Question 1

(a) Suppose that \( p_1/p_2 = 1/3 \) and that (6, 10) is on the budget line. Then

(0, 12), (12, 8), and (36, 0)

are also on the budget line. *(Fill in the missing numbers)*

(b) See the green budget line below. The intercepts are (28, 0) and (0, 21). Since income is 162, this implies \( p_1 = 162/28 \) and \( p_2 = 162/21 \), i.e.,

\[ p_1 = 5.79, \quad p_2 = 7.71 \]

You can use the grid below to help you find the answers.

![Graph showing budget line]

Question 2

The following points are on the budget line

(0, 115), (10, 65), (30, 60), and (270, 0)

*(Fill in the missing numbers)*

The slope of the budget line is \(-\frac{5}{4}\) when \(x_1 < 10\), and \(-\frac{1}{4}\) when \(x_1 > 10\).
Question 3 Note that $\frac{\partial u(x_1, x_2)}{\partial x_1} = x_2^2$ and $\frac{\partial u(x_1, x_2)}{\partial x_2} = 2x_1x_2$. Thus, MRS = $2x_2/x_1$. The equation of the income offer curve is therefore $x_2/(2x_1) = 2/3$, i.e., $x_2 = (4/3)x_1$.

1. Compute the income offer curve and graph it in the grid below. 6 points

2. Now suppose that the person’s income is $m = 36$. Graph the budget line in the grid below. 3 points

3. Thus, the optimal consumption is $x_1 = 6, x_2 = 8$ 3 points
Question 4  The optimal consumption is \[ x_1 = 10, x_2 = 5. \] 14 points

Question 5

1. \[ \frac{\partial u(x_1, x_2)}{\partial x_1} = 4x_1x_2 \] and \[ \frac{\partial u(x_1, x_2)}{\partial x_2} = x_1^4. \]

Thus, \[ \text{MRS} = \frac{4x_2}{x_1}, \]

6 points
2. \[ \frac{\partial u(x_1, x_2)}{\partial x_1} = \frac{-1}{2}(x_1^2 + 2x_2^2)^{-3/2}(-2)(3)(x_1)^{-3} \text{ and } \frac{\partial u(x_1, x_2)}{\partial x_2} = \frac{-1}{2}(x_1^2 + 2x_2^2)^{-3/2}(-2)(x_2)^{-3}. \] Then

\[ \text{MRS} = \frac{3x_2^2}{x_1^3}. \]

Question 6 The equation of the income offer curve is \( \text{MRS} = \frac{x_2}{x_1} = 9 \). Thus, \( x_2 = 3x_1 \).

The budget line equation is \( 9x_1 + x_2 = 240 \). Thus, \( 12x_1 = 240 \) and hence \( x_1 = 20 \) and \( x_1 = 60 \). Then the optimal consumption is \( x_1 = 20, x_2 = 60 \).

Question 7 At prices \( p_1 = 1, p_2 = 4 \) and income \( m \), the optimal consumption is on this indifference curve. Then the optimal consumption is \( x_1 = 30, x_2 = 0 \), and income is \( m = 30 \).
Question 8 His utility function is given by \( u(x_1, x_2) = 14x_1 - x_1^2 + x_2 \). Thus, \( MRS = 14 - 2x_1 \).

(a) Suppose the price of a ride is \( p = 2 \). Then \( 14 - 2x_1 = 2 \). Thus, he will take \( x_1 = 6 \) rides and spend \$12 at the park. 4 points

(b) Since \( p = 0 \), we get \( 0 = 14 - 2x_1 \). Thus, the person will take \( x_1 = 7 \) rides. 4 points

(c) (Difficult) The utility from not going to the park is \( m \), where \( m \) is the person’s income. Given the result from (b), the utility of going to the park is \( 98 - 49 + m - F \). Thus, at the maximum \( F \) we have \( 49 + m - F = m \). Hence, determine the maximum entry fee \( F \) a person with the above preferences would be willing to pay to enter the park (if the person does not pay \( F \) then he cannot enter the park and \( x_1 = 0 \)). Thus, \( F = 49 \). 4 points