Question 1 Recall that $x_1(p, e(p, u)) = \frac{\partial e(p,u)}{\partial p_1}$. Normalizing, $p_2 = 1$, we therefore get

$$\frac{\partial e(p, u)}{\partial p_1} = \frac{1}{p_1^2}.$$

Integrating with respect to p_1 yields $e(p_1, 1, u) = -\frac{1}{p_1} + C$, where C is an integration constant. Next, normalize $e(1, 1, u) = u$. Thus, $-1 + C = u$. Thus, $C = u + 1$, i.e., $e(p_1, 1, u) = 1 - \frac{1}{p_1} + u$. Since $e(p_1, p_2, u) = p_2 e(p_1/p_2, 1, u)$ we get, $e(p_1, p_2, u) = p_2(u + 1) - \frac{p_2^2}{p_1}$.

$$\boldsymbol{e(p, u) = p_2(u + 1) - \frac{p_2^2}{p_1}}$$

Question 2

(a) Demand at original prices is $(20, 20)$. Demand at the new prices is $(5, 20)$. Thus,

The government's tax revenue is 15

Utility after the tax is 100. We must now determine $e(1, 1, 100)$. Since prices are the same, $x_1 = x_2 = 1$. Thus, to obtain a utility of 100, we must have $x_1 = x_2 = 10$. Thus, $e(1, 1, 100) = 20$.

The loss to the consumer using the equivalent variation is 20

Thus, the deadweight loss is 5.

The deadweight loss is 33.3% of the tax revenue

(b) At prices $p_1 = p_2 = 2$ demand is $(10, 10)$.

The government's tax revenue is 20

The consumer's utility is again 100, and we have shown that $e(1, 1, 100) = 20$. Thus,

The loss to the consumer using the equivalent variation is 20

The deadweight loss is 0% of the tax revenue

Question 3 The optimization problem is

$$C(q_1, q_2; w) = \min_z wz, \text{ s.t. } -F(-z, q_1, q_2) \geq 0,$$

which results in the Lagrangean

$$\min_z wz + \lambda F(-z, q_1, q_2).$$

Thus, $\frac{\partial C(q;w)}{\partial q_1} = \lambda \frac{\partial F(-z,q_1,q_2}{\partial q_1}$ and $\frac{\partial C(q;w)}{\partial q_2} = \lambda \frac{\partial F(-z,q_1,q_2}{\partial q_2}$

$\boldsymbol{\frac{\frac{\partial C(q_1,q_2;w)}{q_1}}{\frac{\partial C(q_1,q_2;w)}{q_2}} = MRT(q_1, q_2)}$

(*Note:* The right-hand side should be the simplest possible expression you can find, but it should not contain $C(\cdot)$ or its derivatives. The envelope theorem helps.)

Question 4 Let $\pi(p_1, p_2)$ be the profit function. Then

$\boldsymbol{\pi(2,1) = 0}$	$\boldsymbol{\pi(1,1) = 0}$	$\boldsymbol{\pi(1,2) = 10}$
$\boldsymbol{\pi(1,3) = 23}$	$\boldsymbol{\pi(1,4) = 38}$	$\boldsymbol{\pi(2,5) = 33}$

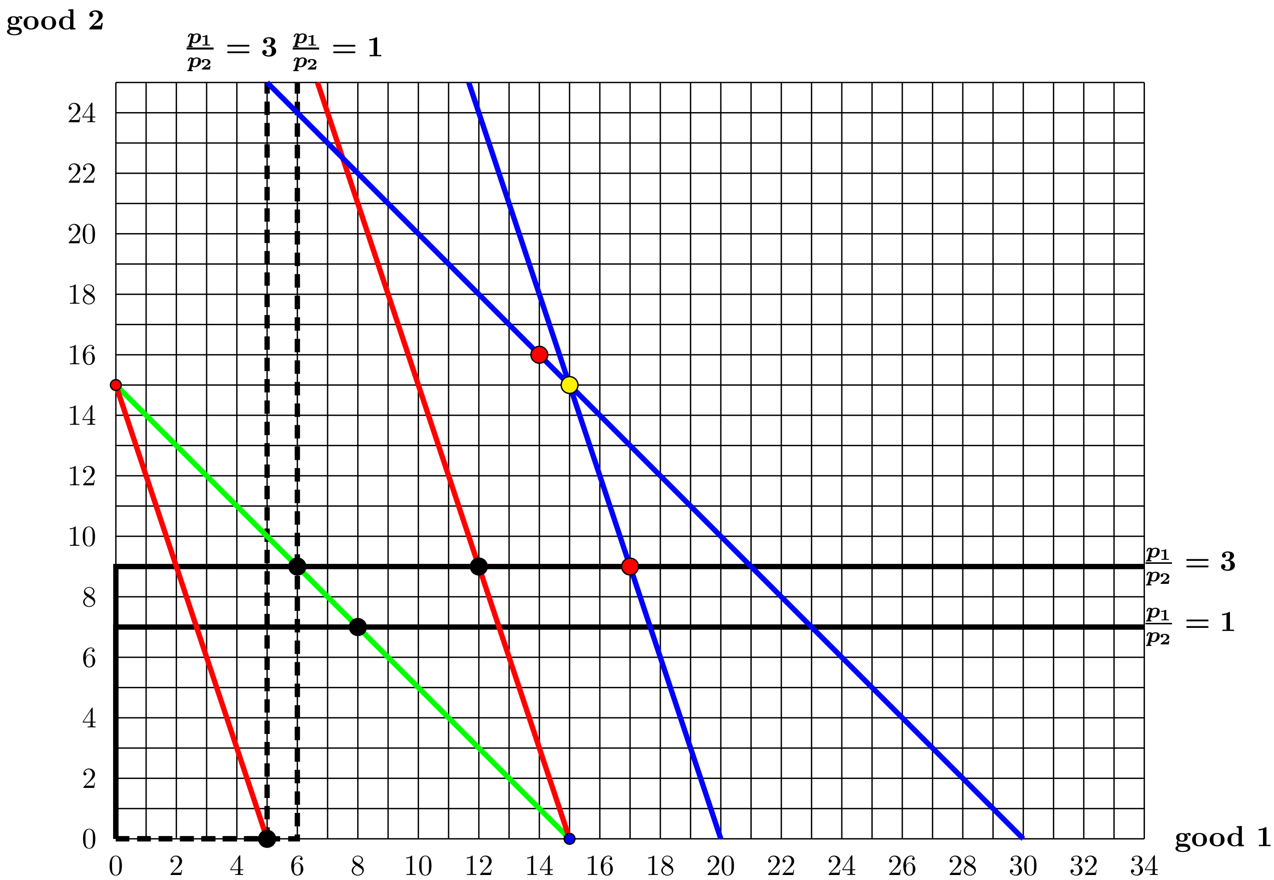
Question 5 MP $= 2 + 1/(2\sqrt{z}) > 2$. Thus, a solution to the profit maximization problem exists

for all $\boldsymbol{p}$ that fulfill $\boldsymbol{p < 2}$.

pMP $= 2p + \frac{p}{2\sqrt{z}} = w$. Solving this for z yields $z = \frac{1}{4(p-2)^2}$. Thus, $f(z) = \frac{1}{2(2-p)} + \frac{1}{2(2-p)^2}$.

for these $\boldsymbol{p}$ supply is $\boldsymbol{z(p) = \frac{1}{2(2-p)} + \frac{1}{2(2-p)^2}}$

Question 6



In the graph, the aggregate buget lines are blue, and the aggregate demand is indicated by the red circles. It follows immediately that the weak Axiom is violated. Hence, aggregate demand cannot be rationalized by preferences.

Question 7

(a) $u(x) = x - \gamma x^2$ implies $u'(x) = 1 - 2\gamma x$ and $u''(x) = -2\gamma$. Thus, Then *10 points*

absolute risk aversion is $\frac{2\gamma}{1-2\gamma x}$, which is

increasing in γ

when $u' \geq 0$

(b) The person solve $\max_{\alpha,\beta} \int_0^3 u(\alpha z + \beta)\, dz$ s.t. $\alpha + \beta = w$, which is equivalent to $\max_{\alpha,\beta} \int_0^3 u(w + \alpha(z-1))\, dz$. The first order condition is $\int_0^3 u'(w + \alpha(z-1))(z-1)\, dz = 0$. Thus, $\int_0^3 (1 - 2\gamma(w + \alpha(z-1)))(z-1)\, dz = 0$, i.e., $1.5(4\alpha\gamma + 2\gamma w - 1) = 0$.

$\boldsymbol{\alpha = \frac{1-2\gamma w}{4\gamma}}$

Question 8 The person solves $\max_{z_1,z_2} p u_i(w - D + z_A^i) + (1-p)(w + z_N^i)$, s.t., $q z_A + (1-q) z_N = 0$. Thus, the first order condition is

$$\frac{p u_i'(w - D + z_A^i)}{(1-p)(w + z_N^i)} = \frac{q}{1-q}. \tag{1}$$

We know that $u_2 = \psi(u_1)$, where ψ is concave and increasing. Thus,

$$\frac{p u_1'(w - D + z_A^2)\psi'(u_1(w - D + z_A^2))}{(1-p)(w + z_N^2)\psi'(u_1(w + z_N^2))} = \frac{q}{1-q}.$$

First, note that $w - D + z_A^i < w + z_N^i$ because of (1) and $p < q$ imply $u_i'(w - D + z_A^i) > u_i'(w + z_N^i)$. Thus, $\psi'(u_1(w - D + z_A^2)) > \psi'(u_1(w + z_N^2))$, which implies

$$\frac{p u_1'(w - D + z_A^2)}{(1-p) u_1'(w + z_N^2)} < \frac{q}{1-q}.$$

Thus,

$\boldsymbol{z_A^2 \geq z_A^1}$