

Name:

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**All questions must be answered on this test form!**

*For each question you must show your work and (or) provide a clear argument.*

**Question 1** Two bidders,  $A$ , and  $B$ , have independent private values for an object which is sold in a simultaneous bid, first price auction. That is,  $A$ 's and  $B$ 's valuations  $V_A$  and  $V_B$  are stochastically independent, and uniformly distributed on  $[0, 1]$ . If  $A$ 's bid is  $b_A$  and  $B$ 's bid is  $b_B$  then  $A$  wins if  $b_A > b_B$  and  $A$  must pay  $b_A$ . If  $b_B > b_A$  then  $B$  wins and pays  $b_B$ . If there is a tie then each person gets the object with probability 0.5. Assume that person  $B$  uses the strategy  $b_B(V_B) = V_B^3$ . Then  $A$ 's best response is given by the strategy

$b_A(V_A) =$

3 points

**Question 2** Assume an object has a value of \$100. This value is known to the two bidders. In the auction both agents make bids simultaneously. Let  $b_i$  and  $b_j$  denote the two bids. Then if  $b_i > b_j$ , person  $i$  receives the object and pays  $3/4$  of his/her bid. The loser, person  $j$ , pays  $1/4$  of his/her bid. In this game, only mixed strategy equilibria exist. Find a mixed strategy equilibrium, where each agent's bid is described by a c.d.f,  $F(b)$  with  $F(0) = 0$ .

(Recall that  $F(b)$  is the probability that the person makes a bid less or equal to  $b$ .)

$F(b) =$

*3 points*

**Question 3** Assume the value of an object is given by  $v = \alpha\beta$ , where  $\alpha$  and  $\beta$  are stochastically independent, and uniformly distributed on  $[0, 1]$ . Person  $A$  knows the value of  $\alpha$  (but not  $\beta$ ), while  $B$  knows  $\beta$  (but not  $\alpha$ ). The object is sold in a Vickrey auction, i.e., both parties submit bids and the winner pays the second highest bid. Assume that  $A$  observes a value  $\alpha = 1/3$ . Assume that both  $A$  and  $B$  bid the expected value of the object given their observed value of  $\alpha$  and  $\beta$ , respectively. Then

**A' expected payoff is**

*3 points*

*(This number could be negative, if A loses money using the above strategy)*