Solutions for Quiz III (White)

Question 1 Bidder A’s expected payoff is

\[ \Pi(b_A) = (v_A - b_A)P(\{v_B|b_B(v_B) < b_A\}). \]

Note that

\[ P(\{v_B|b_B(v_B) < b_A\}) = P(\{v_B|v_B < b_A^{1/3}\}) = b_A^{1/3}, \]

where the last equality follows because \(v_B\) is uniformly distributed on \([0, 1]\). Thus, \(\Pi(b_A) = (v_A - b_A)b_A^{1/3}\). The first order conditions, \(\frac{d}{db} \Pi(b_A) = 0\) yields

\[ \frac{v_A - 4b_A}{3b_A^{2/3}} = 0. \]

Therefore, \(v_A = 4b_A\) which implies \(b_A(v_A) = (1/4)v_A\).

Question 2 The expected payoff of bidder \(i\) is

\[ (100 - (3/4)b_i)F_{-i}(b_i) - (1/4)b_i(1 - F_{-i}(b_i)). \]

In a symmetric equilibrium \(F_{-i} = F_i = F\). Therefore, we get

\[ (100 - (3/4)b_i)F(b_i) - (1/4)b_i(1 - F(b_i)). \]

In a mixed strategy equilibrium, this expected payoff must be the same for all bids. Now note that at \(b_i = 0\) we get \(F(b_i) = 0\). Therefore the expected payoff is 0. As a consequence

\[ (100 - (3/4)b_i)F(b_i) - (1/4)b_i(1 - F(b_i)) = 0. \]

Dropping the index \(i\), this implies

\[ F(b) = \frac{b}{400 - 2b}. \]

Question 3 A bids \(0.5\alpha\) and \(B\) bids \(0.5\beta\). Therefore \(A\) will win if and only if \(\beta < 1/3\). Therefore, \(A\)’s expected payoff is

\[ E[\alpha\beta - 0.5\beta|\alpha = 1/3, \beta < 1/3] = E[(1/3)\beta - 0.5\beta|\beta < (1/3)] \]

\[ = -(1/6)E[\beta|\beta < (1/3)] \]

\[ = -(1/6)(1/6) = -(1/36). \]