

Name:

E-mail:

**All questions must be answered on this test form!**

*For each question you must show your work and (or) provide a clear argument.*

*If you need scratch paper, use the last pages and the back of the form.*

**Question 1** Suppose that preferences are continuous and locally non satiated. Let  $h(p, u)$  be Hicksian demand,  $x(p, w)$  Walrasian demand, and  $e(p, u)$  and  $v(p, w)$  be the expenditure and indirect utility functions, respectively. Simplify each of the following expressions as much as possible:

12 points

(a)  $h(p, v(p, w)) =$  .

(b)  $ph(p, v(p, w)) =$  .

(c)  $px(p, e(p, u)) =$  .

**Question 2** Suppose the Hicksian demand functions are given by

$$h_1(p_1, p_2, u) = \frac{p_2^2}{p_1^2}, \quad h_2(p_1, p_2, u) = u - \frac{2p_2}{p_1}$$

Then Walrasian demand is given by

*13 points*

$x_1(p, w) =$ <span style="float: right;"><math>x_2(p, w) =</math></span>
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*Note:* If you approach it correctly, you can answer this question with very little algebra.

**Question 3** Demand functions are given by

$$x_1(p_1, p_2, w) = \frac{w}{2p_1}, \quad x_2(p_1, p_2, w) = \frac{w}{2p_1}.$$

Suppose that  $p = (1, 1)$  and  $p' = (1, 2)$ . Prove by directly applying the definition that the weak Axiom is satisfied for any wealth levels  $w, w'$ .

*Note:* You will not get credit if you prove this result by using the substitution matrix, or by showing that there exists a utility function that describes the preferences.

*12 points*

*Proof:* Let  $x = x(p, w)$  and  $x' = x(p', w')$ . Then

$x = ( \quad , \quad ), \text{ and } x' = ( \quad , \quad )$

*Complete the argument in the box below*

**Question 4** A person, whose demand satisfies Walras' law, consumes only two goods.  
You have the following information about demand.

*13 points*

- At prices  $p = (4, 2)$  demand is  $x = (5, 8)$
- At prices  $p' = (1, 1)$  demand is  $x' = (4, x'_2)$ .

Then the weak axiom is satisfied if and only if  $x_2$  satisfies the following restrictions:

**Question 5** Suppose a utility function is given by  $u(x_1, x_2, x_3) = x_1 \min\{x_2, x_3\}$ .

Then Hicksian demand for good 2 is given by

$$h_2(p_1, p_2, p_3, u) =$$

As a consequence, goods 2 and 3 are **substitutes** **complements** because

$$\frac{\partial h_2(p_1, p_2, p_3, u)}{\partial p_3} \begin{matrix} > \\ < \\ = \end{matrix} 0.$$

Circle the correct answers. Then

13 points

**Question 6** Suppose a utility function is given by  $u(x_1, x_2) = x_1^2 - 2x_2$ . Then

*12 points*

$$e(p_1, p_2, u) =$$

$$v(p_1, p_2, w) =$$

**Question 7** Below is a substitution matrix for a consumer when prices are  $p_1 = 2$ ,  $p_2 = 1$ ,  $p_3 = 1$ . Fill in the missing entries assuming that *demand is derived from rational preferences*.

12 points

$$\begin{pmatrix} -6 & \boxed{\phantom{00}} & \boxed{\phantom{00}} \\ \boxed{\phantom{00}} & -10 & \boxed{\phantom{00}} \\ \boxed{\phantom{00}} & 4 & \boxed{\phantom{00}} \end{pmatrix}$$

**Question 8** Suppose that utility  $u$  is quasi concave (but not necessarily continuous, e.g., for  $x \in h(p, u)$  it may be the case that  $u(x) > u$ ). Prove that  $h(p, u)$  is a convex set if  $h(p, u) \neq \emptyset$ .

13 points

*Complete the argument in the box below*

*Proof:* Suppose that  $x, x' \in h(p, u)$ . Let  $\alpha \in [0, 1]$ . ...





*Scratch Paper: Not Graded*

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