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All questions must be answered on this form. You must show your work!
Use the back of this form and the last page as scratch paper—do not use your own paper.

Question 1 Suppose there are three commodities and that Walras' law is satisfied. At prices $p^1 = (1, 4, 2)$ demand is $x^1 = (10, 5, 2)$; at prices $p^2 = (4, 3, 2)$, demand is $x^2 = (y, 5, 1)$.

Then the Weak Axiom is satisfied if and only if *5 points*

y satisfies: $y < 10.5$ or $y > 12$.

Note that $w^1 = 34$ and $w^2 = 4y + 17$. x^2 is affordable given p^1, w^1 if $y + 22 \leq 34$, i.e., $y \leq 12$. It then must be the case that x^1 is not affordable given p^2, w^2, i.e., $59 > 4y+17$, which implies $y < 10.5$.

Next, if x_1 is affordable given p^2, w^2 then $4y+17 \geq 39$, i.e., $y \geq 5.5$. Then x_2 should not be affordable given p^1, w^1, i.e., $y+22 > 34$, i.e., $y > 12$.

Question 2

1. Let $X = \{a, b, c\}$ and $\mathfrak{B} = \{\{a, b\}, \{a, c\}, \{b, c\}\}$. Suppose that $C(\{a, b\}) = \{a\}$ and $C(\{b, c\}) = \{b\}$. For each of the following definition of the choice over $\{a, c\}$ determine whether or not the choice structure satisfied the Weak Axiom (*Circle the correct answer. One incorrect, 1 point, more than one incorrect, 0 points*). *2.5 points*

   (a) $C(\{a, c\}) = \{a\}$ **satisfies** **violates** the Weak Axiom.

   (b) $C(\{a, c\}) = \{c\}$ **satisfies** **violates** the Weak Axiom.

   (c) $C(\{a, c\}) = \{a, c\}$ **satisfies** **violates** the Weak Axiom.

2. Let $X = \{a, b, c\}$ and $\mathfrak{B} = \{\{a, b\}, \{b, c\}, \{a, b, c\}\}$. Suppose that $C(\{a, b\}) = \{a\}$ and $C(\{b, c\}) = \{b\}$. For each of the following definition of the choice over $\{a, b, c\}$ determine whether or not the choice structure satisfied the Weak Axiom (*Circle the correct answer. One incorrect, 1 point, more than one incorrect, 0 points*). *2.5 points*

   (a) $C(\{a, b, c\}) = \{a\}$ **satisfies** **violates** the Weak Axiom.

   (b) $C(\{a, b, c\}) = \{b\}$ **satisfies** **violates** the Weak Axiom.

   (c) $C(\{a, b, c\}) = \{a, c\}$ **satisfies** **violates** the Weak Axiom.

   (d) $C(\{a, b, c\}) = \{a, b, c\}$ **satisfies** **violates** the Weak Axiom.

Question 3 Suppose a utility function is given by

$$u(x_1, x_2) = \begin{cases} x_1 + x_2 & \text{if } x_1 + x_2 < 10; \\ 12 & \text{if } 10 \leq x_1 + x_2 < 20; \\ x_1 + x_2 & \text{if } x_1 + x_2 \geq 20. \end{cases}$$

Then $p_1 < p_2$ implies that $x_2 = 0$. Thus, $x_1 = 5$ and *5 points*

$e(2, 3, 5) = 10$

$p_1 = p_2$ implies that x_1 and x_2 are arbitrary as long as $x_1 + x_2 \geq 10$.

$e(2, 2, 11) = 20$

Now $x_2 = 0$. Further, we need $x_1 \geq 20$. Thus,

$h(1, 2, 14) = (20, 0)$

$x_1 = 0$ and $x_2 = 15$. Thus,

$v(5, 2, 30) = 12$

(Note that $e(p_1, p_2, u)$ denotes the expenditure function, $h(p_1, p_2, u)$ the Hicksean demand function/correspondence, and $v(p, w)$ the indirect utility function. *One incorrect: 4 points, 2 incorrect: 2 points, 3 or more incorrect: 0 points*)).

Question 4 Consider two price wealth situations p, w and p', w'. Let x and x' be the unique Walrasian demand at p, w and p', w', respectively. Suppose that p', w' is a compensated price change, i.e., $p'x = w'$ and that $x' \neq x$. We want to prove that $(p' - p).(x' - x) < 0$ if the Weak Axiom and Walras' Law hold.

Complete the proof in the box below. You must continue where the argument stops and you must continue to use the same notation. Do not start an entirely new proof.

5 points

Proof: Note that $(p' - p)(x' - x) = p'x' - px' + px - p'x = w' - px' + w - w' = -px' + w$, since $p'x' = w'$, $px = w$ by Walras' law and $p'x = w'$ by assumption. Since $p'x \leq w'$ where x is the optimal choice at p, w the Weak Axiom implies that x' is not affordable at p, w, i.e., $px' > w$. Thus, $-px' + w < 0$, which concludes the proof.