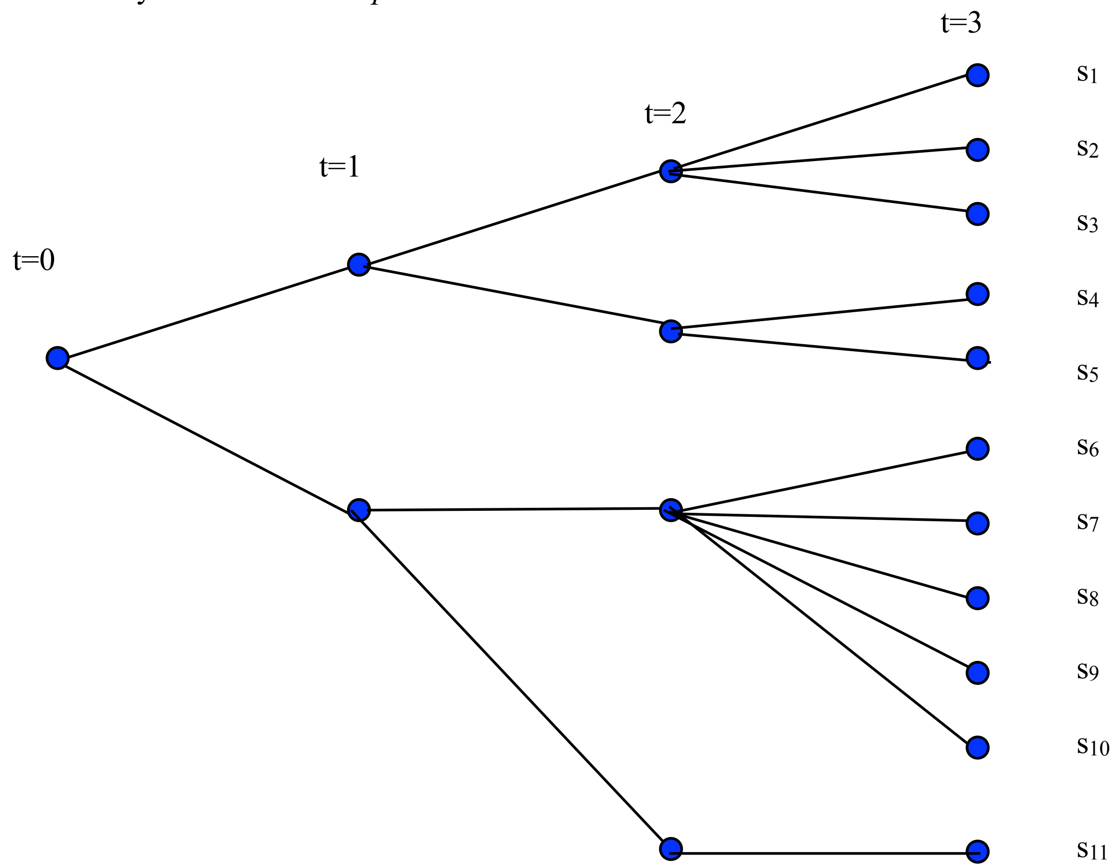
Midterm 1, Financial Economics February 15, 2010

Name:
Email: @illinois.edu

All questions must be answered on this test form.

Question 1: Let S={s_1,…,s_{11}} be the set of states. Suppose that at t=0 the state is unknown. At t=1 the person learns whether the the state is in {s_1,…,s_5} or {s_6,…,s_{11}}. At t=2 the person learns whether the state is in {s_1,…,s_3}, {s_4,s_5}, {s_6,…,s_{10}} or {s_{11}}. Finally, at t=3 all remaining uncertainty is revealed. *Graph the event tree.* *12 points*



Question 2: For each of the following Bernoulli utility functions determine whether or not the person is risk averse (in all cases $x \geq 0$). *To get credit you need to provide a proof, don't just say yes or no.*

12 points

A) $u(x) = 10x - 4x^4$

u'(x)=10-16x^3. Thus, u''(x)=-48x^2<0. Therefore u is concave and the person is risk averse.

B) $u(x) = 4x^2 - 10x^4$

u'(x)=8x-40x^3. Thus, u''(x)=8-120x^2, which is positive for small x, and hence not concave for all x. Thus, the person is not risk averse.

C) $u(x) = 4\sqrt{x} - 10x$

u'(x)=2$x^{-0.5}$. Thus, u''(x)=-$x^{-1.5}$<0. Therefore u is concave and the person is risk averse.

Question 3: Suppose there are four states S={s_1,…,s_4}. The probabilities of the four states are given by 0.2, 0.6, 0.1, and 0.1, respectively.
Suppose there are two investments: Investment A results in payoffs 10, 20, 40, 30, while investment B results in payoffs of 20, 10, 100, and 80, respectively, for each of the four states.

A) Suppose that the person's Bernoulli utility function is $u(x) = \sqrt{x}$. Then *6 points*

The expected utility of Investment A is 4.4959
The expected utility of Investment B is 4.6862

Therefore the investor will select (*mark the correct answer)* Investment B

The person's expected utility from investment A is
$0.2\sqrt{10} + 0.6\sqrt{20} + 0.1\sqrt{40} + 0.1\sqrt{30}$
The person's expected utility from investment B is $0.2\sqrt{20} + 0.6\sqrt{10} + 0.1\sqrt{100} + 0.1\sqrt{80}$

6 points

B) Now suppose that the Bernoulli utility function is $u(x) = -1/x$. Then

The expected utility of Investment A is -0.05583
The expected utility of Investment B is -0.07225

Therefore the investor will select (*mark the correct answer)* Investment A

The person's expected utility from investment A is $-0.2\frac{1}{10} - 0.6\frac{1}{20} - 0.1\frac{1}{40} - 0.1\frac{1}{30}$

The person's expected utility from investment B is $-0.2\frac{1}{20} - 0.6\frac{1}{10} - 0.1\frac{1}{100} - 0.1\frac{1}{80}$

8 points

Question 4: Suppose that asset A has a return of 20% with probability 0.5, 10% with probability 0.3 and -10% with probability 0.2.
Suppose a person's utility is $U(\mu,\sigma)= \mu-4\sigma^2$. Then

the person's utility from asset A is 0.0584

Determine the return of a riskless asset that gives the person exactly the same utility.

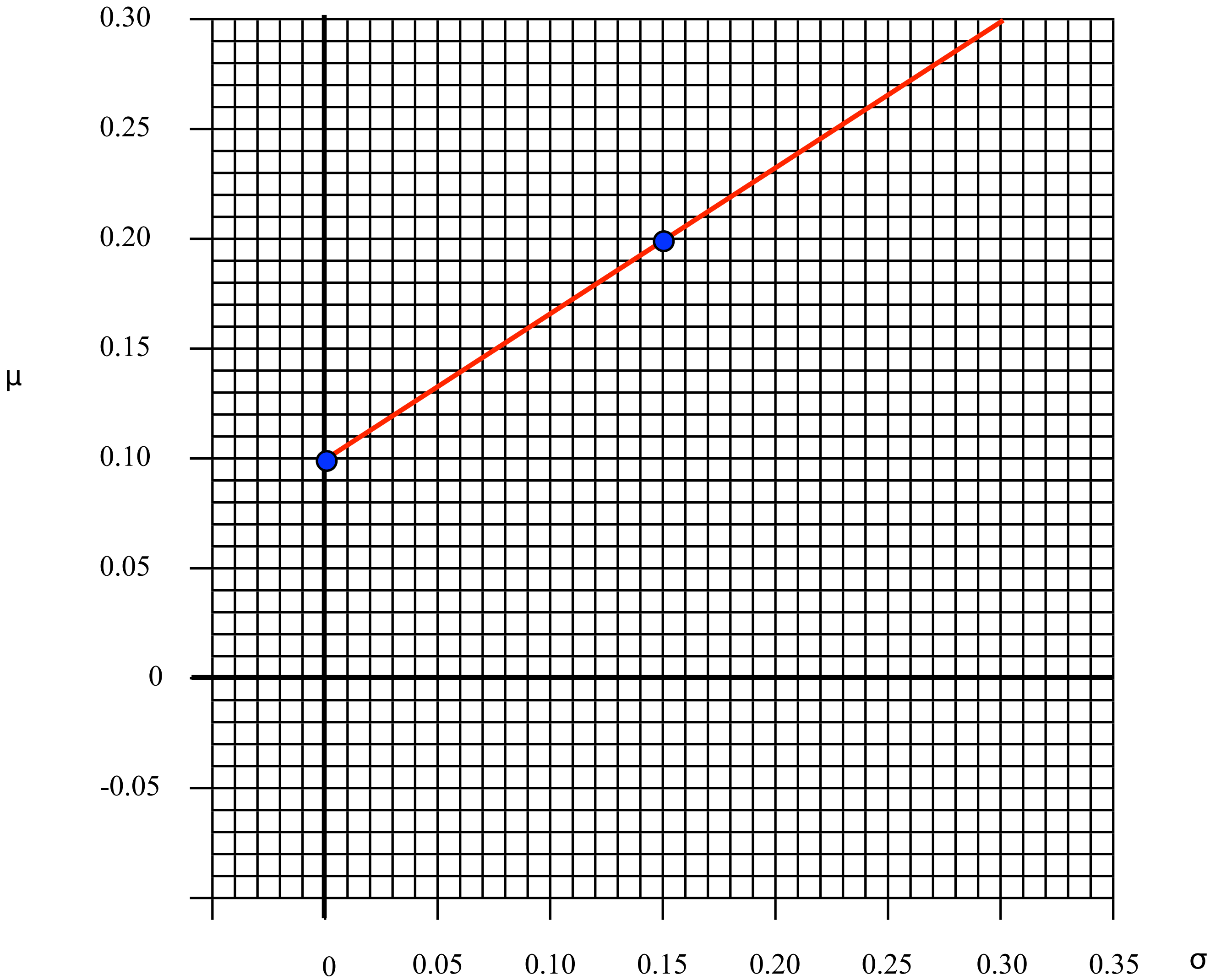
4 points

the return of the riskless asset must be 5.84%

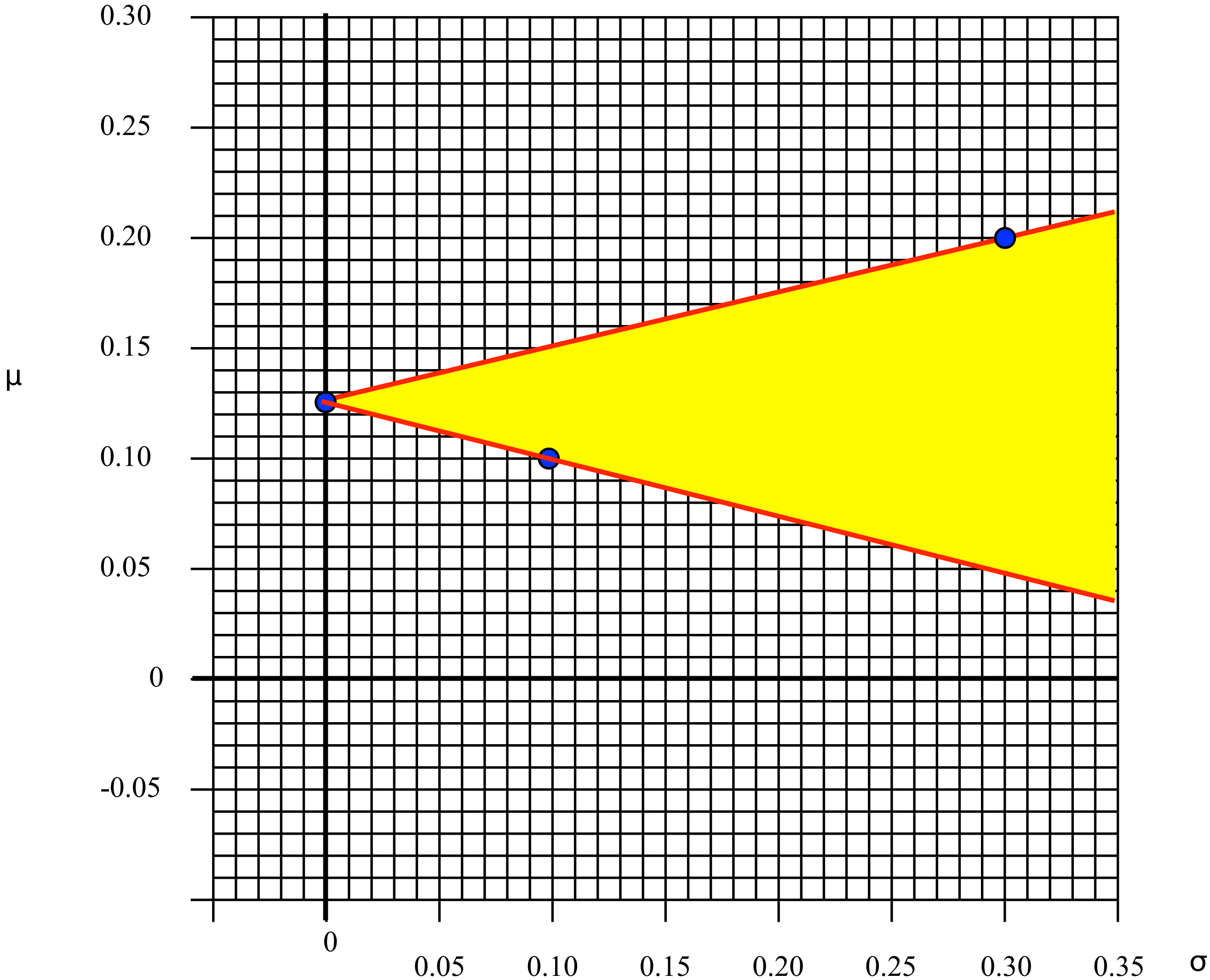
The expected return of A is 0.2(0.5)+0.1(0.3)-0.1(0.2)=0.11
The variance is $0.5(0.2-0.11)^2+0.3(0.1-0.11)^2+0.2(-0.1-0.11)^2=0.0129$.

Question 5: Asset A has a mean return of 20% and a standard deviation of 15% (i.e., μ=0.2 and σ=0.15. In addition, there is a riskless asset that has a return of 10%. Graph the efficient frontier in the grid below:

12 points



Question 6: Suppose there are two risky assets, A, and B. Suppose that μ_A=0.1 and σ_A=0.1, while μ_B=0.2 and σ_A=0.3. Suppose that the correlation between the returns of assets A and B is -1. Graph the set of feasible portfolios in the grid below (*clearly indicate the set by shading it).* *12 points*



The mean return is 0.1a+0.2(1-a). The standard deviation of the portfolio is $0.01a^2-2(0.1)(0.3)a(1-a)+0.09(1-a)^2=(0.1a-0.3(1-a))^2=(0.4a-0.3)^2$. Thus, the standard deviation is $|0.4a-0.3|$. Thus, a=0.75 the standard deviation is 0 and the return is 0.125

Question 7: Suppose there is a risky asset, with return 0.4 and standard deviation 0.2 and a riskless asset with return 0.1. The investor has mean variance preferences given by $G(\mu,\sigma)= \mu-0.6\sigma^2$. The person wants to find the optimal portfolio (a,1-a) where a is the fraction of wealth invested in the risky asset and (1-a) the fraction invested in the riskless asset.

14 points

The optimal value of a is 6.25

The mean return of the optimal portfolio is 197.5%

The mean return is 0.4a+0.1(1-a)=0.1+0.3a. The standard deviation is 0.2a. Thus, utility is $0.1+0.3a-0.6(0.2a)^2=0.1+0.3a-0.024a^2$ To maximize utility, we must have 0.3=0.048a. Thus, a=6.25. Thus, the mean return is 1.975

Question 8: Suppose there are three risky assets A, B, and C. Their returns are μ_A=0.2, μ_B=0.3, and μ_C=0.5. Their standard deviations are σ_A=0.1, σ_B=0.1, and σ_C=0.2. The correlation coefficient are $\rho_{1,2}$=-0.5, while $\rho_{1,3}$=$\rho_{2,3}$=0. Determine the portfolio (a_1,a_2,a_3) that has the lowest standard deviation (i.e., the MRP).

14 points

a_1= 4/9	a_2= 4/9	a_3= 1/9

The portfolio's mean return is 27.78%

The portfolio's standard deviation is 6.67%

The variance of the portfolio is given by

$$0.01a_1^2 + 0.01a_2^2 + 0.04a_3^2$$

Thus, we solve $\max_{a_1,a_2,a_3} 0.01a_1^2 + 0.01a_2^2 + 0.04a_3^2$ subject to $a_1 + a_2 + a_3 = 1$. The Lagrangean is

$$L = 0.01a_1^2 + 0.01a_2^2 + 0.04a_3^2 - \lambda(a_1 + a_2 + a_3 - 1).$$

The first order conditions are $0.02a_1 = \lambda$, $0.02a_2 = \lambda$, and $0.08a_3 = \lambda$. The first two equations yield $a_1 = a_2$. The second and third imply $a_1 = 4a_3$. Thus, the constraint implies that $9a_3 = 1$.

Hence, $a_1 = \frac{4}{9}, a_2 = \frac{4}{9}, a_3 = \frac{1}{9}$.