FINAL EXAM: ECON375, May 10, 2004

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All questions must be answered on this test form!
For each question you must show your work and (or) provide a clear argument.
If you need scratch paper, use the last two pages, or the back of the form.

Question 1 Consider the game below:

		Player 2	
	Actions	L	R
	T	1, -1	-1,1
Player 1	B	-2,2	4,-4

Then *12 points*

Player 1 chooses T with probability , and her expected payoff is
Player 2 chooses L with probability , and her expected payoff is

Question 2 n players must simultaneously decide whether or not to bid for an object that has a value of 10. The cost of participating in the auction is $c = 1$. If only one person participates in the auction, then this person can get the object for free, and the total payoff is $10 - c = 9$. If more than one person participates, then the object will be sold at a price of 10. Thus, each person who participates in the auction receives a payoff of $-c = -1$. A person, who does not participate in the auction gets a payoff of 0.

Determine a symmetric equilibrium in mixed strategies where each person participates with probability p in the auction. Consider any player i. Then $(1 - p)^{n-1}$ is the probability that none of the other players participates in the auction. In equilibrium, *12 points*

the probability $(1 - p)^{n-1}$ that none of the other agents participates is

Question 3 Two players want to divide a cake of size 4. Each player i's action $a_i \in \{1, 2, 3\}$ is a proposal of how much of the cake the player wants to receive. If $a_1 + a_2 = 4$ then each player gets the amount that she proposed, i.e., player i's payoff is a_i. If $a_1 + a_2 > 4$ then both players receive nothing, i.e., payoffs are 0. If $a_1 + a_2 < 4$. Then each player i receives the amount a_i plus half of the remaining cake, i.e., player i's payoff is $a_i + 0.5(4 - a_1 - a_2)$ (e.g., if player 1 proposes 1 and player 2 proposes 2, then player 1 receives 1.5 and player receives 2.5).

Determine a symmetric mixed strategy equilibrium of the game. In equilibrium, each player chooses the actions with the following probabilities: *14 points*

1 with probability	, 2 with probability	, 3 with probability

Question 4 Suppose there are two players who want to split 10 Dollars. The game proceeds as follows:

1. Player 1 makes an offer (m_1, m_2) where m_i is the amount of money player i is proposed to receive (of course, $m_1 + m_2 = 10$ and $m_i \geq 0$).
2. Player 2 can accept or reject. If player 2 accepts, then each agent i's payoff is m_i.
3. If player 2 rejects he makes a counteroffer (m'_1, m'_2).
4. Player 1 can accept or reject the counteroffer. If player 1 accepts then each agent i's payoff is m'_i. If player 1 rejects then each player receives a payoff of 0.

Determine the subgame perfect equilibrium. In the subgame perfect equilibrium *12 points*

Player 1's payoff is	and player 2's payoff is

Question 5 Suppose there are three players who are bidding for an object that has a value of 100 Dollars. The object will be sold to the person with the highest bid. However, there is a fee of 2 Dollars for submitting a bid, and each person must make a bid that is at least 10 Dollars higher than the current highest bid. Formally, the game proceeds as follows.

1. Player 1 can choose whether or not to make a bid b_1. If player 1 makes a bid, her bid must be at least 10, i.e., $b_1 \geq 10$.
2. Player 2 observes player 1's bid and decides whether or not to make a bid b_2. If player 1 made a bid, then player 2's bid must be at least 10 Dollars higher, i.e., $b_2 \geq b_1 + 10$. Otherwise, if player 1 did not make a bid, then player 2's bid must be at least 10, i.e., $b_2 \geq 10$.
3. Player 3 observes the bids of players 1 and 2 and decides whether or not to make a bid b_3. Suppose that one other players made a bid. Let b_i be the highest bid. Then $b_3 \geq b_i + 10$. Otherwise, $b_3 \geq 10$.

The player with the highest bid b_i receives a payoff $100 - b_i - 2$. All players with losing bids b_j receive a payoff of -2. Players who did not bid receive a payoff of 0.

Determine the subgame perfect equilibrium. In the subgame perfect equilibrium *12 points*

Player $\boldsymbol{i} =$	wins and bids $\boldsymbol{b_i} =$

Question 6 Suppose there are two sellers ,S_1, S_2. Each of them has a unit an indivisible good. There are two buyers, B_1, B_2. Buyer B_i's valuation of the objective is $\nu_i(q_i)$, where q_i is the quantity of the good that buyer B_i receives.

Suppose that

$\nu_1(0) = 0$, $\nu_1(1) = 4$, $\nu_1(2) = 7$.

$\nu_2(0) = 0$, $\nu_2(1) = 2$, $\nu_2(2) = 3$.

Buyer B_i's utility is $\nu_i(q_i) - r_i$, where r is the money she pays. Seller S_i's utility is r where r is the money she receives.

We can consider this as a game with transferable payoffs. That is, for each coalition S, the worth $v(S)$ of coalition S is the maximum surplus (sum of the utilities of the coalition members) the coalition can obtain by distributing the good efficiently.

Thus, *12 points*

$\boldsymbol{v(\{B_1, S_1\}) =}$	$\boldsymbol{v(\{B_1, B_2, S_1\}) =}$	$\boldsymbol{v(\{B_2, S_1\}) =}$

$\boldsymbol{v(\{B_2, S_2\}) =}$	$\boldsymbol{v(\{B_1, B_2, S_1, S_2\}) =}$	$\boldsymbol{v(\{S_1, S_2\}) =}$

Question 7 Find the matchings produced by the deferred acceptance procedure with proposals by X. The player's preferences are indicated below:

x_1	x_2	x_3	y_1	y_2	y_3
y_1	y_1	y_3	x_3	x_3	x_1
y_3	y_3	y_1	x_1	x_2	x_2
y_2		y_2	x_2	x_1	x_3

In the box below, indicated with whom the different players are matched. If a player is not matched with another player then write "unmatched." *12 points*

$\mathbf{x_1}$: , $\mathbf{x_2}$: , $\mathbf{x_3}$: .

Question 8 Suppose there are two players bidding for an object which is sold in a first price sealed bid auction. The players' valuations of the object are independent and uniformly distributed on $[0, 1]$. Each player only knows her own valuation, i.e., this is a case of independent private values.

Each of the player has a Bernoulli utility function $\sqrt{x}$, where x is the monetary payoff. Thus, player 1 solves

$$\max_{b_1} \text{Prob}(\{b_1 \geq b_2\})\sqrt{v_1 - b_1}.$$

Player 2 solves

$$\max_{b_2} \text{Prob}(\{b_2 \geq b_1\})\sqrt{v_2 - b_2}.$$

This game has a symmetric equilibrium in which each player i uses a strategy of the form $b_i = \alpha v_i$. Determine α. *14 points*

$\boldsymbol{\alpha =}$

Not graded: Use as Scratch Paper

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