**Question 1**

**(a)** $\boxed{\boldsymbol{h(p, v(p, w)) = x(p, w)}}$.

**(b)** $\boxed{\boldsymbol{ph(p, v(p, w)) = w}}$.

**(c)** $\boxed{\boldsymbol{px(p, e(p, u)) = e(p, u)}}$.

**Question 2** Note that $x_i(p, w) = h_i(p, v(p, w))$. Thus, $x_1(p_1, p_2, w) = \frac{p_2^2}{p_1^2}$. We can use Walras' law to determine the demand of good 2. In particular, $p_1 \frac{p_2^2}{p_1^2} + p_2 x_2 = w$. Thus, $p_2 x_2 = w - \frac{p_2^2}{p_1}$, which implies

$$\boxed{\boldsymbol{x_1(p, w) = \frac{p_2^2}{p_1^2},\; x_2(p, w) = \frac{w}{p_2} - \frac{p_2}{p_1}}}$$

**Question 3** *Proof:* Let $x = x(p, w)$ and $x' = x(p', w')$. Then

$$\boxed{\boldsymbol{x = (w/2.w/2),\ \text{ and } x' = (w'/2, w'/4)}}$$

Thus, $p'x = 3w/2$ and $px' = 3w'/4$. If $p'x \leq w'$ then $w' \geq 3w/2$. Thus, $3w'/4 > 9w/8 > w$.

Next, suppose that $px' \leq w$. Then $3w'/4 \leq w$. Then $p'x = 3w/2 \geq 9w/8 > w$.

**Question 4** Note that $w = px = 36$ and $w' = p'x' = 4 + x_2'$. If $px' \leq w$ then $16 + 2x_2' \leq 36$ (i.e., $x_2' \leq 10$). But then $p'x > w'$, i.e., $13 > 4 + x_2'$, i.e., $x_2' < 9$. If, instead, $p'x \leq w'$, i.e., $13 \leq 4 + x_2'$ implying $x_2 \geq 9$, then $px' > w$, i.e., $16 + 2x_2' > 36$, i.e., $x_2' > 10$. Thus,

$$\boxed{\boldsymbol{x_2' < 9 \text{ or } x_2' > 10.}}$$

**Question 5** Note that $x_2 = x_3$. Let $y = x_2 = x_3$. Then the consumer maximizes $u(x_1, y) = x_1 y$ subject to prices $q_1 = p_1$, $q_2 = p_2 + p_3$ and wealth $w$. The MRS is given by $y/x_1$, the wealth expansion path by $q_1 x_1 = q_2 y$. To get a utility of $u$ we need $x_1 y = u$. Thus, $x_1 q_2 y = q_2 u$ which implies $q_1 x_1^2 = q_2 u$. Thus, $x_1 = \sqrt{(q_2/q_1)u}$ and similarly, $y = \sqrt{(q_1/q_2)u}$. Substituting $q_1$ and $q_2$ yields

$$\boxed{\boldsymbol{h_2(p_1, p_2, p_3, u) = \sqrt{\frac{p_1}{p_2 + p_3} u}}}$$

As a consequence, goods 2 and 3 are $\boxed{\textbf{complements}}$ because $\frac{\partial h_2(p_1,p_2,p_3,u)}{\partial p_3}$ $\boxed{<}$ 0.

**Question 6** Suppose a utility function is given by $u(x_1, x_2) = x_1^2 - 2x_2$. Then $x_2 = 0$. To obtain utility $u$, $x_1 = \sqrt{u}$. Thus, $h_1(p, u) = \sqrt{u}$. Therefore,

$$\boxed{\boldsymbol{e(p_1, p_2, u) = p_1 \sqrt{u}}}$$

$w = p_1 \sqrt{u}$ implies, $u = w^2/p_1^2$.

$$\boxed{\boldsymbol{v(p_1, p_2, w) = w^2/p_1^2}}$$

**Question 7**

$$\begin{pmatrix} -6 & \boxed{3} & \boxed{9} \\ \boxed{3} & -10 & \boxed{4} \\ \boxed{9} & 4 & \boxed{-22} \end{pmatrix}$$

**Question 8** We must show that $x'' = \alpha x + (1 - \alpha)x' \in h(p, u)$. Note that $u(x) \geq u$ and $u(x') \geq u$. Thus, quasi concavity implies $u(x'') \geq \min\{u(x), u(x')\} \geq u$. As a consequence, $x''$ fulfills the constraint of the expenditure minimization problem, when the required utility level is $u$. Further $px'' = \alpha px + (1-\alpha)px' = \alpha e(p, u) + (1-\alpha)e(p, u) = e(p, u)$. Hence, $x''$ minimizes expenditure, which implies $x'' \in h(p, u)$.