**Question 1**  Two bidders, A, and B, have independent private values for an object which is sold in a simultaneous bid, first price auction. That is, A’s and B’s valuations $V_A$ and $V_B$ are stochastically independent, and uniformly distributed on $[0, 1]$. If A’s bid is $b_A$ and B’s bid is $b_B$ then A wins if $b_A > b_B$ and A must pay $b_A$. If $b_B > b_A$ then B wins and pays $b_B$. If there is a tie then each person gets the object with probability 0.5. Assume that person B uses the strategy $b_B(V_B) = V_B^2$. Then A’s best response is given by the strategy

$$b_A(V_A) = \quad 3\text{ points}$$
Question 2 Assume an object has a value of $100. This value is known to the two bidders. In the auction both agents make bids simultaneously. Let $b_i$ and $b_j$ denote the two bids. Then if $b_i > b_j$, person $i$ receives the object and pays $2/3$ of his/her bid. The loser, person $j$, pays $1/3$ of his/her paid. In this game, only mixed strategy equilibria exist. Find a mixed strategy equilibrium, where each agent’s bid is described by a c.d.f, $F(b)$ with $F(0) = 0$. (Recall that $F(b)$ is the probability that the person makes a bid less or equal to $b$.)

$$F(b) = \quad \quad 3 \text{ points}$$
Question 3 Assume the value of an object is given by $v = \alpha \beta$, where $\alpha$ and $\beta$ are stochastically independent, and uniformly distributed on $[0, 1]$. Person $A$ knows the value of $\alpha$ (but not $\beta$), while $B$ knows $\beta$ (but not $\alpha$). The object is sold in a Vickrey auction, i.e., both parties submit bids and the winner pays the second highest bid. Assume that $A$ observes a value $\alpha = 1/4$. Assume that both $A$ and $B$ bid the expected value of the object given their observed value of $\alpha$ and $\beta$, respectively. Then

\[
\text{A' expected payoff is } \quad 3 \text{ points}
\]

(This number could be negative, if A loses money using the above strategy)