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*All questions must be answered on this test form. You must show your work*

**Question 1:** Suppose there are three states at  $t=1$  and two assets. The values of asset 1 in the three states are 10, 2, and 4, respectively. The values of asset 2 are 2, 8, 4. At  $t=0$  the price of asset 1 is 6 Dollars and the price of asset 2 is 5 Dollars.

*10 points*

Denote state prices by  $q_1$ ,  $q_2$ , and  $q_3$ . Specify the two equations that  $q_1$ ,  $q_2$ , and  $q_3$  must satisfy. *Note: You do not have to solve the equations.*

Equation 1:

$$10q_1 + 2q_2 + 4q_3 = 6$$

Equation 2:

$$2q_1 + 8q_2 + 4q_3 = 5$$

**Question 2:** Suppose there are two states  $s_1, s_2$ . Stock A has a return of 10% in state  $s_1$  and 0% in state  $s_2$ . Asset B has a riskless return of 2%.

12 points

Then the probabilities of states  $s_1, s_2$  that allow for risk neutral valuation are given by

$$\pi_1=0.2, \pi_2=0.8$$

$$0.02 = 0.1\pi_1 + 0\pi_2. \text{ Thus, } \pi_1=0.2.$$

Recall that a put option on the stock gives the holder the right to sell the option at  $t=1$  for a price  $K$ , the strike price. Suppose that the price of the stock at  $t=0$  is 20, and that the strike price is  $K=21$ . Determine the price of the option at  $t=0$ .

The price of the put option at  $t=0$  is 78.4 cents

The price of the stock is either 22 or 20. Thus, the option's value is 0 or 1. Thus, the value at  $t=0$  is  $0.8/(1+r)=0.8/1.02=0.784$ .

The stock price at  $t=0$  is again 20. A futures contract for the stock determines a price  $q$  (the futures price) at which the holder of the contract (the long position) must buy the stock at  $t=1$ , and the short position must sell - *no money changes hands at  $t=0$* . The futures price  $q$  is determined by demand and supply. As a consequence, at the equilibrium  $q$  must be such that the value of the futures contract at  $t=0$  is zero (else there would be an arbitrage as the next question indicates).

Therefore the futures price  $q=20.40$ .

We must have  $0=(22-q)\pi_1 + (20-q)\pi_2=0.2(22-q)+0.8(20-q)=20.40-q$ .

**Question 3:** Suppose there are three states. The probabilities that allow risk-neutral valuation are given by  $\pi_1=0.2$ ,  $\pi_2=0.5$ ,  $\pi_3=0.3$ . The riskless rate is 5%.  
 There is a stock with a value of 20, 70, or 10 in each of the three states at  $t=1$ .  
 In addition, there is a call option on the stock with a strike price of 28.

At $t=0$ the price of the stock is      40      , and the price of the option is 20
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10 points

$1.05P=20(0.2)+70(0.5)+10(0.3)=42$ . Thus, the stock price is \$40.

The option is only in the money in state 2, resulting in a payoff of  $70-28=42$ . Thus, the price is  $42(0.5)/1.05$

The payoff with  $q=20$  is 0, 50, and -10. Go short in the stock, resulting in income of 40 at  $t=0$  and payoff of -20, -20, -20. Now buy 20 units of the riskless asset. Cost is  $20/1.05$ , i.e., \$19.05. Thus, the final payoff at  $t=0$  is \$20.95.

20	units of the riskless asset (Assume each unit is 1 Dollars)
-1	units of the stock
0	call options
1	future contracts

**Question 4:** The stock price of Caterpillar (CAT) and of the S&P500 are given below

	A	B	C	D	E	F	G	H	I
1	Date	CAT Stock Price	S&P 500	6 Month T-bill rate	return CAT	return s&P500	riskless rate weekly	$r_j - r_f$	$r_m - r_f$
2	4/06/09	31.07	856.56	4.9	0.0058	0.01522	0.00092	0.004873	0.014303
3	4/13/09	31.25	869.60	4.95	0.0416	-0.00388	0.00093	0.04067	-0.0048
4	4/20/09	32.55	866.23	5.01	0.1078	0.01303	0.00094	0.106894	0.012093
5	4/27/09	36.06	877.52	5.01	0.0713	0.05893	0.00094	0.07033	0.057987
6	5/4/09	38.63	929.23	5.05	xxx	xxx	xxx	xxx	xxx

You want to estimate alpha and beta using weekly data for one year for CAT. On the top is the beginning of your spreadsheet. Label the columns (you don't need to use all of them), fill in the missing entries above, and specify the excel formula for column 3 in the boxes below.

**E3: B4/B3-1**

**F3: C4/C3-1**

**G3: (1+D/100)^(1/52)**

**H3: F3-H3**

**I3: G3-H3**

**In the regression the dependent variable (Y) is: column H**

**In the regression the explanatory variable(s) (X) is (are): column I**

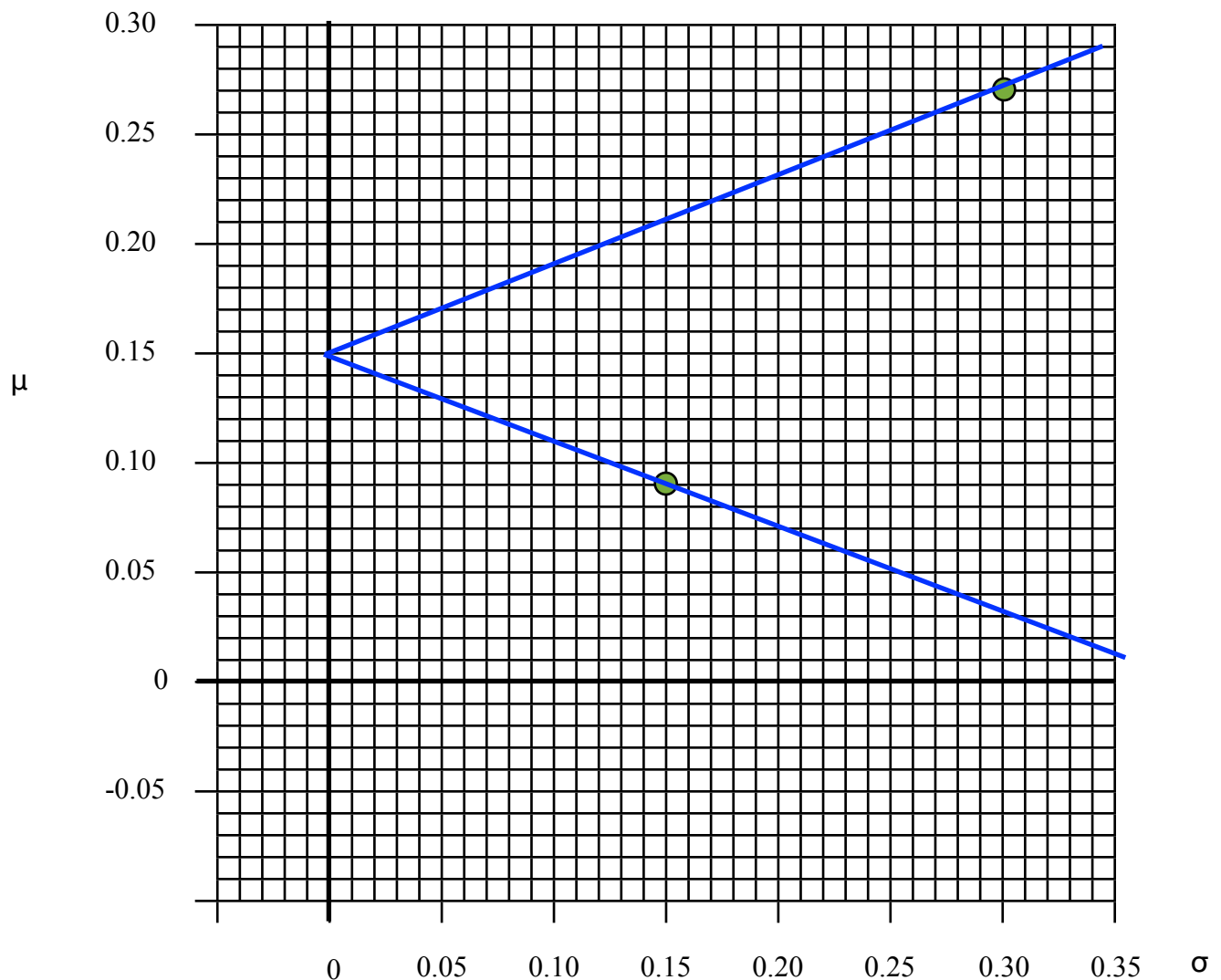
**Question 5:** Suppose there are two assets, A and B. Their mean returns are 0.09, and 0.27, respectively. The standard deviations are 0.15 and 0.30. The correlation between returns of A and B is -1.

Graph the set of all feasible portfolios by shading it. Also clearly indicate the efficient frontier.

12 points

The mean return of a portfolio  $(a, 1-a)$  is  $0.09a + 0.27(1-a)$ .

The variance is  $0.00225a^2 - 0.09a(1-a) + 0.09(1-a)^2 = (0.15a - 0.3(1-a))^2 = (0.45a - 0.3)^2$ . Thus, the standard deviation is  $|0.45a - 0.3|$ . Thus, the MRP has  $a = 2/3$  and a mean return 0.15



12 points

**Question 6:** Suppose there are three assets A, B, C with mean returns 0.1, 0.15, and 0.23. The standard deviations of all assets are identical equal to 0.1. Determine the MRP, i.e., the portfolio  $(a_1, a_2, a_3)$  with the lowest standard deviation (recall that  $a_1 + a_2 + a_3 = 1$ .)

$a_1 =$	$1/3$	$a_2 =$	$1/3$	$a_3 =$	$1/3$
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The return of the MRP = 16%
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The Lagrangian is given by  $L = 0.01a_1^2 + 0.01a_2^2 + 0.01a_3^2 - \lambda(0.01a_1 + 0.01a_2 + 0.01a_3 - 1)$

The first order conditions are  $0.02a_i = \lambda$  for  $i=1,2,3$ . Thus,  $a_1 = a_2 = a_3$ . Thus, the return is 16%

**Question 7:** Suppose the following Government bonds are traded. Face value is always \$100. All coupons are paid semiannually (if applicable).

12 points

- A) A Zero coupon bond that expires in 6 months. Current price \$99.00
- B) A Zero coupon bond that expires in 12 months. Current price \$97.50
- C) A bond with 4% coupon rate (next coupon payment in 6 months) that expires in 18 months. Current price \$101.50
- D) A bond with a 6% coupon rate (next coupon payment in 6 months) that expires in 2 years. Current price \$104.25

Then (report 3 digits after the period, i.e., up to 1/10 of a cent)

The price of a zero coupon bond that expires in 18 month is \$95.657

The price of a zero coupon bond that expires in 2 years is \$92.704

Determine the yield curve (i.e., the spot yield of zero coupon bonds with the indicated maturity)

maturity	6months	1 year	1 ½ years	2 years
yield (%)	2.03	2.56	3.00	3.86

The price of a ZC bond maturity 18months and face value 102 is 97.57.

The price of a ZC bond maturity 2 years and face value 103 is 95.485

**Question 8:** Consider the following prices for zero coupon bonds

12 points

Maturity	½ year	1 year	1 ½ years	2 years
Price	99.10	97.44	94.92	92.31

Determine the price of a bond that pays semiannual coupons with a coupon rate of 6% and face value \$100 that matures in 2 years.

The price of the bond is \$103.823

Determine the coupon rate of a bond with semiannual coupons and face value \$100 that sells at par and matures in 2 years (*report three digits after the period*).

The coupon rate is 4.008 %

Determine the forward rate on a zero coupon with maturity of one year, issued in one year (i.e., if you can enter a contract now that determines the yield to be paid on a zero coupon (issued in one year, maturing in the following year), what must the yield be in order for there not to be an arbitrage?

The forward rate is 5.557%

First questions:  $p = 3(0.991 + 0.9744 + 0.9492 + 0.9231) + 92.31$

Second questions:  $100 = 0.5c(0.991 + 0.9744 + 0.9492 + 0.9231) + 92.31$

Third question: The yield of the 1 ZC bond is  $100/97.44 - 1 = 2.627\%$ . The yield over two years of the 2 year ZC bond is 8.331%. Thus, the forward rate solves  $1.02627(1+r) = 1.08331$ , i.e.,  $r = 5.557\%$ .



**Question 9:** On May 5, 2010 the spread on credit default swaps (CDSs) for 5 year Greek governments bonds reached a record of 865 basis points. That is, suppose you buy a such a CDS and hold a 5 year Greek government bond with face value 100 Euro at the same time, then you must pay Euro 8.65 per year to the issuer of the CDS. In exchange for that, the issuer of the CDS will compensate you for any interest or principal payments that the Greek government defaults on. In other words, if you have a portfolio consisting of a CDS and the underlying bond, then this would be equivalent to holding a bond without default risk (because after paying the fee on the CDS you are promised to receive principal and interest either from the Greek government or the issuer of the CDS).

10 points

The German government is planning to provide a loan of 27 Billion Euro to Greece at an interest rate of 5% annually (one 5% coupon per year), with 5 year maturity. We want to determine the cost to the German government of providing the loan. One way to price the risk of default is to use CDSs. I.e., suppose you hold CDSs in addition to the loan to completely insure against default risk. As a consequence, you pay the annual fee in years 1 to 5 of 8.65% of the principal of 27 Billion to the issuer of CDS in exchange for making the coupon and principal payments riskless.

In the first row of the table below, list the net payments Germany gets/makes in each year - negative numbers if Germany is a net payer, and positive, otherwise (assuming that the loan is completely insured via CDSs). The first number is already filled in. In the second row, determine the present value of each number in the row above, assuming a riskless rate of 3% (flat yield curve).

date	now	1 year	2 years	3 years	4 years	5 years
payment in Billions	-27	-0.9855	-0.9855	-0.9855	-0.9855	26.0145
present value in Billions	-27	-0.9567961	-0.9289283	-0.9018721	-0.875604	22.440336

Thus, you can conclude that present value of the cost to the German government of providing the loans is

-8.223 Billion Euro

Give that the population of Germany is 82 Million, the

per capita cost is Euro 100.28