Solutions: Mid-term Econ500

October 17, 2007

**Question 1** The equivalent variation is  $EV = e(p^0, u^1) - e(p^0, u^0) = e(p^0, u^1) - w$ . Demand after the price change is x = (45, 20). Thus,  $u^1 = 900$ . Therefore, e(1, 1, 900) = 2(30)(1)(1) = 60. Thus, 15 points

## The compensating variation is given by EV = -300.

(Recall that  $e(p, u) = 2\sqrt{up_1p_2}$ .)

**Question 2** Suppose that an expenditure function is given by  $e(p_1, p_2, u) = p_1 u^2$ . Hicksean demand reveals that only  $x_1$  is consumed, i.e.,  $x_1 = u^2$ . Thus, 14 points

The quasiconcave & monotone utility function is  $u(x_1, x_2) = \sqrt{x_1}$ 

The other utility function is  $u(x_1, x_2) = \sqrt{x_1} - x_2$ 

In fact, any utility function  $u(x_1, x_2) = \sqrt{x_1} - g(x_2)$  works where  $g(x_2) \ge g(0)$ .

**Question 3** Let  $X = \{a, b, c, d\}$  and suppose that a choice structure is given by

$$\mathfrak{B} = \{\{a, b, c\}, \{a, b\}, \{a, c, d\}\},\$$

and

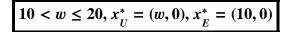
$$C(\{a, b, c\}) = \{c\}, C(\{a, b\}) = \{a\}, C(\{a, c, d\}) = \{c\}.$$

12 points

$$a > b$$
 $a < c$  $a > d$  $b < c$  $b < \text{or} > d$  $c > d$ Or $a > b$  $a < c$  $a < d$  $b < c$  $b < d$  $c > d$ 

**Question 4** Any w with  $10 < w \le 20$  works. For such a w, a solution to the utility maximization problem is (w, 0). The required utility is 10. The solution to the expenditure minimization problem is (10, 0).

12 points



Question 5 The utility function is  $u(x_1, x_2) = \min\{2x_1, 4x_2\}$ . Thus,  $2x_1 = 4x_2 = u$ . Therefore,  $h_1 = u/2$  and  $h_2 = u/4$ . Then 13 points

$$e(p,u)=\left(\tfrac{p_1}{2}+\tfrac{p_2}{4}\right)u$$

## **Question 6**

e(p, u) is not differentiable at all prices  $2p_1 = p_2$ , i.e.,  $p_1/p_2 = 0.5$ 

A monotone and quasiconcave utility function that generates exactly the same ex-

penditure function is  $u(x_1, x_2) = 1/6(x_1 + 2x_2) = x_1/6 + x_2/3$ . 6 points

**Question 7** A utility function is given by  $u(x_1, x_2) = \min\{2x_1 + x_2, x_1 + 2x_2\}$ . Then 10 points

x(2, 2, 20) = (5, 5)
x(2, 3, 60) = (12, 12)
x(2, 5, 60) = (30, 0)
x(2, 10, 60) = (30, 0)
x(5, 3, 60) = (7.5, 7.5)

Question 8 An expenditure function is given by

$$e(p,u) = \frac{p_1(up_2 - p_1)}{p_2}.$$

Differentiating e(p, u) with respect to  $p_2$  gives Hicksean demand  $h_2(p, u) = p_1^2/p_2^2$ . Thus,  $x_2(p, w) = p_1^2/p_2^2$ . Walras' law implies  $p_1x_1 + p_1^2/p_2 = w$ . Thus,

$$x_1(p,w) = \frac{wp_2 - p_1^2}{p_1 p_2} = \frac{w}{p_1} - \frac{p_1}{p_2}.$$

Alternatively, differentiating e(p, u) with respect to  $p_1$  gives Hicksean demand

$$h_1(p,u) = \frac{2p_1 - up_2}{p_2}.$$

Next, solving e(p, u) = w for u yields

$$v(p,w) = \frac{p_1^2 + wp_2}{p_2 p_1}.$$

Since  $x_1(p, w) = h_1(p, v(p, w))$  we get

14 points