Question 1 The equivalent variation is EV $= e(p^0, u^1) - e(p^0, u^0) = e(p^0, u^1) - w$. Demand after the price change is $x = (45, 20)$. Thus, $u^1 = 900$. Therefore, $e(1, 1, 900) = 2(30)(1)(1) = 60$. Thus, *15 points*

The compensating variation is given by $EV = -300$.

(Recall that $e(p, u) = 2\sqrt{up_1p_2}$.)

Question 2 Suppose that an expenditure function is given by $e(p_1, p_2, u) = p_1u^2$. Hicksean demand reveals that only x_1 is consumed, i.e., $x_1 = u^2$. Thus, *14 points*

The quasiconcave & monotone utility function is $u(x_1, x_2) = \sqrt{x_1}$

The other utility function is $u(x_1, x_2) = \sqrt{x_1} - x_2$

In fact, any utility function $u(x_1, x_2) = \sqrt{x_1} - g(x_2)$ works where $g(x_2) \geq g(0)$.

Question 3 Let $X = \{a, b, c, d\}$ and suppose that a choice structure is given by

$$\mathfrak{B} = \{\{a, b, c\}, \{a, b\}, \{a, c, d\}\},$$

and

$$C(\{a, b, c\}) = \{c\},\ C(\{a, b\}) = \{a\},\ C(\{a, c, d\}) = \{c\}.$$

12 points

$a \succ b$ **$a \prec c$** **$a \succ d$** **$b \prec c$** **$b \prec$ or $\succ d$** **$c \succ d$**

Or

$a \succ b$ **$a \prec c$** **$a \prec d$** **$b \prec c$** **$b \prec d$** **$c \succ d$**

Question 4 Any w with $10 < w \leq 20$ works. For such a w, a solution to the utility maximization problem is $(w, 0)$. The required utility is 10. The solution to the expenditure minimization problem is $(10, 0)$. *12 points*

$10 < w \leq 20$, $x^*_U = (w, 0)$, $x^*_E = (10, 0)$

Question 5 The utility function is $u(x_1, x_2) = \min\{2x_1, 4x_2\}$. Thus, $2x_1 = 4x_2 = u$. Therefore, $h_1 = u/2$ and $h_2 = u/4$.Then *13 points*

$e(p, u) = \left(\frac{p_1}{2} + \frac{p_2}{4}\right) u$

Question 6

$e(p, u)$ is not differentiable at all prices $2p_1 = p_2$, i.e., $p_1/p_2 = 0.5$

A monotone and quasiconcave utility function that generates exactly the same expenditure function is $\boxed{\boldsymbol{u(x_1, x_2) = 1/6(x_1 + 2x_2) = x_1/6 + x_2/3.}}$ *6 points*

Question 7 A utility function is given by $u(x_1, x_2) = \min\{2x_1 + x_2, x_1 + 2x_2\}$. Then *10 points*

$\boxed{\boldsymbol{x(2, 2, 20) = (5, 5)}}$

$\boxed{\boldsymbol{x(2, 3, 60) = (12, 12)}}$

$\boxed{\boldsymbol{x(2, 5, 60) = (30, 0)}}$

$\boxed{\boldsymbol{x(2, 10, 60) = (30, 0)}}$

$\boxed{\boldsymbol{x(5, 3, 60) = (7.5, 7.5)}}$

Question 8 An expenditure function is given by

$$e(p, u) = \frac{p_1(up_2 - p_1)}{p_2}.$$

Differentiating $e(p, u)$ with respect to p_2 gives Hicksean demand $h_2(p, u) = p_1^2/p_2^2$. Thus, $x_2(p, w) = p_1^2/p_2^2$. Walras' law implies $p_1x_1 + p_1^2/p_2 = w$. Thus,

$$\boxed{\boldsymbol{x_1(p, w) = \frac{wp_2 - p_1^2}{p_1p_2} = \frac{w}{p_1} - \frac{p_1}{p_2}.}}$$

Alternatively, differentiating $e(p, u)$ with respect to p_1 gives Hicksean demand

$$h_1(p, u) = \frac{2p_1 - up_2}{p_2}.$$

Next, solving $e(p, u) = w$ for u yields

$$v(p, w) = \frac{p_1^2 + wp_2}{p_2p_1}.$$

Since $x_1(p, w) = h_1(p, v(p, w)$ we get *14 points*