Question 1 Firm 1 maximizes $800(h_1 + h_2) - h_1^2$. The derivative with respect to h_1 is $800 - 2h_1 = 0$. Therefore, the best response is $h_1 = 400$ independent of the action of the other firm.

Firm 2 maximizes $800(h_1 + h_2) - 2h_2^2$. The derivative with respect to h_1 is $800 - 2h_2 = 0$. Therefore, the best response is $h_2 = 200$ independent of the action of the other firm. Therefore $\boldsymbol{h_1 = 400}$, $\boldsymbol{h_2 = 200}$ is an equilibrium in strictly dominant strategies.

Question 2 If Mary bids 60 on the first unit then Joe will bid 61. Otherwise, if Mary bids less than 60, Joe will bid 60. If Mary bids 80 on the first unit then Joe will bid 81. Otherwise, if Mary bids less than 80, Joe will bid 80.

Question 3 Joe will bid 1,400, Mary will bid 800, and Paul will bid 1,200. Therefore, Joe will win and pay 1,200.

Question 4 In the last round 1 will offer $(1, 0, 0)$. Therefore, in the previous round, 2 will offer $(\delta, 1-\delta, 0)$. In the first round, 3 will therefore offer $\boldsymbol{(\delta^2, \delta(1-\delta), 1-\delta)}$.

Question 5

(a) $m_A = m_B = 0$.

(b) Candidates A,and B maximize

$$\frac{m_A}{m_A + m_B} - 0.05m_A, \text{ and } \frac{m_B}{m_A + m_B} - 0.05m_B.$$

The first order conditions are

$$\frac{m_B}{(m_A + m_B)^2} = 0.05, \text{ and } \frac{m_A}{(m_A + m_B)^2} = 0.05.$$

This implies $m_A = m_B$. Hence, the probability of winning is the same for A and B. Finally, $m_A = m_B$ implies $\boldsymbol{m_A = m_B = 5}$

(c) A's probability of winning Illinois is $\frac{m_A}{m_A+m_B}$, in which case (a) implies that no further money will be spend. With probability $\frac{m_B}{m_A+m_B}$ candidate A loses the primary. In this case we are in the subgame discussed in (b). Thus, A's chance of winning is 0.5 and A will spend 5. The net-benefit of winning the candidacy is therefore $1 - 0.05(5) = 0.75$. Therefore, A's payoff is

$$\frac{m_A}{m_A + m_B} + \frac{m_B}{m_A + m_B}\frac{1}{2}0.75 - 0.05m_A.$$

In contrast, candidate B will only win if he wins Illinois and California. Therefore, B's payoff is

$$\frac{m_B}{m_A + m_B}\frac{1}{2}0.75 - 0.05m_A.$$

The first order conditions are

$$\frac{5m_B}{8(m_A + m_B)^2} = 0.05, \text{ and } \frac{3m_A}{8(m_A + m_B)^2} = 0.05.$$

Therefore, $3m_A = 5m_B$. Now substitute this in $\frac{m_A}{m_A+m_B}$. Then the probability of winning Illinois is $\mathbf{\frac{5}{8}}$ for candidate A and $\mathbf{\frac{3}{8}}$ for candidate B.

If we substitute $3m_A = 5m_B$ into the first order condition for agent B we get $\boldsymbol{m_B = 1.758}$ and $\boldsymbol{m_A = 2.930}$.

The candidate who wins the first primary has a significantly higher chance to win further primaries. Early primary states therefore have much more impact on the selection of the candidate than states with later primaries. Candidates should therefore campaign more heavily and spend more resources in early primary states.

Question 6

(a) Joe maximizes $40s - s^2 - 200h$ we get $\boldsymbol{s = 20}$. Paul maximizes $10h - h^2 - 50s$ we get $\boldsymbol{h = 5}$. Joe's payoff is therefore -600, and Paul's payoff is -975.

(b) Assume Joe deviates. Then $s = 20$ and Joe's payoff is 400. Similarly, if Paul deviates, he will choose $h = 5$ and his payoff is 25.

In order to induce Joe to cooperate we need

$$0 \geq 400 - 600\delta - 600\delta^2 - \ldots = 400 - \frac{600\delta}{1-\delta},$$

which implies $\delta \geq 0.4$.

In order to induce Paul to cooperate we need

$$0 \geq 25 - 975\delta - 975\delta^2 - \ldots = 25 - \frac{975\delta}{1-\delta},$$

which implies $\delta \geq 0.025$. Therefore, we need $\boldsymbol{\delta \geq 0.4}$.

Question 7 Mark will announce voluntary compliance, and then log the optimal amount, i.e., $x = 85$.