Solutions: Mid-term Econ500, 12:00 October 16, 2006

Name:

E-mail:

All questions must be answered on this test form!
For each question you must show your work and (or) provide a clear argument.
All graphs must be accurate to get credit.
If you need scratch paper, use the back of the form.

Short Questions Which of the following is true or false, where $e(p,u)$ is the expenditure function, $h(p,u)$ Hicksean demand, $x(p,w)$ Walrasian demand, $v(p,w)$ indirect utility. (*mark the correct box. Be careful, some of the questions are a little bit tricky.*) *30 points*
Each question is worth 1.5 points. No explanation is needed

1. $e(\lambda p, \lambda u) = \lambda e(p,u)$ for all $\lambda > 0$. true [✓ **false**]
2. $e(\lambda p, \lambda u) = \lambda e(p,u)$ for all $\lambda \in \mathbb{R}$. true [✓ **false**]
3. $e(\alpha p + (1-\alpha)p', u) \geq \alpha e(p,u) + (1-\alpha)e(p',u)$ for all $0 < \alpha < 1$. [✓ **true**] false
4. $e(\alpha p + (1-\alpha)p', u) \geq \min\{e(p,u), e(p',u)\}$ for all $0 < \alpha < 1$. [✓ **true**] false
5. If e is differentiable then $\frac{\partial e(p,u)}{\partial p_i} > 0$ for all i. true [✓ **false**]
6. If e is differentiable and preferences are locally non-satiated then $\frac{\partial e(p,u)}{\partial u} > 0$. true [✓ **false**] [1]
7. Let $u(x_1,x_2) = \min\{x_1,x_2\}$. Then $e(p,u)$ is not differentiable with respect to prices. true [✓ **false**]
8. Let $u(x_1,x_2) = x_1 + x_2$. Then $e(p,u)$ is not differentiable with respect to prices when $p_1 = p_2$. [✓ **true**] false
9. Suppose that preferences are lexicographic. Then $e(p,u)$ exists. true [✓ **false**]
10. $v(\lambda p, \lambda w) = v(p,w)$ for all $\lambda > 0$. [✓ **true**] false

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[1] Suppose that there is only one commodity and $u(x) = \text{sign}(x-64)|x-64|^{1/3} + 4$. Then $e(p,u) = p(u-4)^3 + 64p$. The derivative at $u = 4$ is 0.

11. v is convex in p and w. true [✓ **false**]

12. If preferences are continuous and locally non satiated then $v(p, e(p, u)) = w$. true [✓ **false**]

13. If preferences are continuous and locally non satiated then $h(p, v(p, w)) = x(p, w)$. [✓ **true**] false

14. Let $x^* = x(p^*, w^*)$. If there are two commodities, Walras' law is satisfied and $x_1^* = 0$ then $\text{MRS}(x_1^*, x_2^*) \geq \frac{p_1^*}{p_2^*}$. true [✓ **false**]

15. Suppose that $u(x_1, x_2) = (x_1 + 10)(x_2 + 10)$. Then $x(p, w)$ satisfies Walras' law. [✓ **true**] false

16. Suppose that $u(x_1, x_2) = (x_1 - 10)(x_2 - 10)$. Then $x(p, w)$ satisfies Walras' law. true [✓ **false**]

17. Suppose that $u(x_1, x_2) = x_1 - x_2^2$. Then $x(p, w)$ satisfies Walras' law. [✓ **true**] false

18. $v(\alpha p + (1 - \alpha)p', \alpha w + (1 - \alpha)w') \leq \max\{v(p, w), v(p'w')\}$ for all $0 < \alpha < 1$. [✓ **true**] false

19. Let $v(p, w)$ be the indirect utility function for $u(x)$ where $u(x) > 0$. Then $\ln(v(p, w))$ is the indirect utility function for $\ln(u(x))$. [✓ **true**] false

20. Let $e(p, u)$ be the expenditure function for $u(x)$ where $u(x) > 0$. Then $\ln(e(p, u))$ is the expenditure function for $\ln(u(x))$. true [✓ **false**]

Other Questions

Question 1 Suppose that X is finite. Let $\geq$ be preferences on X that are transitive and reflexive (i.e., $x \geq x$) but not necessarily complete. We want to prove that there exists an optimal choice x^*. An optimal choice means that for all $y \in X$ one of the following is the case: (1) $x^* \geq y$, or (2) x^* and y are not comparable.

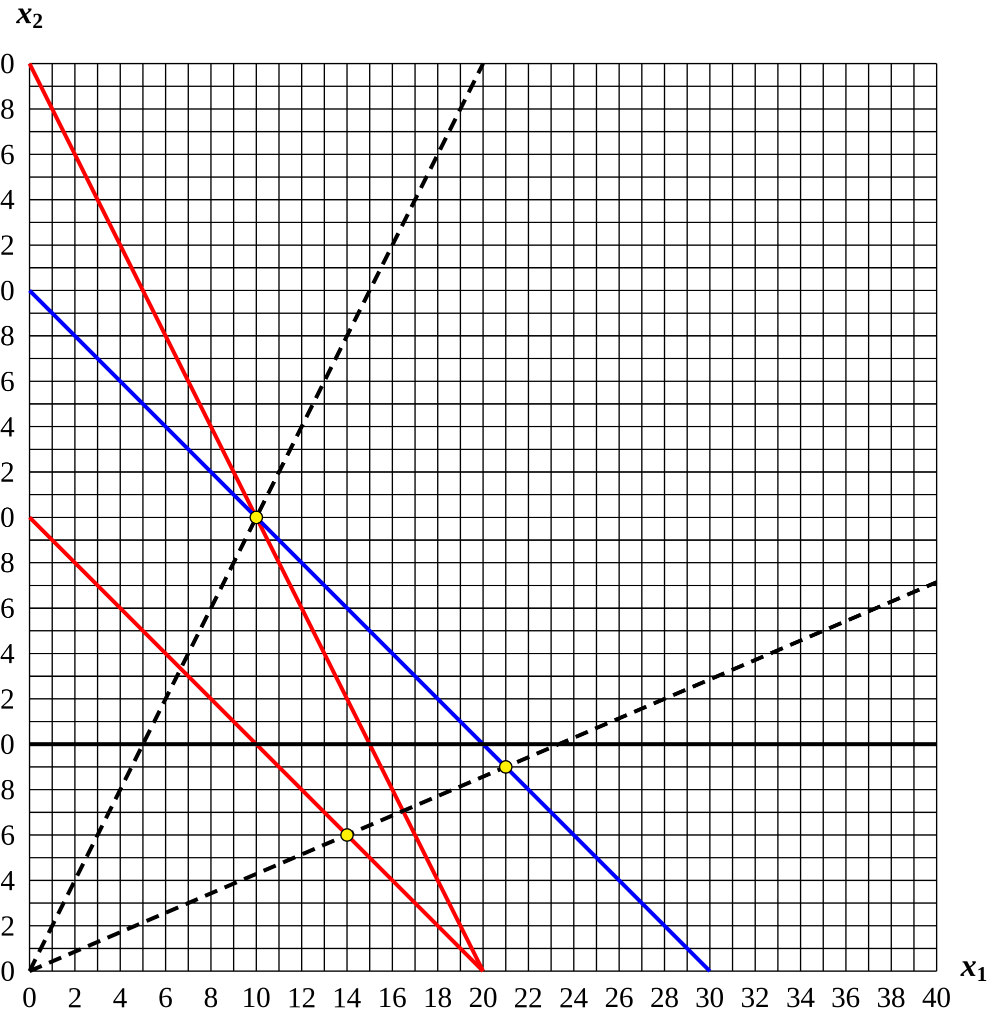
We proceed by way of induction. That is, suppose X consists of a single element. Then the result is obvious. Thus, suppose that we have proved the result for all sets with strictly less than n elements. Now suppose that X has n elements. (*Write the proof in the box below. The argument must be clear and concise to get credit.*) *15 points*

Let $X = X' \cup \{x_n\}$, where $X' = X \backslash \{x_n\}$. Then X' contains $n-1$ elements. Thus, by the induction hypothesis there exists $\tilde{x}$ that is optimal in X'. If $\tilde{x} \geq x_n$ or $\tilde{x}$ and x_n are not comparable then $\tilde{x}$ is also optimal in X. Thus, suppose that $x_n \geq \tilde{x}$. It remains to prove that x_n is optimal. Clearly, $x_n \geq y$ for $y = x_n$. Thus, let $y \in X'$. Suppose by way of contradiction that $y \succ x_n$. Because $x_n \geq \tilde{x}$, transitivity implies $y \succ \tilde{x}$, a contradiction to the optimality of $\tilde{x}$ in X'.

Question 2 Suppose preferences are homothetic (recall, that this implies that all wealth expansion paths are straight lines starting at 0). Suppose that at prices $p_1 = 2$, $p_2 = 1$ and wealth $w = 40$ Walrasian demand for commodity 1 is 10 units. When the price of commodity 2 increases to $p_2 = 2$ (p_1 and w do not change) demand for commodity 1 increases to 14 units. Then using the Slutzky method of compensation (where we adjust wealth such the original consumption remains afordable) the change in demand due to substitution, i.e, the substitution effect, for the commodities are

Commodity 1: 11 units, Commodity 2: -11 units

(*Write the exact quantity of change in the box above. Indicate a decrease in demand by a negative sign. Solve the problem graphically by using the grid below*). *15 points*



Question 3 Consider the utility function $u(x_1, x_2) = \min\{2x_1, 3x_2\}$. Then *15 points*

$\boxed{e(p,u) = u(p_1/2 + p_2/3)}$ $\boxed{v(p,w) = 6w/(2p_2 + 3p_1)}$

$\boxed{h_1(p,u) = u/2,\ h_2(p,u) = u/3}$

The wealth expansion path is $2x_1 = 3x_2$. In order to optain utility level u, $u = 2x_1 = 3x_2$. Thus, $h(p,u) = (u/2, u/3)$. The expenditure function is therefore $e(p,u) = u(p_1/2+p_2/3)$. The indirect utility function is $v(p,w) = 6w/(2p_2 + 3p_1)$.

Question 4 Suppose there are two commodities. Let

$$g(x_1, y_1) = \begin{cases} 0 & \text{if } |x_1 - y_1| < 2; \\ x_1 - y_1 - 2 & \text{if } |x_1 - y_1| \geq 2;. \end{cases}$$

Suppose that $(x_1, x_2) \succeq (y_1, y_2)$ if and only if $g(x_1, y_1) + x_2 - y_2 \geq 0$.

(*Before you answer the questions below, you should try to understand intuitively what kind of behavior these preferences describe. Once you understand this, the solutions are very short.*)

(a) Let $\bar{x} = (20, 30)$ and $\bar{y} = (19, 30.5)$. Then mark all that apply *5 points*

$\bar{x} \succeq \bar{y}$ $\boxed{\bar{y} \succeq \bar{x}}$

Find a $\bar{z}$ such that $\bar{x}$, $\bar{y}$ (from above) and $\bar{z}$ violate transitivity.

$$\boxed{\textbf{Let } \bar{z} = (18, 31)\textbf{. Then } z \succeq y, y \succeq x \textbf{ but } x \succ z.}$$

(b) Suppose prices are $p_1 = 2$, $p_2 = 1$ and $w = 10$. Then *5 points*

$$\boxed{x(p, w) = (0, 10)}$$

(c) Suppose prices are $p_1 = 4$, $p_2 = 5$ and $w = 15$. Then *5 points*

$$\boxed{x(p, w) = (0, 3)}$$

Question 5 An indirect utility function is given by

$$v(p, w) = \frac{\sqrt{w}}{\sqrt{p_1} + \sqrt{p_2}}.$$

Then *10 points*

$$\boxed{e(p, u) = u^2(\sqrt{p_1} + \sqrt{p_2})^2}$$

$$\boxed{h_1(p, u) = u^2 \frac{\sqrt{p_1} + \sqrt{p_2}}{\sqrt{p_1}}}$$