Question 1  Consider the game below:

<table>
<thead>
<tr>
<th></th>
<th>L</th>
<th>R</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>T</strong></td>
<td>1,-1</td>
<td>-1,1</td>
</tr>
<tr>
<td><strong>B</strong></td>
<td>-2,2</td>
<td>4,-4</td>
</tr>
</tbody>
</table>

Then

| Player 1 chooses T with probability \( p \), and her expected payoff is \( p \times 1 + (1-p) \times (-1) \) |

| Player 2 chooses L with probability \( q \), and her expected payoff is \( q \times -1 + (1-q) \times 1 \) |
Question 2 $n$ players must simultaneously decide whether or not to bid for an object that has a value of 10. The cost of participating in the auction is $c = 1$. If only one person participates in the auction, then this person can get the object for free, and the total payoff is $10 - c = 9$. If more than one person participates, then the object will be sold at a price of 10. Thus, each person who participates in the auction receives a payoff of $-c = -1$. A person, who does not participate in the auction gets a payoff of 0.

Determine a symmetric equilibrium in mixed strategies where each person participates with probability $p$ in the auction. Consider any player $i$. Then $(1 - p)^{n-1}$ is the probability that none of the other agents participates in the auction. In equilibrium,

\[
(1 - p)^{n-1}
\]

the probability $(1 - p)^{n-1}$ that none of the other agents participates is
Question 3 Two players want to divide a cake of size 4. Each player $i$’s action $a_i \in \{1, 2, 3\}$ is a proposal of how much of the cake the player wants to receive. If $a_1 + a_2 = 4$ then each player gets the amount that she proposed, i.e., player $i$’s payoff is $a_i$. If $a_1 + a_2 > 4$ then both players receive nothing, i.e., payoffs are 0. If $a_1 + a_2 < 4$. Then each player $i$ receives the amount $a_i$ plus half of the remaining cake, i.e., player $i$’s payoff is $a_i + 0.5(4 - a_1 - a_2)$ (e.g., if player 1 proposes 1 and player 2 proposes 2, then player 1 receives 1.5 and player receives 2.5).

Determine a symmetric mixed strategy equilibrium of the game. In equilibrium, each player chooses the actions with the following probabilities: \[\begin{array}{c|c|c|c}
1 & 2 & 3 \\
\hline
\text{with probability} & \text{with probability} & \text{with probability} \\
\end{array}\]
Question 4 Suppose there are two players who want to split 10 Dollars. The game proceeds as follows:

1. Player 1 makes an offer \((m_1, m_2)\) where \(m_i\) is the amount of money player \(i\) is proposed to receive (of course, \(m_1 + m_2 = 10\) and \(m_i \geq 0\)).
2. Player 2 can accept or reject. If player 2 accepts, then each agent \(i\)’s payoff is \(m_i\).
3. If player 2 rejects he makes a counteroffer \((m'_1, m'_2)\).
4. Player 1 can accept or reject the counteroffer. If player 1 accepts then each agent \(i\)’s payoff is \(m'_i\). If player 1 rejects then each player receives a payoff of 0.

Determine the subgame perfect equilibrium. In the subgame perfect equilibrium \hspace{1cm} 12 points

Player 1’s payoff is \hspace{1cm} and player 2’s payoff is
**Question 5**  Suppose there are three players who are bidding for an object that has a value of 100 Dollars. The object will be sold to the person with the highest bid. However, there is a fee of 2 Dollars for submitting a bid, and each person must make a bid that is at least 10 Dollars higher than the current highest bid. Formally, the game proceeds as follows.

1. Player 1 can choose whether or not to make a bid $b_1$. If player 1 makes a bid, her bid must be at least 10, i.e., $b_1 \geq 10$.

2. Player 2 observes player 1’s bid and decides whether or not to make a bid $b_2$. If player 1 made a bid, then player 2’s bid must be at least 10 Dollars higher, i.e., $b_2 \geq b_1 + 10$. Otherwise, if player 1 did not make a bid, then player 2’s bid must be at least 10, i.e., $b_2 \geq 10$.

3. Player 3 observes the bids of players 1 and 2 and decides whether or not to make a bid $b_3$. Suppose that one other players made a bid. Let $b_i$ be the highest bid. Then $b_3 \geq b_i + 10$. Otherwise, $b_3 \geq 10$.

The player with the highest bid $b_i$ receives a payoff $100 - b_i - 2$. All players with losing bids $b_j$ receive a payoff of $-2$. Players who did not bid receive a payoff of 0.

Determine the subgame perfect equilibrium. In the subgame perfect equilibrium  

| Player $i$ = | wins and bids $b_i$ = | 12 points |
**Question 6** Suppose there are two sellers, $S_1, S_2$. Each of them has a unit an indivisible good. There are two buyers, $B_1, B_2$. Buyer $B_i$’s valuation of the objective is $v_i(q_i)$, where $q_i$ is the quantity of the good that buyer $B_i$ receives.

Suppose that

$v_1(0) = 0, v_1(1) = 4, v_1(2) = 7.$
$v_2(0) = 0, v_2(1) = 2, v_2(2) = 3.$

Buyer $B_i$’s utility is $v_i(q_i) - r_i$, where $r$ is the money she pays. Seller $S_i$’s utility is $r$ where $r$ is the money she receives.

We can consider this as a game with transferable payoffs. That is, for each coalition $S$, the worth $v(S)$ of coalition $S$ is the maximum surplus (sum of the utilities of the coalition members) the coalition can obtain by distributing the good efficiently.

Thus,

<table>
<thead>
<tr>
<th>$v({B_1, S_1})$</th>
<th>$v({B_1, B_2, S_1})$</th>
<th>$v({B_2, S_1})$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$v({B_2, S_2})$</td>
<td>$v({B_1, B_2, S_1, S_2})$</td>
<td>$v({S_1, S_2})$</td>
</tr>
</tbody>
</table>

12 points
Question 7 Find the matchings produced by the deferred acceptance procedure with proposals by $X$. The player’s preferences are indicated below:

<table>
<thead>
<tr>
<th></th>
<th>$x_1$</th>
<th>$x_2$</th>
<th>$x_3$</th>
<th>$y_1$</th>
<th>$y_2$</th>
<th>$y_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y_1$</td>
<td>$y_1$</td>
<td>$y_3$</td>
<td>$x_1$</td>
<td>$x_3$</td>
<td>$x_2$</td>
<td>$x_3$</td>
</tr>
<tr>
<td>$y_3$</td>
<td>$y_3$</td>
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<td>$x_1$</td>
<td>$x_2$</td>
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<td>$y_2$</td>
<td>$y_2$</td>
<td>$y_2$</td>
<td>$x_2$</td>
<td>$x_1$</td>
<td>$x_3$</td>
<td>$x_3$</td>
</tr>
</tbody>
</table>

In the box below, indicated with whom the different players are matched. If a player is not matched with another player then write "unmatched." 

$\begin{array}{ccc}
\text{x}_1: & \quad \text{x}_2: & \quad \text{x}_3: \\
\end{array}$
Question 8 Suppose there are two players bidding for an object which is sold in a first price sealed bid auction. The players’ valuations of the object are independent and uniformly distributed on $[0, 1]$. Each player only knows her own valuation, i.e., this is a case of independent private values.

Each of the player has a Bernoulli utility function $\sqrt{x}$, where $x$ is the monetary payoff. Thus, player 1 solves

$$\max_{b_1} \text{Prob}(\{b_1 \geq b_2\}) \sqrt{v_1 - b_1}.$$ 

Player 2 solves

$$\max_{b_2} \text{Prob}(\{b_2 \geq b_1\}) \sqrt{v_2 - b_2}.$$ 

This game has a symmetric equilibrium in which each player $i$ uses a strategy of the form $b_i = \alpha v_i$. Determine $\alpha$. 

$$\alpha =$$
Not graded: Use as Scratch Paper
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