SOLUTIONS FOR QUIZ III (YELLOW)

Question 1  Bidder A’s expected payoff is

\[ \Pi(b_A) = (v_A - b_A) P((v_B | b_B(v_B) < b_A). \]

Note that

\[ P((v_B | b_B(v_B) < b_A) = P((v_B^2 < b_A) = P((v_B < \sqrt{b_A}) = \sqrt{b_A}, \]

where the last equality follows because \( v_B \) is uniformly distributed on [0, 1]. Thus, \( \Pi(b_A) = (v_A - b_A)\sqrt{b_A} \). The first order conditions, \( \frac{d}{db_A} \Pi(b_A) = 0 \) yields

\[ \frac{v_A - 3b_A}{2\sqrt{b_A}} = 0. \]

Therefore, \( v_A = 3b_A \) which implies \( b_A(v_A) = (1/3)v_A. \)

Question 2  The expected payoff of bidder \( i \) is

\[ (100 - (2/3)b_i) F_i(b_i) - (1/3)b_i(1 - F_i(b_i)). \]

In a symmetric equilibrium \( F_i = F \). Therefore, we get

\[ (100 - (2/3)b_i) F(b_i) - (1/3)b_i(1 - F(b_i)). \]

In a mixed strategy equilibrium, this expected payoff must be the same for all bids. Now note that at \( b_i = 0 \) we get \( F(b_i) = 0 \). Therefore the expected payoff is 0. As a consequence

\[ (100 - (2/3)b_i) F(b_i) - (1/3)b_i(1 - F(b_i)) = 0. \]

Dropping the index \( i \), this implies

\[ F(b) = \frac{b}{300 - b}. \]

Question 3  \( A \) bids 0.5\( \alpha \) and \( B \) bids 0.5\( \beta \). Therefore \( A \) will win if and only if \( \beta \leq 1/4 \). Therefore, \( A \)'s expected payoff is

\[ E[\alpha\beta - 0.5\beta|\alpha = 1/4, \beta < 1/4] = E[(1/4)\beta - 0.5\beta|\beta < 1/4] \]

\[ = -(1/4)E[\beta|\beta < (1.4)] \]

\[ = -(1/4)(1/8) = -(1/32). \]