Question 1 The production function is $f(z) = 2\min\{z_1, z_2\} + z_3$.

Clearly $z_1 = z_2$. In order to produce q units of output, one can use $z_1 = z_2 = q/2$ or $z_3 = q$, whichever has lower costs. Thus, $z_1 = z_2 = q/2$ will be used if $w_1 + w_2 < w_3$, resulting in costs $(w_1 + w_2)q$ and $z_3 = q$ will be used if $w_1 + w_2 > w_3$, resulting in costs $w_3 q$. Thus,

$$\boxed{c(w,q) = q\min\left\{\frac{w_1+w_2}{2}, w_3\right\}.}$$

Question 2 Suppose a production function $f(z_1, z_2)$ has the following properties.

1. The marginal product of input i only depends on the level of input i, not on that of the other input.
2. Changing both inputs by a factor of α changes output by α^2.

Let $\frac{\partial f(z_1,z_2)}{\partial z_i} = g_i(z_i)$. Then it follows that $f(z_1, z_2) = G_1(z_1) + G_2(z_2)$, where $G_i' = g_i$ (the integration constant k can be included into G_i). We can also define G_i such that $G_1(0) = G_2(0)$. Since $f(\alpha z_1, \alpha z_2) = \alpha^2 f(z_1, z_2)$, we get $G_1(\alpha z_1) + G_2(\alpha z_2) = \alpha^2 G(z_1) + \alpha^2 G(z_2)$. Choosing $\alpha = 0$ implies $G_1(0) + G_2(0) = 0$, and therefore $G_1(0) = G_2(0) = 0$. Further, $G_1(z_1) = G_1(z_1) + G_2(0) = z_1^2 G_1(1) + z_2^2 G_2(0) = z_1^2 G(1)$. Similarly, $G_2(z_2) = z_2^2 G_2(1)$. Thus,

The general form of a production function that satisfies both conditions is

$$\boxed{f(z_1, z_2) = az_1^2 + bz_2^2}$$

Use a, b, . . . to denote general parameters.

The parameters must satisfy the following conditions:

$$\boxed{a, b \geq 0.}$$

Question 3 `Statement 1 implies that white sugar must have a perfect substitute.`

`Statement 2: The profit function must be affine linear in` p`. As a consequence, the supply function (which is the derivative of the profit function with respect to` p`) is constant. This can happen if the transformation frontier is not smooth (i.e., has a ‘‘corner’’).`

Question 4 Note that $\frac{z_1}{z_2} = \sqrt{\frac{w_2}{w_1}}$. Thus,

$$z_1 = q\left(1 + \frac{z_1}{z_2}\right).$$

Solving for q yields the production function.

Therefore, the production function is given by

$$\boxed{f(z_1, z_2) = \frac{z_1 z_2}{z_1+z_2} = \left(z_1^{-1} + z_2^{-1}\right)^{-1}}$$

Question 5

(a) Suppose by way of contradiction that $k < 0$. Then $u(x) = Eu(x + k + X) \le u(x + k + E[X]) = u(x + k)$. Since $x > x + k$, this violates monotonicity of u, a contradiction.

(b) Let Y be an arbitrary random variable with mean $\bar{y}$. We must prove that $E[u(Y)] \le u(\bar{y})$. Let $x = \bar{y}$ and $X = Y - \bar{y}$. Then $u(\bar{y}) = E[u(k + Y)]$ for some $k \ge 0$. Since u is increasing, $u(\bar{y}) = E[u(k + Y)] \ge E[u(Y)]$.

Question 6 The portfolio optimization problem is given by

$$\max_{\alpha} \int 2(\alpha x + 1.1(1-\alpha)) - (\alpha x + 1.1(1-\alpha))^2 \, dF(x).$$

$$\int 2x - 2.2 - 2(\alpha x + 1.1(1-\alpha))(x - 1.1) \, dF(x) = 0.$$

Thus,

$$\int [0.22 - 0.2x - 2\alpha x^2 + 4.4\alpha x - 2.42\alpha] \, dF(x) = 0.$$

Therefore,

$$0.22 - 0.2E[X] - 2\alpha E[X^2] + 4.4\alpha E[X] - 2.42\alpha = 0.$$

Thus, $2.74\alpha = 0.06$.

$$\boxed{\textbf{The optimal portfolio is } \alpha = 0.0219, \beta = 0.9781.}$$

Question 7 Let $w'' = \alpha w + (1-\alpha)w$ and let $\tilde{q}$ be the optimal output at prices w''. Then concavity of the cost function implies $c(w'', \tilde{q}) \ge \alpha c(w, \tilde{q}) + (1-\alpha)c(w', \tilde{q})$. Thus, $\pi(w'') = p(\tilde{q})\tilde{q} - c(w'', \tilde{q}) \le \alpha(p(\tilde{q})\tilde{q} - c(w, \tilde{q})) + (1-\alpha)(p(\tilde{q})\tilde{q} - c(w', \tilde{q})) \le \alpha\pi(w) + (1-\alpha)\pi(w')$.

Question 8 Since $E[X] = E[Y]$ we get $\int_a^b F(x)\,dx = \int_a^b G(x)\,dx$. Now suppose that there exists t such that $\int_a^y F(x)\,dx > \int_a^t G(x)\,dx$. Then $F(t) \le G(t)$ for all $t \le \bar{t}$ implies that $t > \bar{t}$. Since $F(t) \ge G(t)$ for all $t \ge \bar{t}$, we have $\int_t^b F(x)\,dx > \int_t^b G(x)\,dx$. This, however, implies $\int_a^b F(x)\,dx > \int_a^b G(x)\,dx$, a contradiction.

Question 9 The cdfs on $[0, 1]$ are $F(x) = x^3$ and $G(x) = x^4$. Thus, $F(x) \ge G(x)$, i.e., Y second order stochastically dominates X.