Name:

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All questions must be answered on this test form!
For each question you must show your work and (or) provide a clear argument.
If you need scratch paper, use the last two pages or the back of the form.

Question 1 Suppose there are two goods. If $p_2 = 1$ then the demand for good 1 is given by $x(p_1, 1, w) = 20 - 4p_1$, for $p_1 \leq 5$. Then a utility function that describes these preferences is *10 points*

$$\boxed{\boldsymbol{u(x_1, x_2) =} \qquad\qquad\qquad\qquad}$$

Question 2 A firm's production function is $f(z_1, z_2) = \sqrt{z_1 + 2z_2}$. Let w_1, w_2 denote the factor prices and p the price of the output. Then the firm's profit function is

$$\boxed{\boldsymbol{\pi(w_1, w_2, p) =} \qquad\qquad\qquad\qquad\qquad\qquad}$$

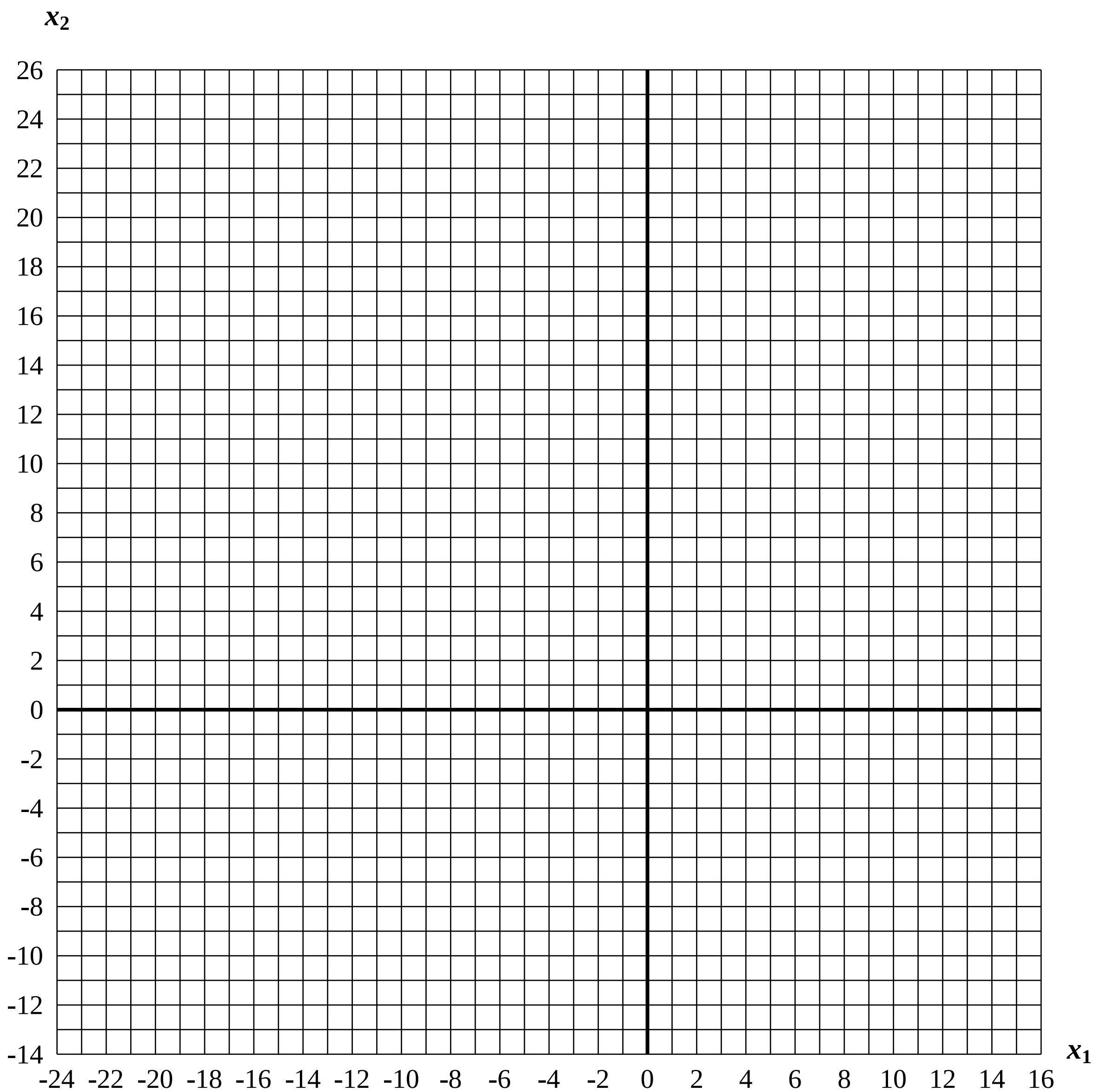
10 points

Question 3 Suppose that a firm can operate at two locations, using one input at each location (the input also has the same price at the two locations). The production functions at the two locations are given by $f(z)$ and $g(z)$. Clearly, $f(0) = g(0) = 0$. Further, suppose that f has constant returns to scale and that g is strictly concave (and has therefore decreasing returns to scale). When producing q units of output overall, the firm wants to allocate production at the two locations by minimizing the total use of input. In the box below write a simple mathematical conditions that specifies exactly when the firm produces a strictly positive amount of output using g. *12 points*

$g > 0$ if and only if

Now determine a sufficient conditions such that the firm only produces at the second location, i.e., $g > 0$ and $f = 0$.

Question 4 Consider a technology Y that can be described by the production function $f(z) = 4\sqrt{z+1} - 4$. Depict the additive closure $\overline{Y}$ of this technology in the grid below. *10 points*



Question 5 Suppose a firm uses two inputs. The cost function of a firm is of the form $c(w, q) = q^2(\phi(w_1) + \phi(w_2))$. Then the production function is of the form *10 points*

$$\boxed{\boldsymbol{f(z_1, z_2) =} \qquad\qquad\qquad\qquad}$$

(*Hint:* First, use the properties of the cost function to determine the structure of ϕ.)

Question 6 A consumer's wealth is $w = 400$. The utility function is $u(x_1, x_2) = x_1 x_2$. Prices are originally $p_1 = p_2 = 1$. However, the government introduces a tax of 3 Dollars on good 1, thereby raising p_1 to 4. Determine the amount of tax revenue per costumer, and deadweight loss of taxation, measured by the Equivalent Variation. *10 points*

Tax revenue is

The deadweight loss using EV is

Question 7 A person with wealth w wants to start a firm. There is one input and one output. The firm has constant returns to scale production function given by $f(z) = z$. The price of the input is 1 Dollar per unit. The price of the output, however, is determined by the state of nature. In particular, there are two states ω_1 and ω_2, where ω_1 occurs with probability q and ω_2 with probability $1-q$. The prices of the output in states ω_1 and ω_2 are given by p_1 and p_2. Let u be the person's Bernoulli utility function.

(a) Specify the person's optimization problem (the choice variable is obviously z): *4 points*

(b) Specify the most general condition on the parameters such that no person with a strictly monotone utility function will start a firm (i.e., not starting a firm means choosing $z = 0$). *4 points*

(c) Now suppose that $u(x) = \ln(x)$, that $p_1 > 1$, $p_2 < 1$ and $p_1 + p_2 > 1$. Determine the optimal amount of input z. *8 points*

$\boldsymbol{z =}$

Question 8 Let X be a random variable, with $EX = 0$. We define the *compensating risk premium* k by the equation $u(x) = E[u(x + k + X)]$. In the following assume that u is strictly increasing.

(a) Prove that if u is concave then $k \geq 0$. Complete the proof in the box below: *6 points*

Suppose by way of contradiction that $k < 0$:

(b) Suppose that $k \geq 0$ for all lotteries X and for all x. Prove that the utility function is $u(x)$ is concave. Complete the proof in the box below: *6 points*

Let x, x' and $0 \leq \alpha \leq 1$. We must prove that

Question 9 Let u_1 and u_2 be two utility functions on $[0, \overline{x}]$. We say that u_1 is strongly more risk averse than u_2 if there exists $k > 0$ and a non-increasing concave function $v(x)$ such that $u_1(x) = ku_2(x) + v(x)$.

Prove that if u_1 is strongly more risk averse than u_2, then $r_A(x, u_1) \geq r_A(x, u_2)$ for all x. *10 points*

Scratch Paper: Not Graded

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