SOLUTIONS FOR QUIZ III (YELLOW)

Question 1 Bidder A's expected payoff is

$$\Pi(b_A) = (v_A - b_A) P(\{v_B | b_B(v_B) < b_A).$$

Note that

$$P(\{v_B | b_B(v_B) < b_A) = P(\{v_B | v_B^2 < b_A) = P(\{v_B | v_B < \sqrt{b_A}) = \sqrt{b_A},$$

where the last equality follows because v_B is uniformly distributed on $[0, 1]$. Thus, $\Pi(b_A) = (v_A - b_A)\sqrt{b_A}$. The first order conditions, $\frac{d}{db_A}\Pi(b_a) = 0$ yields

$$\frac{v_A - 3b_A}{2\sqrt{b_A}} = 0.$$

Therefore, $v_A = 3b_A$ which implies $\boldsymbol{b_A(v_A) = (1/3)v_A}$.

Question 2 The expected payoff of bidder i is

$$(100 - (2/3)b_i)F_{-i}(b_i) - (1/3)b_i(1 - F_{-i}(b_i)).$$

In a symmetric equilibrium $F_{-i} = F_i = F$. Therefore, we get

$$(100 - (2/3)b_i)F(b_i) - (1/3)b_i(1 - F(b_i)).$$

In a mixed strategy equilibrium, this expected payoff must be the same for all bids. Now note that at $b_i = 0$ we get $F(b_i) = 0$. Therefore the expected payoff is 0. As a consequence

$$(100 - (2/3)b_i)F(b_i) - (1/3)b_i(1 - F(b_i)) = 0.$$

Dropping the index i, this implies

$$\boldsymbol{F(b) = \frac{b}{300 - b}}.$$

Question 3 A bids 0.5α and B bids 0.5β. Therefore A will win if and only if $\beta \leq 1/4$. Therefore, A's expected payoff is

$$\begin{aligned} E[\alpha\beta - 0.5\beta | \alpha = 1/4, \beta < 1/4] &= E[(1/4)\beta - 0.5\beta | \beta < 0.25] \\ &= -(1/4)E[\beta | \beta < (1.4)] \\ &= -(1/4)(1/8) = -(1/32). \end{aligned}$$