

**Midterm 1, Financial Economics**

February 15, 2010

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*All questions must be answered on this test form.*

**Question 1:** Let  $S=\{s_1,\dots,s_{11}\}$  be the set of states. Suppose that at  $t=0$  the state is unknown. At  $t=1$  the person learns whether the state is in  $\{s_1,\dots,s_5\}$  or  $\{s_6,\dots,s_{11}\}$ . At  $t=2$  the person learns whether the state is in  $\{s_1,\dots,s_3\}$ ,  $\{s_4,s_5\}$ ,  $\{s_6,\dots,s_{10}\}$  or  $\{s_{11}\}$ . Finally, at  $t=3$  all remaining uncertainty is revealed. *Graph the event tree.*

*12 points*

**Question 2:** For each of the following Bernoulli utility functions determine whether or not the person is risk averse (in all cases  $x \geq 0$ ). *To get credit you need to provide a proof, don't just say yes or no.*

*12 points*

A)  $u(x) = 10x - 4x^4$

B)  $u(x) = 4x^2 - 10x^4$

C)  $u(x) = 4\sqrt{x} - 10x$

**Question 3:** Suppose there are four states  $S=\{s_1,\dots,s_4\}$ . The probabilities of the four states are given by 0.2, 0.6, 0.1, and 0.1, respectively.

Suppose there are two investments: Investment A results in payoffs 10, 20, 40, 30, while investment B results in payoffs of 20, 10, 100, and 80, respectively, for each of the four states.

A) Suppose that the person's Bernoulli utility function is  $u(x) = \sqrt{x}$ . Then

6 points

The expected utility of Investment A is

The expected utility of Investment B is

Therefore the investor will select (*mark the correct answer*)

Investment A

Investment B

6 points

B) Now suppose that the Bernoulli utility function is  $u(x) = -1/x$ . Then

The expected utility of Investment A is

The expected utility of Investment B is

Therefore the investor will select (*mark the correct answer*)

Investment A

Investment B

**Question 4:** Suppose that asset A has a return of 20% with probability 0.5, 10% with probability 0.3 and -10% with probability 0.2.  
Suppose a person's utility is  $U(\mu, \sigma) = \mu - 4\sigma^2$ . Then

*8 points*

the person's utility from asset A is

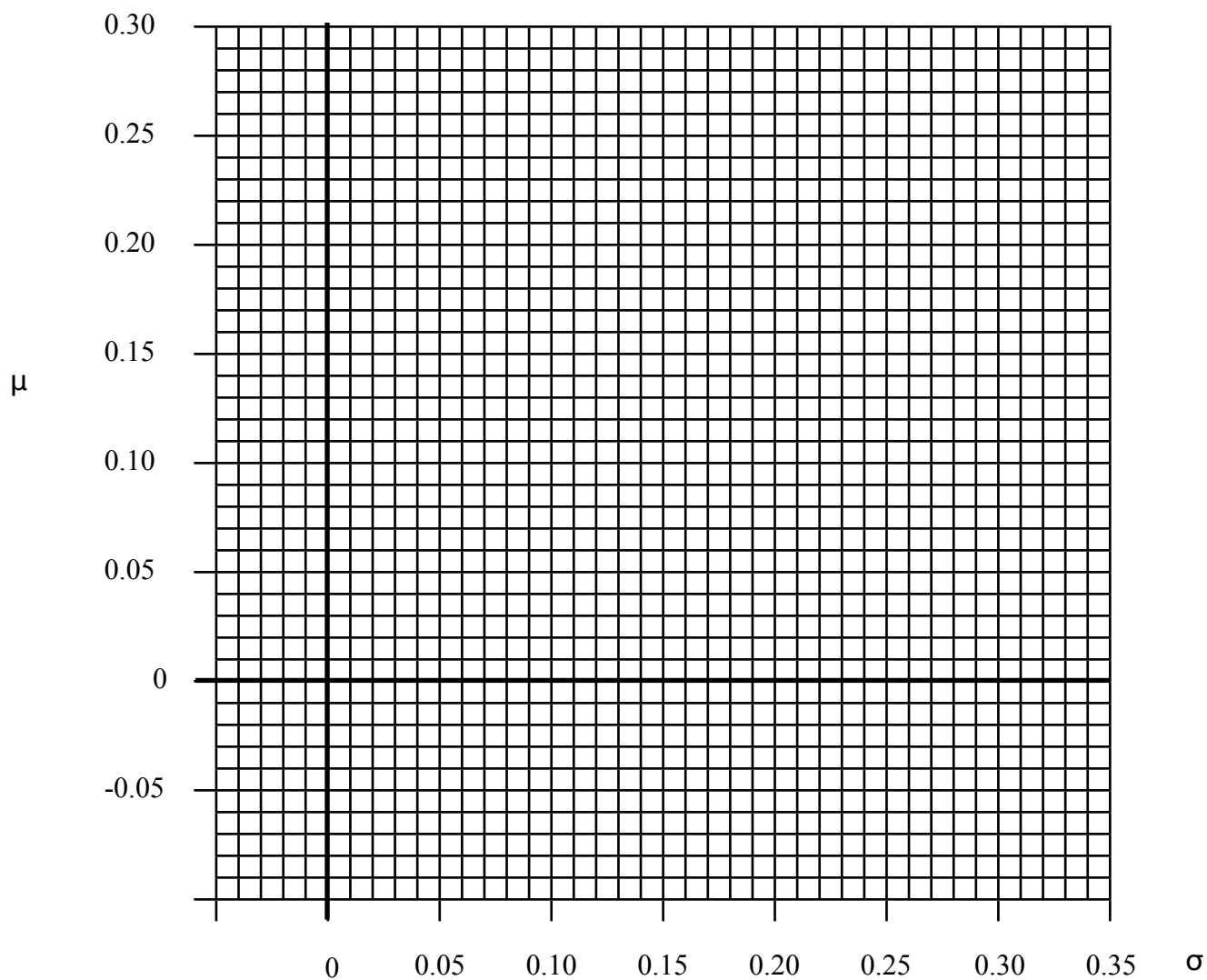
Determine the return of a riskless asset that gives the person exactly the same utility.

the return of the riskless asset must be

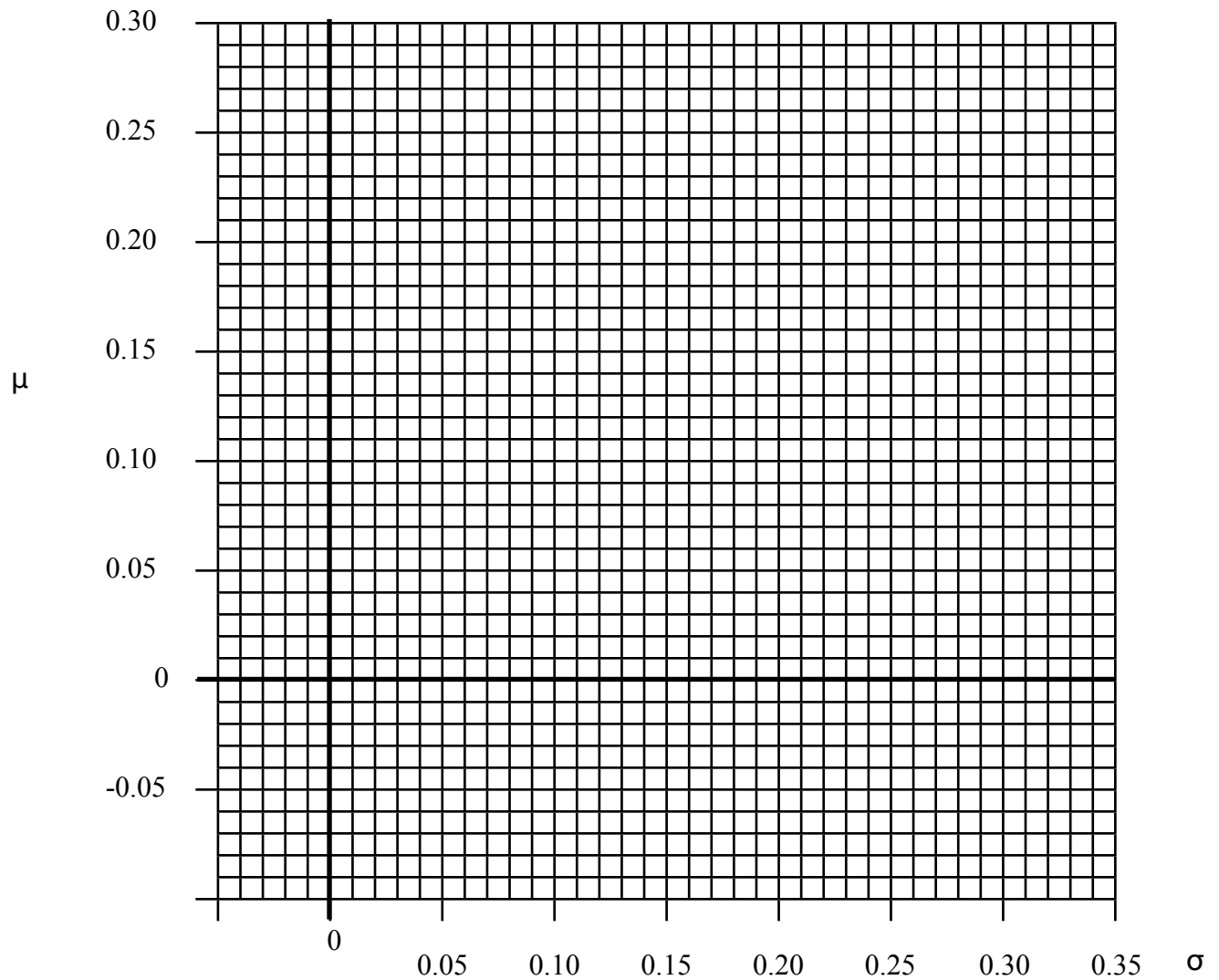
*4 points*

**Question 5:** Asset A has a mean return of 20% and a standard deviation of 15% (i.e.,  $\mu=0.2$  and  $\sigma=0.15$ ). In addition, there is a riskless asset that has a return of 10%. Graph the efficient frontier in the grid below:

12 points



**Question 6:** Suppose there are two risky assets, A, and B. Suppose that  $\mu_A=0.1$  and  $\sigma_A=0.1$ , while  $\mu_B=0.2$  and  $\sigma_B=0.3$ . Suppose that the correlation between the returns of assets A and B is -1. Graph the set of feasible portfolios in the grid below (*clearly indicate the set by shading it*). 12 points



**Question 7:** Suppose there is a risky asset, with return 0.4 and standard deviation 0.2 and a riskless asset with return 0.1. The investor has mean variance preferences given by  $G(\mu, \sigma) = \mu - 0.6\sigma^2$ . The person wants to find the optimal portfolio  $(a, 1-a)$  where  $a$  is the fraction of wealth invested in the risky asset and  $(1-a)$  the fraction invested in the riskless asset.

*14 points*

The optimal value of  $a$  is

The mean return of the optimal portfolio is

**Question 8:** Suppose there are three risky assets A, B, and C. Their returns are  $\mu_A=0.2$ ,  $\mu_B=0.3$ , and  $\mu_C=0.5$ . Their standard deviations are  $\sigma_A=0.1$ ,  $\sigma_B=0.1$ , and  $\sigma_C=0.2$ . The correlation between the assets is zero. Determine the portfolio  $(a_1, a_2, a_3)$  that has the lowest standard deviation (i.e., the MRP).

14 points

$a_1=$

$a_2=$

$a_3=$

The portfolio's mean return is

The portfolio's standard deviation is