Question 1 Let \( p \) and \( q \) be the probabilities of \( T \) and \( L \), respectively. Then player 1 must be indifferent between \( T \) and \( B \). Thus, \( q - (1 - q) = -2q + 4(1 - q) \) which implies \( q = 5/8 = 0.625 \). The expected payoff is therefore 1/4. Player 2 must be indifferent between \( L \) and \( R \). Thus, \( -p + 2(1 - p) = p - 4(1 - p) \) which implies \( p = 3/4 = 0.75 \). The expected payoff is \(-1/4\).

Player 1 chooses \( T \) with probability \( 3/4 \), and her expected payoff is \( 1/4 \)

Player 2 chooses \( L \) with probability \( 5/8 \), and her expected payoff is \(-1/4\).

Question 2 Player \( i \) must be indifferent between participating and not participating. The payoff from not participating is 0. Thus, \((1 - p)^{n-1}10 - c = 0\), i.e., \((1 - p)^{n-1} = c/10\). Therefore, the probability \((1 - p)^{n-1}\) that none of the other agents participates is \(1/10\).

Question 3 In the mixed strategy equilibrium, each person must be indifferent between actions 1, 2, and 3. Let \( p_i \) be the probability that the opponent chooses action \( i \). Then

- Payoff from \( a_1 = 1 \): \( 2p_1 + 1.5p_2 + p_3 \).
- Payoff from \( a_1 = 2 \): \( 2.5p_1 + 2p_2 \).
- Payoff from \( a_1 = 3 \): \( 3p_1 \).

Since all payoffs must be the same we get \( 2.5p_1 + 2p_2 = 3p_1 \) which implies \( p_1 = 4p_2 \). Further, \( 2p_1 + 1.5p_2 + p_3 = 3p_1 \). Thus, \( p_1 = 4p_2 \) implies \( 2.5p_2 = p_3 \).

Finally, \( p_1 + p_2 + p_3 = 1 \). Thus,

1 with probability \( 8/15 \), 2 with probability \( 2/15 \), 3 with probability \( 1/3 \).

Question 4 The subgame starting in period 3 is a standard ultimatum game: player 2 will offer \( m'_1 = 0, m'_2 = 0 \) and player 1 will accept. Thus, in period 2 player 2 will accept any bid \( m_2 = 100 \). Therefore, in period 1 player 1 will bid \( m_1 = 0, m_2 = 100 \), which will be accepted by player 2.

Player 1’s payoff is 0 and player 2’s payoff is 100.

Question 5 In the last period of the game, agent 3 will make a bid unless \( \max\{b_1, b_2\} \geq 88 \). Thus, person 2 will bid \( b_2 = 88 \) unless \( b_1 > 78 \). Therefore,

Player \( i = 1 \) wins and bids \( b_i = 88 \).
Question 6  \[ v(\{B_1, S_1\}) = 4, \ v(\{B_1, B_2, S_1\}) = 4 \ v(\{B_2, S_1\}) = 2 \]

\[ v(\{B_2, S_2\}) = 2, \ v(\{B_1, B_2, S_1, S_2\}) = 7, \ v(\{S_1, S_2\}) = 0 \]

Question 7  \( x_1 \) proposes \( y_1 \), \( x_2 \) proposes \( y_1 \), and \( x_3 \) proposes \( y_3 \). Both \( x_1 \) and \( x_3 \) are matched. Unmatched \( x_2 \) proposes \( y_3 \), which is accepted. Thus, \( x_1 \) is matched to \( y_1 \), \( x_2 \) to \( y_3 \) and \( x_3 \) is now unmatched. \( x_3 \) proposes to \( y_1 \), which is accepted. Thus, \( x_2 \) is now matched to \( y_3 \) and \( x_3 \) to \( y_1 \). \( x_1 \) proposes to \( y_3 \) which is accepted. Thus, \( x_1 \) is matched to \( y_3 \), and \( x_3 \) to \( y_1 \). \( x_2 \) has not further proposals and remains unmatched.

\[ x_1: y_3 \ , \ x_2: \text{unmatched} \ , \ x_3: y_1 \]

Question 8  Let \( b_2 = \alpha v_2 \). Then \( b_1 \geq b_2 \) if and only if \( v_2 < b_1/\alpha \). Thus, \( \text{Prob}(\{b_1 \geq b_2\}) = b_1/\alpha \). Player 1 therefore solves

\[ \max_{b_1} \frac{b_1}{\alpha} \sqrt{v_1 - b_1}. \]

The first order condition is

\[ \frac{\partial}{\partial b_1} : \frac{\sqrt{v_1 - b_1}}{\alpha} - \frac{b_1}{2\alpha \sqrt{v_1 - b_1}} = 0. \]

This implies \( 2(v_1 - b_1) = b_1 \), and hence \( b_1 = (2/3)v_1 \).

\[ \alpha = 2/3 \]