,  $P(\{X \geq x_D\})$ .

Constraints (5) and (6) are analogues of constraints (2) and (3) in problem 1. Constraint (5) ensures that the informed lender's expected payoff net of his investigation cost  $c$  is at least  $\bar{u}$ . Constraint (6) ensures that it is optimal for the lender to investigate the project.

We now provide general conditions under which debt gives rise to over-financing. That is, with debt, an informed lender extends funding to negative NPV projects. While the lender expects to cover his capital costs on these projects, the entrepreneur does not expect to cover his sweat equity investment  $(1 + r_i)w$ , and hence would prefer not to have these negative NPV projects funded.

**Proposition 2** *Suppose that  $Y$  has support  $[\underline{y}, \bar{y}]$ . Then a weak sufficient condition for informed debt to give rise to over-financing, i.e.  $x_D < 1 + r_i$ , is that  $-\underline{y} < w(1 + r_i)$ . In this instance, an informed lender funds negative NPV projects that the entrepreneur would prefer to remain unfunded.*

To understand why debt gives rise to over-financing, suppose that the lender can evaluate a project perfectly, i.e.,  $Y \equiv 0$ . Then equation (4) immediately implies that  $x_D = (1 + r_i)(1 - w) < (1 + r_i)$ , so that projects with payoffs  $E[x + Y] \in [(1 + r_i)(1 - w), (1 + r_i)]$  are funded to the entrepreneur's detriment. Suppose, instead, that  $Y$  is non-degenerate so that there is some project uncertainty that the lender cannot resolve. Then the lender will increase  $x_D$ , reducing the over-financing. However, unless the unresolved uncertainty is inordinately large or  $w$  is very small, it is still the case that  $x_D < (1 + r_i)$ .

Our calibration analysis reveals that for any reasonable parameterization, informed debt are far too willing to fund projects that should not receive finance: Even when a lender's investigation only resolves half of all uncertainty, about one-third of projects that would receive debt finance have negative expected NPVs.

We next show that to reduce this over-financing, the entrepreneur will select an interest rate that provides the lender his minimum required payoff,  $\bar{u}$ .

**Proposition 3** *Suppose that informed debt gives rise to over-financing. Then if the entrepreneur pursues debt finance, he will choose the interest rate that gives the lender an expected payoff net of his investigation costs of  $\bar{u}$ .*

The intuition is simple. Reducing  $r$  both reduces the entrepreneur's payment to the investor, and decreases over-financing by increasing  $x_D$ . As a result, the entrepreneur will choose  $r$  as low as possible, so that the lender receives  $\bar{u}$ . We next show that increasing the intrinsic uncertainty that is not resolved by a lender's project evaluation, reduces the magnitude of over-financing.

**Proposition 4** *Let  $Y$  be a mean-preserving spread of  $\tilde{Y}$ . Suppose that informed debt gives rise to over-financing when the unresolved uncertainty is  $Y$ . Then, raising the unresolved uncertainty from  $\tilde{Y}$  to  $Y$  reduces over-financing (i.e., raises  $x_D$ ), and raises both the entrepreneur's payoff and the interest rate.*

Due to the concavity of the debt function, the lender responds to increased unresolved uncertainty about a project by funding fewer negative NPV projects. Because over-financing is reduced and both parties are risk neutral, it follows that adding mean zero uncertainty makes the entrepreneur better off. If, instead, we fix the total project uncertainty, and increase the unresolved uncertainty, then, in general, the entrepreneur is made worse off, as the lender learns less from his investigation. For example, if  $X \equiv E[X]$  and  $Y$  contains all project uncertainty, then investigating the project is pointless, and the entrepreneur is worse off relative to a situation in which information about the project can be acquired.

Next, we detail how over-financing is affected by key parameters of the economy.

**Proposition 5** *Suppose that informed debt gives rise to over-financing. Then the following parameter changes reduce the quality of the marginal-funded project,  $x_D$ , and hence increase over-financing:*

1. *reducing the risk adjusted rate of interest,  $r_i$ ;*
2. *increasing the entrepreneur's sweat equity,  $w$ ;*
3. *increasing the cost of investigation,  $c$ ;*
4. *increasing the lender's bargaining strength,  $\bar{u}$ .*

Reducing  $r_i$  lowers the lender's opportunity costs of funds, which raises the attractiveness of funding any project. As a consequence, the marginal funded project must have a lower return, which means that  $x_D$  falls. Similarly, raising the entrepreneur's sweat equity  $w$  raises over-financing, because the lender does not internalize  $w$  in his funding decision. Both raising  $\bar{u}$  and raising  $c$  increase the equilibrium interest rate that the lender receives, making him more willing to fund marginal projects.

#### 4.1 Debt versus Equity

Propositions 1 and 2 reveal that informed equity and informed debt give rise to very different distortions from the entrepreneur's perspective. Informed debt leads to over-financing, because the lender does not internalize the entrepreneur's sweat equity,  $w$ . Hence, the lender funds projects for which the entrepreneur's expected return is less than  $w(1 + r_i)$ . In contrast, informed equity/venture capital leads to under-financing, because the venture capitalist does not internalize the entrepreneur's share of the project's payoff when deciding whether or not to provide finance. Further, to reduce the magnitude of under-financing associated with equity finance, the entrepreneur willingly gives the venture capitalist an equity share that more than compensates for his cost of capital.

The question then becomes: when would the entrepreneur prefer informed debt finance, and when would venture capital finance be more attractive? Qualitatively, the answer is clear. If there is substantial probability mass on marginally negative NPV projects,  $x \in [x_D, (1 + r_i)]$ , then informed debt finance gives rise to significant over-financing, making informed equity more attractive. The density  $f(x)$  in Figure 1 illustrates such a project distribution. Analogously, if there is substantial probability mass on projects  $x \in [(1 + r_i), x_E]$ , then informed equity finance gives rise to significant under-financing, which makes debt more attractive. Qualitatively, if a project has a small positive ex ante NPV, as density  $\tilde{f}(x)$  in Figure 1 illustrates, then debt finance dominates. However, if the project is quite risky, with a negative ex ante NPV, then, as density  $f(x)$  illustrates, the over-financing associated with debt would be large relative to the under-financing costs associated with equity, and hence venture capital finance will tend to dominate. Finally, if projects are likely

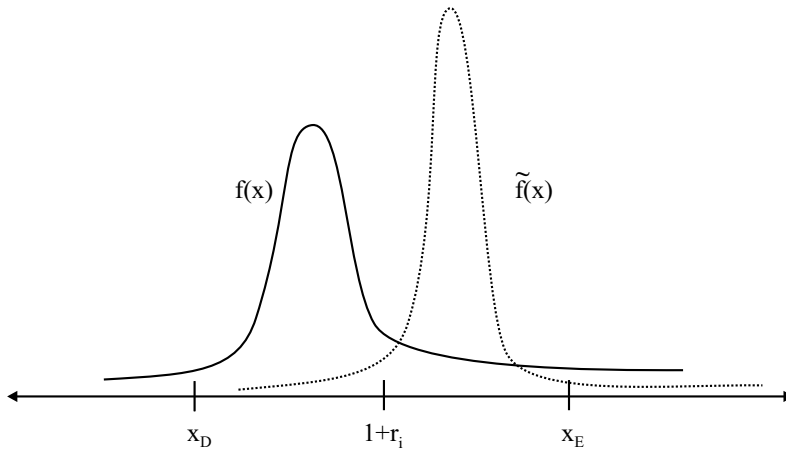


Figure 1: Comparing Debt and Equity Finance

to have a positive ex ante NPV and investigation costs are large relative to the capital required for finance, then it may be optimal to pursue uninformed finance.

Observe also that increasing unresolved uncertainty about a project has no effect on the venture capitalist’s funding decisions (because of risk neutrality), and hence on attractiveness of equity. However, Proposition 4 reveals that increasing unresolved uncertainty raises  $x_D$ , and hence the attractiveness of informed debt. Hege, Palomino and Schwienbacher [14] provide evidence that a financier’s screening in the U.S. resolves more uncertainty than in Europe. This could be one of the reasons why venture capital finance is so prevalent in the U.S., while informed debt finance is used in Europe.

## 5 Calibration Analysis

Our model has six parameters:  $\mu$ ,  $\sigma$ ,  $w$ ,  $r_i$ ,  $c$  and  $\bar{u}$ . The data pin down  $w$ ,  $r_i$  and  $c$ . Further neither  $c$  nor  $\bar{u}$  affect the predictions of equity finance. This effectively leaves us with two free parameters—the mean and variance of ex-ante project returns. With these two parameters, we match five moments that characterize the key features of venture capital finance: the venture capitalist’s equity share, the mean and standard deviation of the venture capitalist’s returns from funded projects, the probability that a project is funded, and the probability that the venture capitalist loses money.

We now detail how we derive these key moments from the data.

**VC’s equity share:** Kaplan and Stromberg [16] have a data set with 213 investments by 14 venture capital firms. The data include the contractual agreements governing each financing round in which the firm participated. In practice, the venture capitalist’s share is contingent on performance: if the firm does well, then the founders receive stock options that dilute the venture capitalist’s share. Kaplan and Stromberg find a median

*minimum* share of 41% for first round finance (i.e., the share the venture capitalist receives when the firm meets all performance standards), and a median *maximum* share of 50.5% for first round finance. Because the venture capitalist's share is less when the firm does well, this suggests an average venture capital share of 43-44%. Although venture capital contracts are only approximately linear,<sup>5</sup> our abstraction of a perfectly linear contract does a remarkable job of explaining the data.

**Mean VC Return:** Ljungqvist and Richardson [18] have a data set that contains the exact timing of investments and distribution of cash flows for 73 private equity funds and hence are free of sample selection bias (see Cochrane [8]). They find that “on a risk-adjusted basis, the excess value of the typical private equity fund is on the order of 24 percent relative to the present value of the invested capital.”

**Standard Deviation of VC return:** Cochrane [8] computes the standard deviation of the financier's return per unit capital invested for different financing rounds. His estimated standard deviation for the first round of finance, which is the appropriate measure for our model, is 120%. This is slightly higher than the standard deviation across all rounds of funding which is 107%. Peng Chen, Gary Baierl and Paul Kaplan [7] find a slightly higher standard deviation over all rounds of finance of 116%.

**Probability of Funding:** There are essentially two stages of evaluation for first-stage funding. In the first stage, the venture capitalist does a very cursory (few minute) skim of the abstract of the business plan, discarding the overwhelming majority. The remaining projects receive serious scrutiny—and this is the investigation that we model. Blumberg Capital (<http://www.sba.gov/INV/vc101.pdf>) asserts that “a typical \$100 million venture capital firm receives at least 1,500 business plans per year. Perhaps 50 result in serious due diligence, and 5 ultimately will obtain funding.” Sherman McCorkle, CEO of Technology Venture Corporation asserts that about 10% of all projects that are subjects of detailed investigations are funded ([http://www.abqtrib.com/archives/business00/050100\\_tvcsymp.shtml](http://www.abqtrib.com/archives/business00/050100_tvcsymp.shtml)). Other informal sources also suggest a rate of about 10% or slightly higher.<sup>6</sup>

**Probability VC loses money:** The probability the venture capitalist loses money corresponds to the probability that his return is less than  $1 + r_i$ . Using Cochrane's data set we compute the loss probability for venture capital projects with a first round of finance between 1987 and 1992. Cochrane's data set extends to

---

<sup>5</sup>This limited non-linearity reduces the inefficiency of a pure equity contract, and reduces the return to venture capitalists by lowering their claims to the best projects.

<sup>6</sup>Our assumption that only one venture capitalist seriously investigates a given project approximates practice. While rejection at the first stage may not reflect on the project's intrinsic merit, “Once a [project] is rejected [after serious scrutiny], it is very difficult to get it reconsidered... if the proposal is rejected... it may get an ‘overshopped’ reputation. Venture capitalists trade information quite freely and a turndown by one firm influences others” (Paul Keaton, <http://www.i2m.org/ftp/freepubs/0501.pdf>). If one could eliminate firms from the sample that were rejected after serious scrutiny, one would get a higher acceptance probability, which is consistent with our numerical analysis. Indeed, for reasonable distributional and investigation cost assumptions in our model, it is not optimal for an entrepreneur to consider a two-stage strategy in which he offers a higher equity share to a venture capitalist on the second round if rejected in the first round. This is because a first-round rejection precludes a high upside for the project, and the share must compensate the second-round venture capitalist for his investigation given the project's lower potential.

2000, and by using this earlier sample we ensure that more uncertainty about project returns is resolved. 33 percent of these firms went public with IPOs, 35 percent were acquired by another firm, 19 percent went out of business and 13 percent remained private by the year 2000 (and hence we cannot compute their returns). Dropping negative and zero returns for IPOs, 11 percent of IPOs lose money; 32 percent of acquisitions lose money; and we assume all bankrupt firms lose money. Assuming that projects that are still private in 2000 have the same failure rate as acquired projects—they have been active for at least 7 years, so they are not clearly lemons—then 38 percent of venture capital projects generate returns that fail to earn the venture capitalist’s required return of 11 percent.

We next detail how we identify the model parameters:

**Financier’s Required Return,  $r_i$ :** Mehra and Prescott [19] document that the historical average real return on the S&P 500 is 7 percent. For the real risk free rate we use 1 percent. The average annual rate of inflation between 1987 and 2001 was 3.25 percent. Ljungqvist and Richardson [18] estimate  $\beta = 1.09$ . Thus, the risk-adjusted rate is 11 percent, which we use for  $r_i$ .

**Sweat Equity,  $w$ :** For our model, the relevant measure is the sweat equity plus financial assets provided by the founders and key personnel. The opportunity costs of founders are high, as they take large salary cuts in exchange for equity/options. The median first round finance is 2 Million Dollars for 15 months in our 1987-1992 sample from Cochrane. If there are 8 key personnel/founders each of whom contributes \$200,000 through their personal capital plus the opportunity cost of their time on an annual basis, then  $w$  is 50 percent.

**Distribution,  $\mu$ ,  $\sigma$ :** To minimize the free parameters at our disposal, we assume that  $X$  and  $Y$  are normally distributed with a common standard deviation,  $\sigma_X = \sigma_Y = \sigma$ . One cannot observe this ex-ante distribution, but the high rejection rate of projects and high rate of unprofitable ventures indicate that ex-ante projects have a mean return that is far less than one, with a large standard deviation.

We now summarize the exogenous parameter choices that we use in our baseline model.

$w$	$r_i$	$\mu$	$\sigma$
50%	11%	0.7	0.5

Note that we do not specify how costly it is for the venture capitalist to investigate the project. We only assume that  $c$  does not exceed the value detailed in proposition 1. In this case,  $c$  effectively becomes a “sunk” cost that is covered by the venture capitalist’s return and does not affect the predictions of the model.

Given these parameter values, we solve for equilibrium values of endogenous variables numerically. From proposition 1, as long as  $c$  and  $\bar{u}$  are not too high, the equilibrium share  $k$  that the entrepreneur offers the financier is obtained by solving the unconstrained optimization problem for informed equity (problem 1),

and the precise values of  $c$  and  $\bar{u}$  are therefore irrelevant. Similarly, for debt finance, proposition 3 reveals that as long as informed debt gives rise to over-financing, then equilibrium interest rate will just provide the financier his reservation payoff. Thus, we only have to solve constraint (5) for  $r$ . With the endogenous variables in hand, we determine the remaining model parameters.

Table 1: Endogenous variables: model predictions versus estimated values from data.

	Share to financier	Financier excess return	Standard Deviation financier return	Probability of funding	Probability of loss	Equity: under-finance
Model	43.7%	21.8%	142%	12.7%	32.6%	61.8%
Data	43-44%	24%	120%	10%	38%	—

Table 1 details the predictions of the model about the key endogenous variables and how they compare with their real world counterparts obtained from the sources cited above. Inspection reveals that the model does an astonishingly good job at matching the key empirical regularities of the data. We almost perfectly match the project share that the venture capitalist receives, and fall only slightly short of the venture capitalist’s excess return. Ljungqvist and Richardson [18] propose that this excess return is a premium for the illiquidity of the investments. However, our model explains essentially the entire surplus without appealing to an illiquidity premium. That is, the entrepreneur willingly gives the venture capitalist an equity share sufficient to generate substantial “excess” returns. The entrepreneur does this to mitigate the severe under-investment problem, i.e., to increase his chances of being funded. Even with this premium, our model predicts that the venture capitalist does not fund about 60 percent of all projects that have positive NPVs conditional on the venture capitalist’s investigation.<sup>7</sup> Consistent with this prediction, 81 percent of surveyed venture capitalists felt that a significant number of viable early-stage investments go unfunded (Meyer et al. [20]).

Figure 2 provides another measure of the extent to which the entrepreneur transfers surplus to the venture capitalist. The figure illustrates the share of the total surplus that accrues to the venture capitalist. The correct measure of total surplus must incorporate the investigation costs that the venture capitalist incurs. Returns must be non-trivial to cover investigation costs, as funded projects are but 12.7 percent of all projects investigated. We find that returns are far higher than that. If the cost of investigating one project is one percent of the size of the venture capitalist’s investment in those projects that funds (i.e., about \$20,000, if the median annual investment is \$2,000,000), then the venture capitalist’s share of the surplus is a substantial 18.4 percent.<sup>8</sup> If, instead, the investigation cost is 0.5 percent, the venture capitalist gains all of the

<sup>7</sup>Again we note that these predictions are not affected by the venture capitalist’s cost of investigation  $c$ , as long as venture capitalist’s “excess returns” cover  $c$ . The level of excess returns that we find ensure that this is so for any reasonable value of  $c$ .

<sup>8</sup>In practice, a venture capitalist both devotes resources to evaluating projects and to hands-on management of funded projects. The “true” level of  $c$  reflects both of these expenditures.

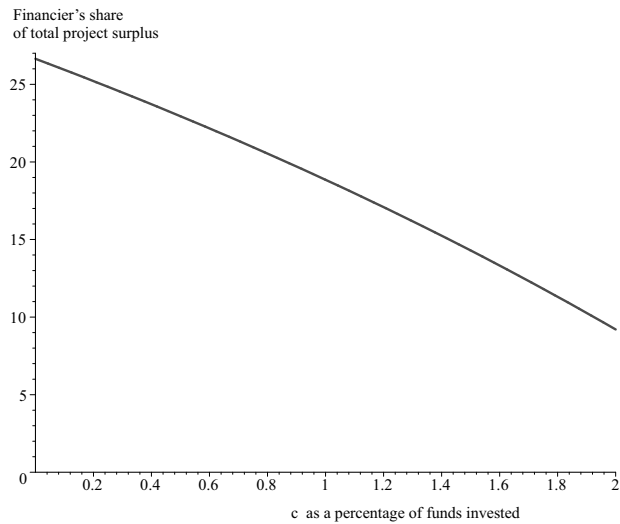


Figure 2: Financier's share of total expected project surplus

surplus increase associated with this lower cost, raising his share of the surplus to 22.9 percent. That is, the entrepreneur cedes almost a quarter of the total surplus to the venture capitalist. This is a huge amount—it represents the amount by which the equity compensation to the venture capitalist exceeds the level necessary to compensate him for his investigation costs plus his opportunity cost of funds.

The only predicted value that the model misses to any extent is the funding probability. Even then, we do not over shoot the estimated value by that much. Further, there is considerable uncertainty about the true funding rate, which may well be closer to our prediction.

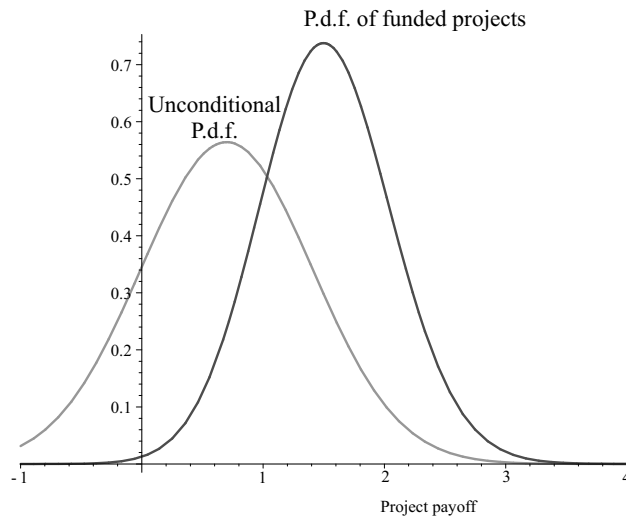


Figure 3: Project payoff distributions: unconditional vs. funded projects

We now explore other features of our baseline model. Figure 3 highlights the importance of the venture



capitalist's investigation for the quality of projects funded. The unconditional (pre-investigation) distribution of project payoffs reflects that most projects are bad: the mean project loss is 30 percent, with a standard deviation for  $X + Y$  of 71 percent. The venture capitalist's investigation leads him to discard low expected NPV projects, raising the conditional mean project return to 52 percent. Nevertheless, funded projects remain quite variable: the standard deviation of a project's return, conditional on being funded, is 54 percent.

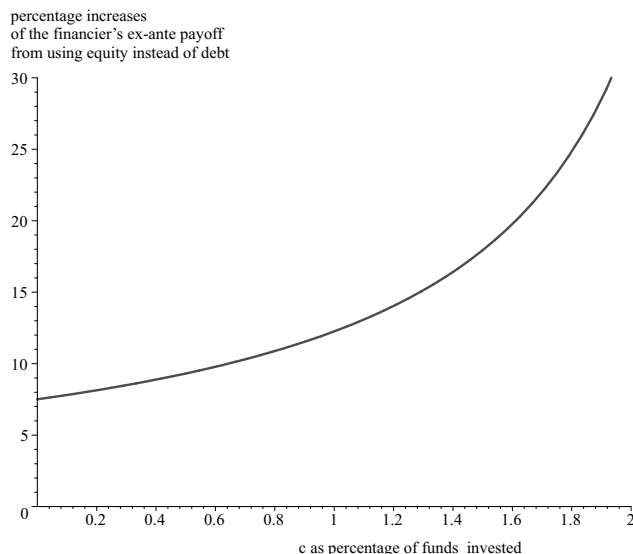


Figure 4: Financier's ex-ante expected payoff from equity relative to debt

Figure 4 sheds light on why equity finance dominates debt finance in the United States. To compare debt and equity finance, we set the financier's reservation value  $\bar{u}$  so that the entrepreneur is indifferent between debt and equity. With informed debt, the financier would receive  $\bar{u}$ , because the reservation utility constraint binds. With equity finance, total surplus is increased, so that the reservation utility constraint is slack, and the financier's payoff strictly exceeds  $\bar{u}$ . Figure 4 shows the percentage by which the financier's payoff is increased by using equity instead of debt as a function of  $c$ . We see that the financier's gain from equity ranges from 9.3 percent to 12.3 percent as  $c$  is increased from 0.5 percent to one percent. That is, there is substantial surplus to be gained from equity finance because debt finance gives rise to such excessive funding of negative NPV projects. In particular, suppose that only debt finance were feasible. Then, because the lender does not internalize the entrepreneur's sweat equity, 30.8 percent of projects that would receive informed debt finance have negative ex-ante NPVs, i.e., project payoffs are not expected to cover the value of entrepreneur's sweat equity. Of these projects hypothetically funded with debt, 12.1 percent would ultimately be non-performing projects that would fail to generate the entire required interest payment, and would lead either to bankruptcy or contract renegotiation.

Our analysis also sheds light on the prevalence of informed debt in developing countries. If a financier's

investigation leaves more unresolved uncertainty in these countries, then informed debt becomes more attractive. Property rights are also less well-enforced in emerging markets, Asia and Europe than in the United States (see, Laporta et al. [17], or Beim and Calomiris [3]); because debt contracts are easier to enforce than equity contracts, the difference in property rights enforcement does not have to be great for informed debt to dominate equity. Further, in some countries, governments effectively subsidize business loans, for example, by providing loan guarantees. Again, these subsidies do not have to be large for debt to dominate equity.

## 5.1 Sensitivity Analysis

We now highlight a crucial feature of our analysis: our theoretical model robustly generates the key empirical regularities in the data. Our quantitative predictions are not sensitive to the model parameterizations. This is important in its own right. In addition, it suggests that the model implications that cannot be directly verified from the data (e.g., the magnitudes of under-financing associated with equity, or the surplus generated by equity relative to debt finance) should be taken seriously. Indeed, this robustness suggests a role of our theoretical model for policy analysis and as the core feature of macroeconomic models that link the type of informed finance to economic growth rates and volatility.

Figure 5 displays how the key predictions of Table 1 are affected by the level of the entrepreneur’s sweat equity,  $w$ . Inspection of the scales of each of the six graphs reveals that only the financier’s equilibrium share and the probability of under-financing are remotely sensitive to the level of  $w$ . A one percentage point increase in the entrepreneur’s sweat equity  $w$  reduces the financier’s equity share by about 0.8 of a percent. The intuition is clear: as sweat equity  $w$  is increased, the financier’s required contribution,  $1 - w$  is decreased, so the share necessary to cover the financier’s investment is reduced. The reduction in the share  $k$  falls by less than one-for-one with an increase in  $w$ , because greater reductions in  $k$  make under-financing problems more severe. The bottom right figure illustrates that the probability of under-financing increases about 20 percent as sweat equity rises from  $w = 0.4$  to  $w = 0.6$ .

We emphasize how insensitive the other predictions are to the level of sweat equity. In particular, raising  $w$  by fifty percent from  $w = 0.4$  to  $w = 0.6$  affects the moments of the financier’s returns, the funding probability, and probability that the financier loses money by less than five percent.

Figure 6 reveals that the ex-ante mean of the project distribution primarily affects only the probability that a project is funded. Raising the ex-ante project mean from  $\mu = 0.6$  to  $\mu = 0.8$  raises the probability of funding from about 0.9 to 0.17. None of the other predicted values are sensitive to  $\mu$ : the effect of this 33 percent change in  $\mu$  on all other predicted model parameter values is less than three percent.

Figure 7 details how  $\sigma$ , the standard deviation of  $X$  and  $Y$  for the ex-ante project, affects model predictions. Again, most predictions are not sensitive to  $\sigma$ . Raising project uncertainty by 50 percent from

$\sigma = 0.4$  to  $\sigma = 0.6$  reduces the financier's equilibrium share by only 6 percent, raises his excess return by only 7 percent, reduces the probability that he loses money by only 3 percent, and reduces the probability of under-financing by less than 1 percent. That is,  $\sigma$  primarily affects the probability of funding, and the standard deviation of the financier's return. Funding rates rise by slightly more than 50 percent, and the standard deviation of the financier's return rises by a little less than 20 percent. Even so, this rise in the standard deviation of the return is far less than the raw increase in the project's riskiness.

Figure 8 highlights how the financier's required return  $r_i$  affects the model's predictions. Remarkably, increasing  $r_i$  from eight to fourteen percent reduces the financier's excess return by far less than one percentage point. Recall that the financier's excess return is net of  $r_i$ . Hence, one might expect that raising  $r_i$  would have a more profound effect on excess returns. What drives this insensitivity is that increasing  $r_i$  raises slightly the share of the project that the entrepreneur is willing to give the financier. The standard deviation of the return is slightly more sensitive to  $r_i$ , rising by about 20 percent in response to the 75 percent increase in  $r_i$ . This reflects the fact that fewer marginal projects are funded, and that the financier's project share (which effectively scales project payoffs) is increased. The other model predictions are completely insensitive to  $r_i$ .

## 6 Conclusion

This paper develops the simple analytical implications of informed finance: debt gives rise to over-financing because lenders do not internalize the sweat equity of key personnel in the firm, while equity gives rise to under-financing. Our quantitative analysis indicates that the under-financing with equity is severe: venture capitalists do not fund about 60 percent of all projects that have positive NPVs conditional on the venture capitalist's investigation. To reduce this under-financing and increase the chances of funding, the entrepreneur willingly cedes a large equity share of the project to a venture capitalist. In fact, our quantitative analysis indicates that essentially all of the surplus that venture capitalists receive (see Ljungqvist and Richardson [18]) is due to the optimal decision by entrepreneurs to reduce the extent of under-financing.

More generally, our calibration analysis reveals that our model closely predicts the key statistics associated with venture capital finance in the United States. Our model's explanatory power indicates that one must take seriously the model's predictions about parameters that cannot be readily identified from the data. For example, our model identifies why informed finance in the United States is primarily equity finance. The model also quantifies the surplus gained from using informed equity instead of informed debt finance.

The model also suggests why informed debt finance is used in Europe and Asia. Our hope is that integrating financing choices into a macroeconomic model will be able to explain the high rate of economic activity and growth in Asia, as well as the high rate of default in the so-called Asian debt crisis.

## 7 Appendix

**Proof of Proposition 1.** Let  $[\underline{x}, \bar{x}]$  be the (possibly unbounded) support of  $X$ . Let  $k^*$  be the solution to Problem 1 ignoring the constraints. Let  $x_E^* = (1 - w)(1 + r_i)/k^*$  be the cutoff value for funding. Note that  $x_E^* < \bar{x}$ . Otherwise, no project is funded, and the entrepreneur's payoff is zero. Because the density  $f(x)$  is strictly positive, the definition of  $x_E^*$  immediately implies that

$$\gamma_1 = \int_{x_E^*}^{\bar{x}} [k^*x - (1 + r_i)(1 - w)] dF(x) > 0, \quad (7)$$

Similarly,

$$\gamma_2 = \int_{x_E^*}^{\bar{x}} [k^*x - (1 + r_i)(1 - w)] dF(x) - \int_{\underline{x}}^{\bar{x}} [k^*x - (1 + r_i)(1 - w)] dF(x) > 0. \quad (8)$$

Then the constraints to Problem 1 are slack if and only if  $c + \bar{u} \leq \gamma_1$  and  $c \leq \gamma_2$ .

Now suppose that the constraints are slack. Then the first-order condition to Problem 1 is

$$-\int_{x_E^*}^{\bar{x}} x dF(x) + x_E^* f(x_E^*) \left[ \frac{1 - k^*}{k^*} x_E^* - \frac{w(1 + r_i)}{k^*} \right] = 0, \quad (9)$$

Because  $x_E^* < \bar{x}$  and  $f(x) > 0$  for  $x \in [\underline{x}, \bar{x}]$  it follows that  $-\int_{x_E^*}^{\bar{x}} x dF(x) < 0$ . Thus,  $x_E^* f(x_E^*) > 0$  and (9) imply

$$(1 - k^*)x_E > w(1 + r_i). \quad (10)$$

Adding equation  $k^*x_E = (1 - w)(1 + r_i)$  to (10) yields  $x_E^* > 1 + r_i$ , i.e., there is under-financing. ■

**Proof of Proposition 2.** If  $x \geq (1 + r_i)(1 - w) - \underline{y}$  then  $E[\min\{x + Y, (1 + r)(1 - w)\}] \geq (1 + r_i)(1 - w)$  (since  $r > r_i$ , as the financier must recover costs  $c$ ). Hence,  $x_D \leq (1 + r_i)(1 - w) - \underline{y}$ . Because  $-\underline{y} < w(1 + r_i)$ , we get  $x_D < 1 + r_i$ , i.e., there is over-financing. ■

**Proof of Proposition 3.** Suppose that constraint (5) is slack and  $x_D < 1 + r_i$ . Now decrease  $r$  marginally to  $r'$ . Then equation (4) implies that  $x_D$  increases marginally to  $x'_D$ , such that  $x'_D < 1 + r_i$ . Let  $x \in [x_D, x'_D]$ . Then equation 4 implies  $E[\min\{x + Y, (1 + r)(1 - w)\}] \geq (1 + r_i)(1 - w)$ . Because  $x'_D < 1 + r_i$  it follows that  $E[x + Y - \min\{x + Y, (1 + r)(1 - w)\}] - w(1 + r_i) < 0$  for all  $x \in [x_D, x'_D]$ . Thus, the objective of problem 2 is increased if projects are not funded for which  $x \in [x_D, x'_D]$ . If the interest rate is  $r'$  then only projects with  $x \geq x'_D$  are funded. This, and  $r' > r$  implies that the objective of problem 2 is strictly increased.

It remains to prove that constraint (6) is satisfied. Note that

$$\begin{aligned} & E\left[\min\{X + Y, (1 + r)(1 - w)\} - (1 + r_i)(1 - w)\right] \\ &= P(X < x_D)E\left[\min\{X + Y, (1 + r)(1 - w)\} - (1 + r_i)(1 - w) \mid X < x_D\right] \\ &\quad + P(X \geq x_D)E\left[\min\{X + Y, (1 + r)(1 - w)\} - (1 + r_i)(1 - w) \mid X \geq x_D\right] \end{aligned} \quad (11)$$

Subtracting  $P(\{X \geq x_D\})E[\min\{X + Y, (1 + r)(1 - w)\} - (1 + r_i)(1 - w) \mid X \geq x_D]$  on both sides of (6) and using (11) implies

$$-c \geq P(X < x_D)E\left[\min\{X + Y, (1 + r)(1 - w)\} - (1 + r_i)(1 - w) \mid X < x_D\right]. \quad (12)$$

When  $r$  is decreased to  $r'$  then  $E[\min\{X + Y, (1 + r)(1 - w)\} - (1 + r_i)(1 - w) \mid X < x_D]$  is decreased. Moreover,  $E[\min\{X + Y, 1 + r'\} - (1 + r_i)(1 - w) \mid X < x'_D] < 0$  for all  $x \leq x'_D$ . Thus the increase of  $x_D$  to  $x'_D$  further decreases the right-hand side of (12). Thus, constraint (6) is satisfied. ■

**Proof of Proposition 4.** By the the definition of second order stochastic dominance,  $E[u(\tilde{Y})] \geq E[u(Y)]$  for any non-decreasing concave function  $u$ . Thus,  $E[\min\{x + \tilde{Y}, (1 + r)(1 - w)\}] \geq E[\min\{x + Y, (1 + r)(1 - w)\}]$ . This, and (4) implies

$$E[\min\{x_D + \tilde{Y}, (1 + r)(1 - w)\}] \geq (1 + r_i)(1 - w) \quad (13)$$

From Proposition 3 we can conclude that constraint (5) must hold with equality when the noise is  $Y$ . Let  $\tilde{x}_D$ , and  $\tilde{r}$  be the cutoff value for providing finance and the interest rate, respectively, when the noise is  $\tilde{Y}$ .

First, suppose that  $\tilde{x}_D < x_D$  and  $\tilde{r} > r$ . Then it follows immediately constraint (5) holds with a strict inequality. However,  $\tilde{x}_D < x_D$  implies that there is also over-financing under  $\tilde{Y}$ . Because of Proposition 3, constraint (5) must therefore hold with equality, a contradiction.

Next, suppose that  $\tilde{x}_D > x_D$  and  $\tilde{r} < r$ . Then the left-hand side of constraint (5) strictly increases. Hence, constraint (5) is violated, again a contradiction. Inequality (13) therefore implies that  $\tilde{x}_D < x_D$  and  $\tilde{r} < r$ .

Finally, note that  $\tilde{x}_D < x_D$  implies that the surplus decrease because over-financing is increased. Because the financier's payoff does not change, the entrepreneur's payoff must therefore decrease. ■

**Proof of Proposition 5.** We first provide the proof for changes of  $r_i$ . If there is over-financing, then Proposition 3 implies that constraint (5) binds. The optimal values  $x_D(r_i)$ , and  $r(r_i)$  are therefore given by equation (4) and constraint (5), i.e.,

$$\int_{x_D(r_i)}^{\infty} \left[ \int_{-\infty}^{(1+r(r_i))(1-w)-x} (x + y) dG(y) + \int_{(1+r(r_i))(1-w)-x}^{\infty} (1 + r(r_i))(1 - w) dG(y) \right] dF(x) = c + \bar{u}; \quad (14)$$

and

$$\int_{-\infty}^{(1+r(r_i))(1-w)-x_D(r)} (x_D(r) + y) dG(y) + \int_{(1+r(r_i))(1-w)-x_D(r_i)}^{\infty} (1 + r(r_i))(1 - w) dG(y) = (1 + r_i)(1 - w). \quad (15)$$

Differentiating (14) with respect to  $r_i$  and using (15) yields

$$\int_{x_D(r_i)}^{\infty} \left[ r'(r_i)(1 - w) \left( 1 - G((1 + r)(1 - w) - x) \right) - (1 - w) \right] dF(x) = 0,$$

which implies

$$r'(r_i) = \frac{1 - F(x_D(r_i))}{\int_{x_D(r_i)}^{\infty} [1 - G((1 + r)(1 - w) - x)] dF(x)}. \quad (16)$$

We can assume that  $G((1 + r)(1 - w) - x_D) > 0$ . Otherwise, if  $G((1 + r)(1 - w) - x_D) = 0$  then (15) implies  $x_D = (1 + r_i)(1 - w)$  and the result follows, i.e.,  $x_D$  increases when  $r$  increases. Thus,  $G((1 + r)(1 - w) - x) < G((1 + r)(1 - w) - x_D)$ , for some  $x > x_D$ . This, and (16) imply

$$r'(r_i) < \frac{1 - F(x_D(r_i))}{\int_{x_D(r_i)}^{\infty} [1 - G((1 + r)(1 - w) - x_D(r_i))] dF(x)} = \frac{1}{1 - G((1 + r)(1 - w) - x_D(r_i))}. \quad (17)$$

Differentiating (15) with respect to  $r_i$  yields

$$\int_{-\infty}^{(1+r)(1-w)-x_D(r)} x'_D(r_i) dG(y) = (1 - w) \left[ 1 - (1 - G((1 + r)(1 - w) - x_D(r)))r'(r_i) \right] > 0, \quad (18)$$

where the inequality follows from (17). Hence,  $x'_D(r_i) > 0$ .

We sketch the proof for  $w$ , as the key steps are qualitatively identical. Differentiating (14) with respect to  $w$  and using (15) yields

$$r'(w) = \frac{1 + r}{1 - w} - \frac{(1 + r_i)(1 - F(x_D))}{\int_{x_D(w)}^{\infty} [1 - G((1 + r)(1 - w) - x)] dF(x)},$$

which again implies

$$r'(w) > \frac{1 + r}{1 - w} - \frac{(1 + r_i)(1 - F(x_D(w)))}{1 - G((1 + r)(1 - w) - x_D(w))}. \quad (19)$$

Differentiating (15) with respect to  $w$  yields

$$\int_{-\infty}^{(1+r)(1-w)-x_D(w)} x'_D(w) dG(y) = \left[ (1 + r) - r'(w)(1 - w) \right] \left[ 1 - G((1 + r)(1 - w) - x_D(w)) - (1 + r_i) \right] \quad (20)$$

Substituting (19) for  $r'(w)$  in (20) reveals that  $x'_D(w) < 0$ .

Increasing  $c$  or  $\bar{u}$  raises  $r$ . The result then follows immediately. ■

## References

- [1] P. Aghion and P. Bolton. An incomplete contracts approach to financial contracting. *Review of Economic Studies*, 59:473–494, 1992.
- [2] S. Ambec and M. Poitevin. Organizational design of r&d activities. working paper: University of Montreal, 2001.
- [3] D. Beim and C. Calomiris. *Emerging Financial Markets*. McGraw-Hill Irwin, New York, NY, 2001.
- [4] D. Bergemann and U. Hege. Dynamic venture capital financing, learning and moral hazard. *Journal of Banking and Finance*, 22:703–735, 1998.
- [5] D. Bernhardt and S. Krasa. Informed finance? working paper: University of Illinois, 2004.
- [6] B. Biais and E. Perotti. Entrepreneurs and new ideas. working paper: CEPR, 2002.
- [7] P. Chen, G. Baierl, and P. Kaplan. Venture capital and its role in strategic asset allocation. *Journal of Portfolio Management*, 28:83–90, 2001.
- [8] J. H. Cochrane. The risk and return of venture capital. working paper: GSB Chicago, 2003.
- [9] M. Dewatripont and J. Tirole. A theory of debt and equity: Diversity of securities and manager-shareholder congruence. *Quarterly Journal of Economics*, 109:1027–1054, 1994.
- [10] G. Fenn, G. N. Liang, and S. Prowse. The economics of private equity markets. Staff Study 168, Board of Governors of the Federal Reserve System, 1995.
- [11] V. H. Fried and R. D. Hisrich. Toward a model of venture capital investment decision making. *Financial Management*, 23:28–37, 1994.
- [12] M. Garmaise. Informed investors and the financing of entrepreneurial projects. working paper: GSB Chicago, 2001.
- [13] O. Hart and J. Moore. Default and renegotiation: A dynamic model of debt. *Quarterly Journal of Economics*, 3:1–41, 1998.
- [14] U. Hege, F. Palomino, and A. Schwienbacher. Determinants of venture capital performance: Europe and the united states. working paper: LSE, 2003.
- [15] S. Kaplan and P. Stromberg. How do venture capitalists choose and manage their investments? working paper: University of Chicago, 2000.
- [16] S. Kaplan and P. Stromberg. Financial contracting theory meets the real world: An empirical analysis of venture capital contract. *Review of Economic Studies*, 70:281–315, 2003.
- [17] R. La Porta, F. L. de Silanes, A. Shleifer, and R. Vishny. Law and finance. *Journal of Political Economy*, 106:1113–1155, 1998.
- [18] A. Ljungqvist and M. Richardson. The cash flow, return and risk characteristics of private equity. working paper: NYU, 2003.
- [19] R. Mehra and E. C. Prescott. The equity premium: A puzzle. *Journal of Monetary Economics*, 15:145–161, 1996.
- [20] R. Meyer, R. Radosевич, E. Carayannis, M. David, and J. G. Butler. The 1995 national census of early-stage capital financing. <http://www.oriontechnical.com/95report.html>, 1995.
- [21] R. Repullo and J. Suarez. Venture capital finance: A security design approach. working paper, CEMFI, 1999.
- [22] M. Ueda. Banks versus venture capital: Project evaluation, screening, and expropriation. *Journal of Finance*, 59:601–621, 2004.
- [23] A. Winton and V. Yerramilli. A model of entrepreneurial finance. working paper: University of Minnesota, 2004.

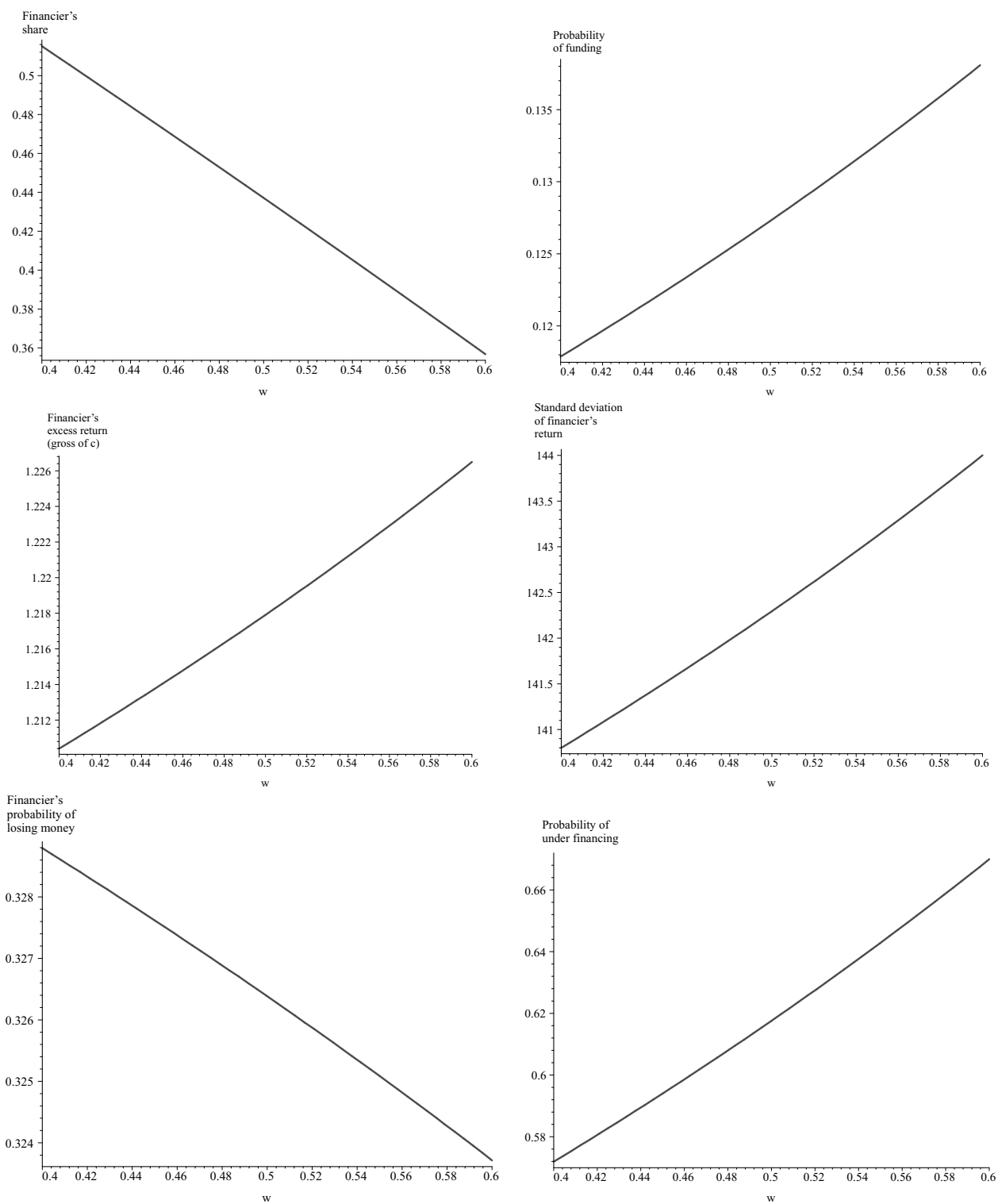


Figure 5: Sensitivity with respect to  $w$



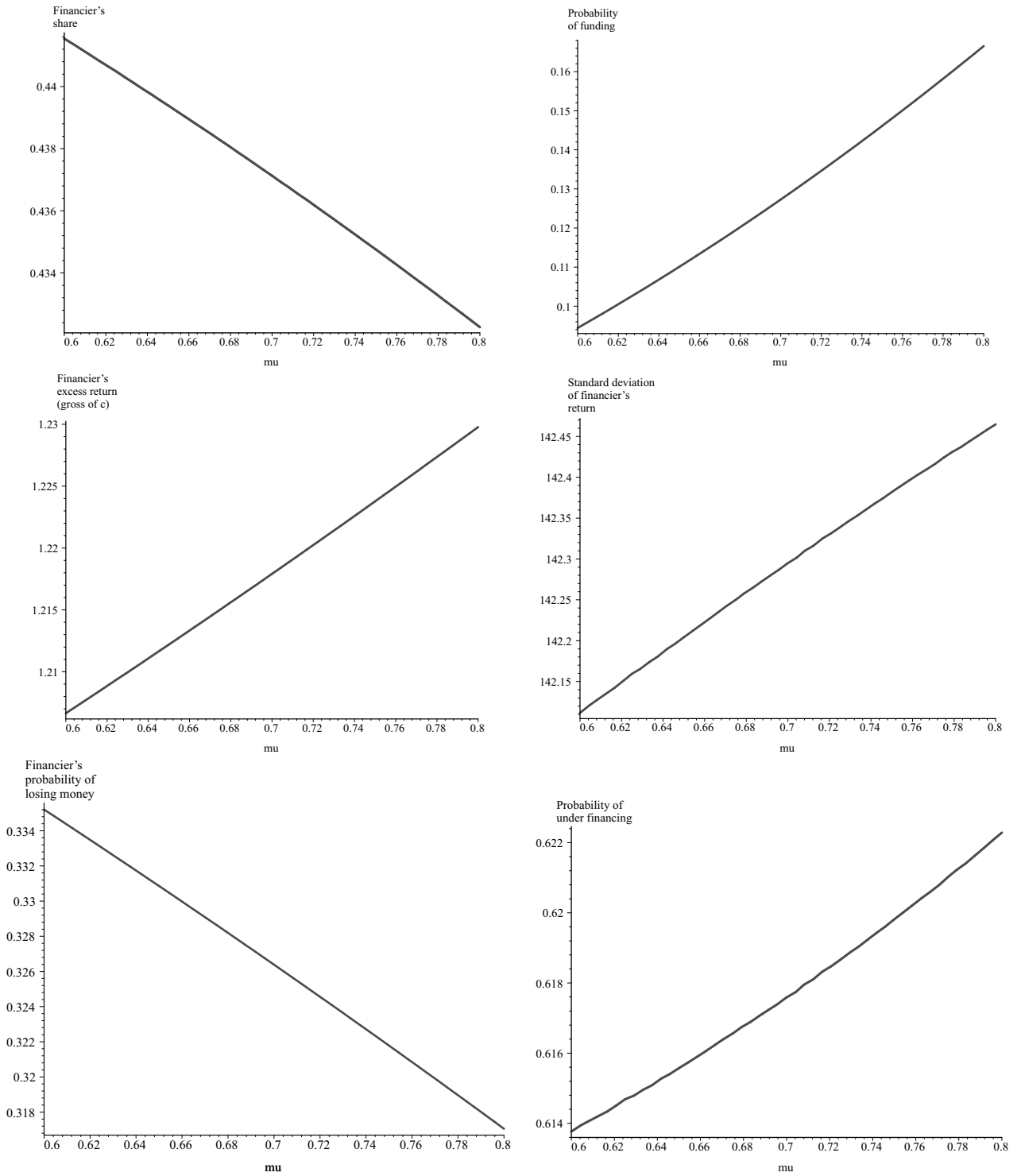


Figure 6: Sensitivity with respect to  $\mu$

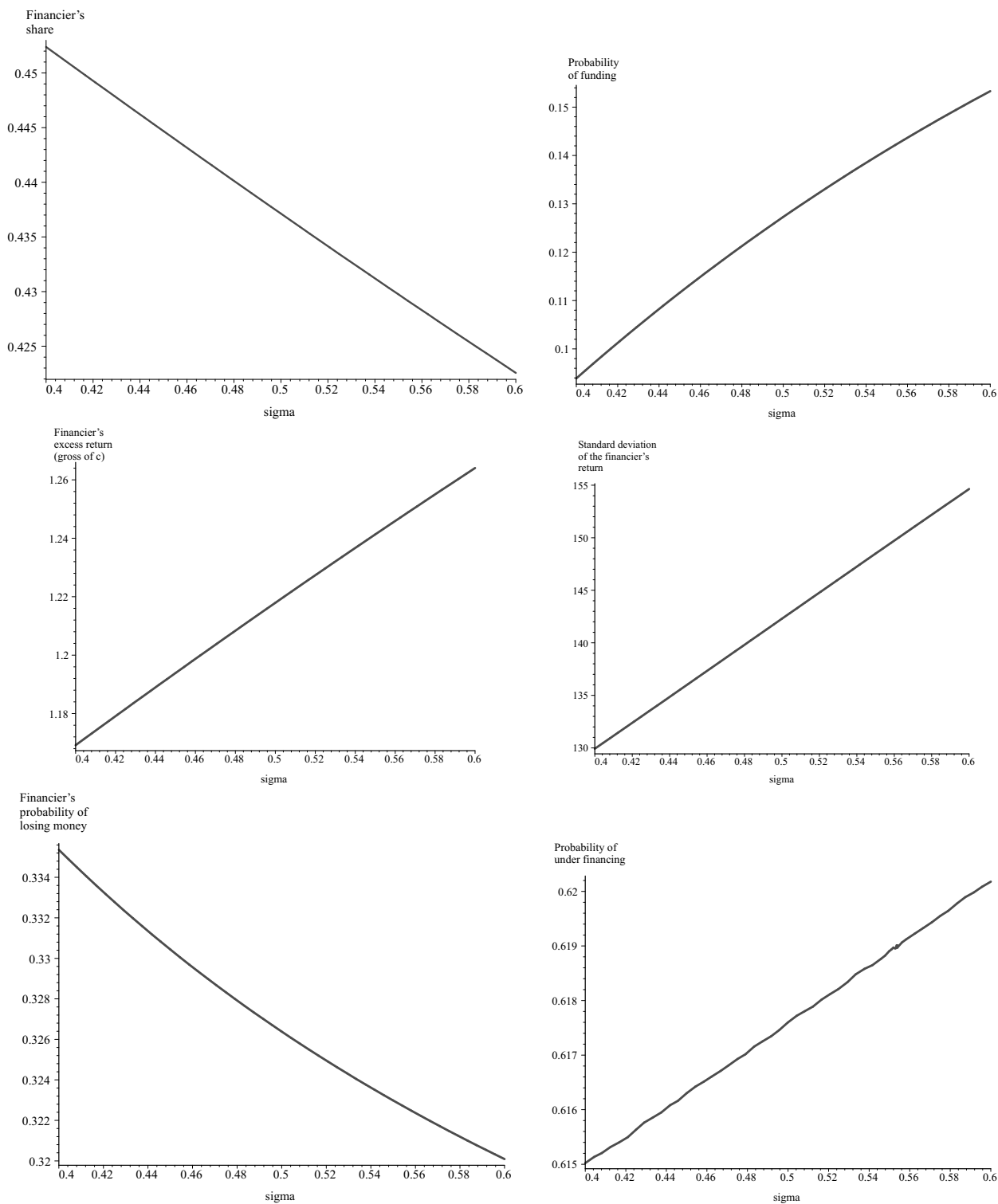


Figure 7: Sensitivity with respect to  $\sigma$

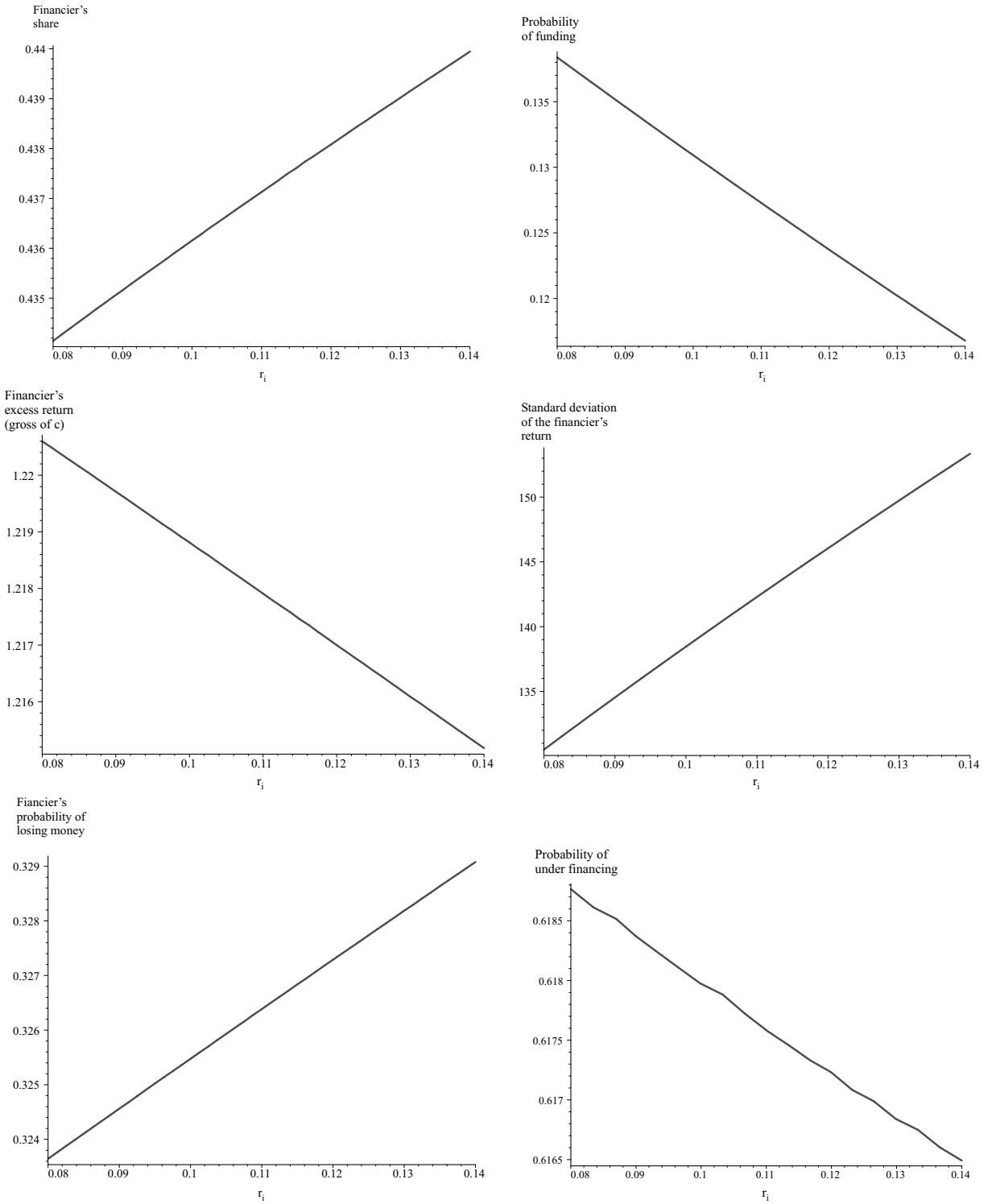


Figure 8: Sensitivity with respect to  $r_i$