Final Econ500, 8:00am December 12, 2007

Name:

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All questions must be answered on this test form!
For each question you must show your work and (or) provide a clear argument.
If you need scratch paper, use the last two pages or the back of the form.

Question 1 A production function is given by $f(z) = 2\min\{z_1, z_2\} + z_3$. Then the cost function is given by *10 points*

$$\boxed{c(w, q) = \qquad\qquad}$$

Question 2 Suppose a production function $f(z_1, z_2)$ has the following properties.

1. The marginal product of input i only depends on the level of input i, not on that of the other input.
2. Changing both inputs by a factor of $\alpha \geq 0$ changes output by α^2.

The general form of a production function that satisfies both conditions is *10 points*

$\boxed{\boldsymbol{f(z_1, z_2) =} \qquad\qquad\qquad\qquad}$

Use a, b, ... to denote general parameters.

The parameters must satisfy the following conditions:

$\boxed{\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad}$

No detailed proof needed.

Question 3 A bakery tells you the following two facts about its profit function:

1. "Currently, we use 100 units of white sugar in our production process. However, if the current price of this factor increased from 10 to 20, our profit would remain the same."
2. "Currently, our output price is \$10 and our profit is \$ 1000. If, instead, our output price were \$8 with probability 50% and \$12 with probability 50%, our expected profit would still be \$1000."

Assuming that these statements are true and that the firm behaves optimally, what can you conclude from these statements? *10 points*

Statement 1 implies:

Statement 2 implies:

Question 4 Conditional factor demand functions are given by

$$z_1(w,q) = q\frac{\sqrt{w_1} + \sqrt{w_2}}{\sqrt{w_1}}$$
$$z_2(w,q) = q\frac{\sqrt{w_1} + \sqrt{w_2}}{\sqrt{w_2}}$$

Therefore, the production function is given by *10 points*

$$\boxed{\boldsymbol{f(z_1, z_2) =}}$$

Question 5 Let X be a random variable, with $EX = 0$. We define the *compensating risk premium* k by the equation $u(x) = E[u(x + k + X)]$. In the following assume that u is strictly increasing.

(a) Prove that if u is concave then $k \geq 0$. Complete the proof in the box below: *10 points*

Suppose by way of contradiction that $k < 0$:

(b) Suppose that $k \geq 0$ for all lotteries X and for all y. Prove that the person is risk averse (You need to prove directly that the definition of risk aversion from class holds. Don't prove one of the other implications and say that according to the Theorem this is equivalent to risk aversion). Complete the proof in the box below: *10 points*

Let Y be an arbitrary random variable with mean $\bar{y}$. We must prove that...

Question 6 Assume a Bernoulli utility function is given by $u(x) = 2x - x^2$. The person has 1 units of wealth that can be allocated between a risky asset, X with $E[X] = 1.4$ and $E[X^2] = 0.5$. There is a riskless asset, a bond, with an interest rate of 10%. Let α be amount of money invested in the risky asset, and $\beta = 1 - \alpha$ be the amount invested in the riskless asset. (We allow for the possibility of short sales, i.e., α and β can also be negative.) Then the portfolio optimization problem is given by *10 points*

The optimal portfolio is α = , β = . 8%

Question 7 Suppose a firm produces one output with many inputs. The price of the output is given by $p(q)$, where q is the quantity produced. Let w be the vector of input prices. Then the maximum profit is given by

$$\pi(w) = \max_{q \geq 0} p(q)q - c(w, q).$$

Prove that $\pi(w)$ is convex in w. *10 points*

Question 8 Let X and Y be random variables with values in $[a, b]$. The cdfs are F and G, respectively. Suppose $E[X] = E[Y]$.

Suppose that there exists $\bar{t}$ such that $F(t) \leq G(t)$ for all $t \leq \bar{t}$ and $F(t) \geq G(t)$ for all $t \geq \bar{t}$. Prove that X second order stochastically dominates Y. *10 points*

Question 9 Lotteries X and Y are described by the density functions

$$f(x) = \begin{cases} 3x^2 & \text{if } 0 \le x \le 1 \\ 0 & \text{otherwise;} \end{cases} \qquad g(x) = \begin{cases} 4x^3 & \text{if } 0 \le x \le 1 \\ 0 & \text{otherwise.} \end{cases}$$

Then *10 points*

_____ first order stochastically dominates _____

Provide a proof:

Scratch Paper: Not Graded

Scratch Paper: Not Graded