Question 1 Suppose preferences $\succeq$ are rational and convex, but not necessarily continuous.

Let $x(p, w)$ be the Walrasian demand at prices $p$ and $w$. Prove that $x(p, w)$ is a convex set if $x(p, w) \neq \emptyset$.  

Proof Recall that the preferences are convex if $\{z \in \mathbb{R}_{+}^L | z \succeq y\}$ is a convex set for all $y \in \mathbb{R}_{+}^L$. Let $x, x' \in x(p, w)$ and $\alpha \in [0, 1]$. 

Complete the proof in the box below. That is, you must use the same notation. Also, you must specify the choice of $y$ (in the definition of convexity). Otherwise you will not get credit.

Let $y =$
Question 2 Let $X = \{a, b, c\}$ and $\mathcal{B} = \{\{a, b\}, \{a, c\}, \{b, c\}\}$.

Suppose that $C(\{a, b\}) = \{b\}$. Define the remaining choices below such that the choice structure satisfies the weak Axiom, such that $C(B) \neq \emptyset$ for all $B \in \mathcal{B}$ but that the choice is not rationalizable, i.e., there don’t exist rational preferences $\succeq$ that generate $C(\cdot)$.

\[
\begin{align*}
C(\{a, c\}) &= \\
C(\{b, c\}) &=
\end{align*}
\]
Question 3  Suppose a utility function is given by \( u(x_1, x_2) = \max\{2x_1, x_2\} \). Then

\[ x(5, 2, w) = (\quad,\quad) \]

(Note that \((5, 2)\) is the price vector).
Scratch paper: Anything on this page will not be graded.