Question 1 Utility is quasilinear, i.e., $u(x) = v(x_1) + x_2$. Thus, $v'(x_1) = p_1$. Thus, $20 - 4p_1 = x_1$ implies $p_1 = 5 - (1/4)x_1$. Hence, $v'(x_1) = 5 - (1/4)x_1$, and therefore $v(x_1) = 5x_1 - (1/8)x_1^2$. Thus,

$$\boxed{\boldsymbol{u(x_1, x_2) = 5x_1 - (1/8)x_1^2 + x_2}}$$

Question 2 To produce q units of output, we must have $z_1 + 2z_2 = q^2$. Thus, costs are $c(w, q) = \min\{w_1, 0.5w_2\}q^2$. The first order conditions for profit maximization is $p = \min\{w_1, 0.5w_2\}2q$. Thus,

$$q = \frac{p}{\min\{2w_1, w_2\}}.$$

Hence profit is

$$\frac{p^2}{\min\{2w_1, w_2\}} - \min\{w_1, 0.5w_2\}\frac{p^2}{4\min\{w_1, 0.5w_2\}^2} = \frac{p^2}{\min\{2w_1, w_2\}} - \frac{p^2}{2\min\{2w_1, w_2\}}.$$

Then the firm's profit function is $\boxed{\boldsymbol{\pi(w_1, w_2, p) = \frac{p^2}{2\min\{2w_1, w_2\}}}}$

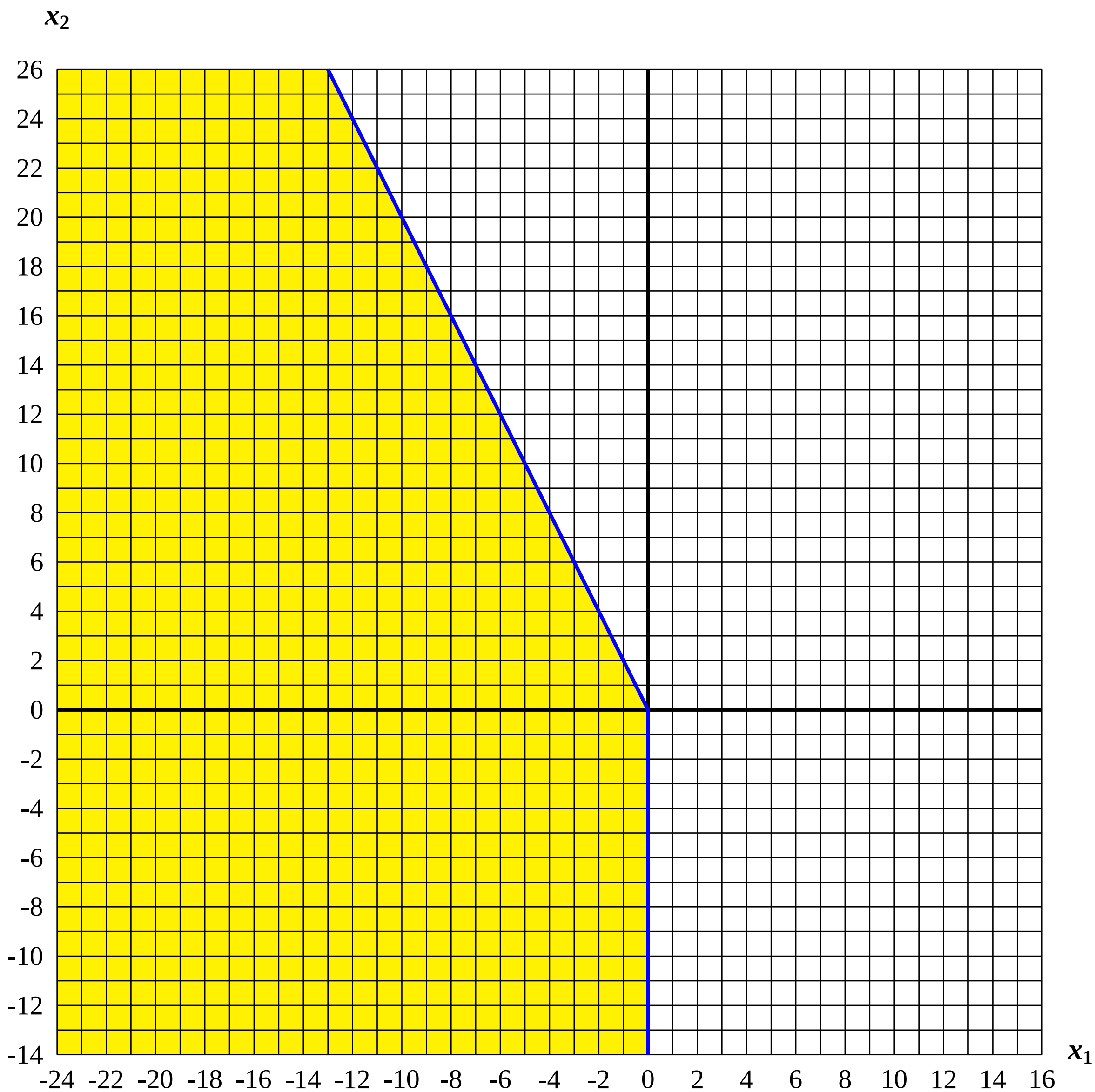
Question 3 Suppose that a firm can operate at two locations, using one input at each location (the input also has the same price at the two locations). The production functions at the two locations are given by $f(z)$ and $g(z)$. Clearly, $f(0) = g(0) = 0$. Further, suppose that f has constant returns to scale and that g is strictly concave (and has therefore decreasing returns to scale). When producing q units of output overall, the firm wants to allocate production at the two locations by minimizing the total use of input. In the box below write a simple mathematical conditions that specifies exactly when the firm produces a strictly positive amount of output using g. *12 points*

$$\boxed{\boldsymbol{g > 0 \text{ if and only if } g'(0) > f'(0) \text{ and } pg'(0) > w.}}$$

Now determine a sufficient conditions such that the firm only produces at the second location, i.e., $g > 0$ and $f = 0$.

$$\boxed{\boldsymbol{g'(0) > f'(0), pg'(0) > w, \text{ and } pf'(0) < w.}}$$

Question 4 Note that f is concave and that $f'(0) = 2$. the additive closure $\bar{Y}$ of this technology is: *10 points*



Question 5 Note that $c(\lambda w_1, \lambda w_2, q) = \lambda c(w_1, w_2, q)$ for any cost function. Let $w_1 = w_2 = w$. Then $(\phi(\lambda w) + \phi(\lambda w)) = 2\lambda\phi(w)$. Thus, $\phi(\lambda w) = \lambda\phi(w)$ which implies that $\phi(w) = aw$ for some $a > 0$. Hence the cost function is $c(w_1, w_2, q) = aq^2(w_1 + w_2)$. Thus, the production function is of the form

$$\boxed{\boldsymbol{f(z_1, z_2) = b\sqrt{\min\{x_1, x_2\}}}}$$

Question 6 Demand functions are

$$x_1 = \frac{w}{2p_1}, \quad x_2 = \frac{w}{2p_2}.$$

Thus, demand after the price change is $(50, 200)$. Therefore,

$\boxed{\textbf{Tax revenue is 150}}$

The utility $u(50, 200) = 10,000$. To get this utility at the original prices with the lowest possible expenditures, $x_1 = x_2$ and $x_1 x_2 = 10,000$, i.e., $x_1 = x_2 = 100$. Thus, expenditures are 200. The tax therefore corresponds to a loss of 200 Dollars. Thus, the deadweight loss is 50.

$\boxed{\textbf{The deadweight loss using EV is 50}}$

Question 7 A person with wealth w wants to start a firm. There is one input and one output. The firm has constant returns to scale production function given by $f(z) = z$. The price of the input is 1 Dollar per unit. The price of the output, however, is determined by the state of nature. In particular, there are two states ω_1 and ω_2, where ω_1 occurs with probability q and ω_2 with probability $1 - q$. The prices of the output in states ω_1 and ω_2 are given by p_1 and p_2. Let u be the person's Bernoulli utility function.

(a) $\max_z qu(w - z + p_1 z) + (1 - q)u(w - z + p_2 z)$.

(b) $p_1, p_2 < 1$.

(c) The first order conditions is

$$\frac{q(p_1 - 1)}{w + (p_1 - 1)z} = \frac{(1 - q)(1 - p_2)}{w + (p_2 - 1)z}.$$

Thus,

$$\boxed{z = \frac{q(p_1 - p_2) + p_1 - 1}{2(p_1 - 1)(1 - p_2)} w}$$

Question 8

(a) Suppose by way of contradiction that $k < 0$. Then $u(x) = E[u(x + k + X)] < E[u(x + X)] \leq u(E[x + X]) = u(x)$, a contradiction.

(b) Let x, x' and $0 \leq \alpha \leq 1$. We must prove that $u(\alpha x + (1 - \alpha)x') \geq \alpha u(x) + (1 - \alpha)u(x')$. Let $\overline{x} = \alpha x + (1 - \alpha)x'$ and let X be a lottery with outcome $x - \overline{x}$ with probability α and $x' - \overline{x}$ with probability $1 - \alpha$. Then $E[X] = 0$. Thus, $u(\alpha x + (1 - \alpha)x') = u(\overline{x}) = E[u(x + k + X)] \geq E[u(x + X)] = \alpha u(x) + (1 - \alpha)u(x')$.

Question 9

$$r_A(x, u_1) = -\frac{ku_2''(x) + v''(x)}{ku_2'(x) + v'(x)} \geq -\frac{ku_2''(x)}{ku_2'(x) + v'(x)} \geq -\frac{ku_2''(x)}{ku_2'(x)} = r_A(x, u_2).$$