Question 1

(a) Suppose that $p_1/p_2 = 2$ and that $(6, 14)$ is on the budget line. Then

(0, 26), (10, 6), and (13, 0)

are also on the budget line. (*Fill in the missing numbers*) *6 points*

(b) See the green budget line below. The intercepts are $(18, 0)$ and $(0, 27)$. Since income is 540, this implies $p_1 = 540/18$ and $p_2 = 540/27$, i.e., *6 points*

$\mathbf{p_1 = 30,\ p_2 = 20}$

You can use the grid below to help you find the answers.

Question 2 The following points are on the budget line

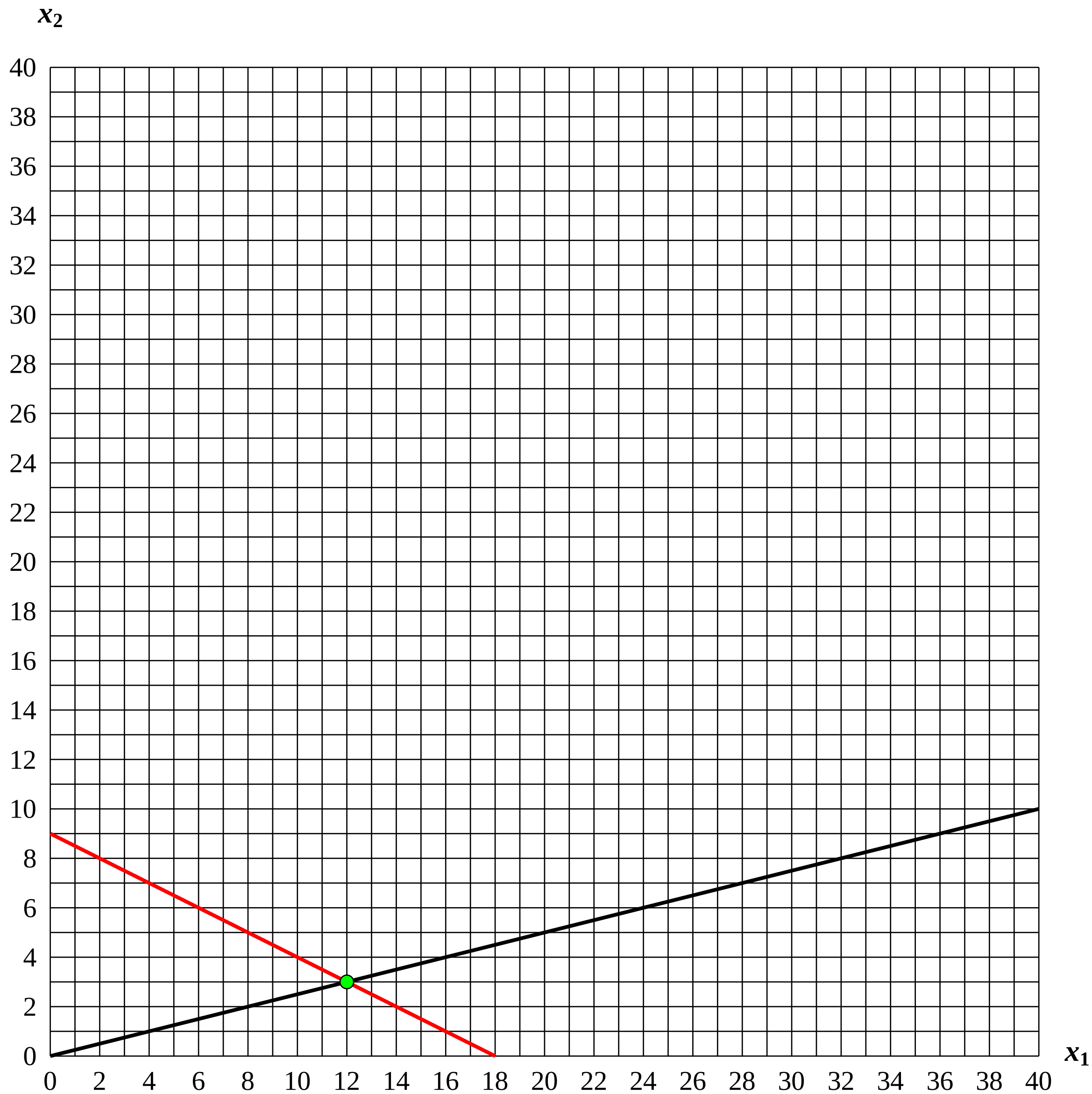
(0, 100), (10, 75), (30, 70), and (310, 0)

(*Fill in the missing numbers*) *8 points*

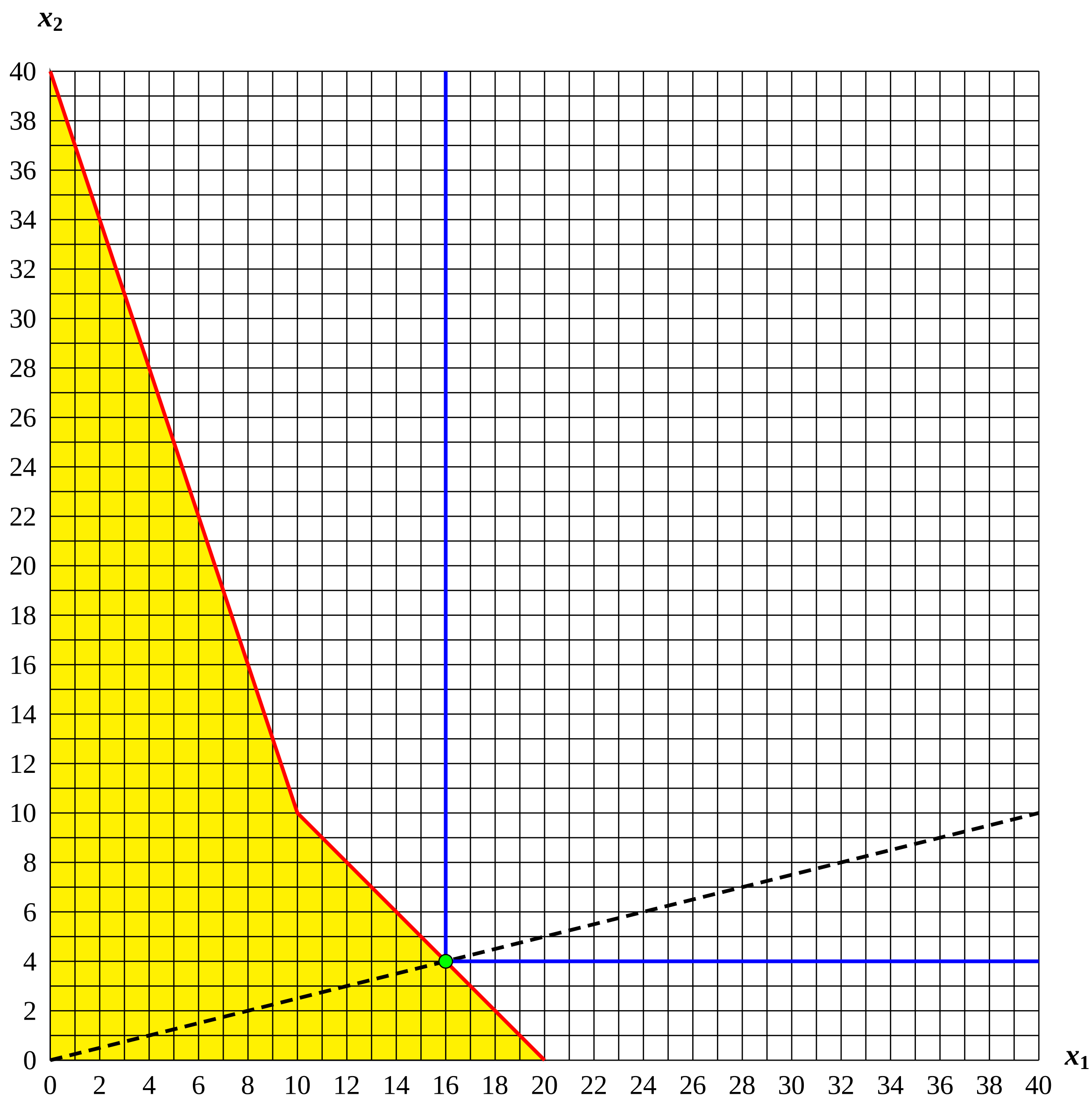
The slope of the budget line is **−2.5** when $x_1 < 10$, and **−1/4** when $x_1 > 10$. *4 points*

Question 3 Note that $\frac{\partial u(x_1,x_2)}{\partial x_1} = 2x_1x_2$ and $\frac{\partial u(x_1,x_2)}{\partial x_2} = x_1^2$. Thus, MRS $= 2x_2/x_1$. The equation of the income offer curve is therefore $2x_2/x_1 = 1/2$, i.e., $x_1 = 4x_2$.

1. Compute the income offer curve and graph it in the grid below. *6 points*
2. Now suppose that the person's income is $m = 18$. Graph the budget line in the grid below. *3 points*
3. Thus, the optimal consumption is **$x_1 = 12, x_2 = 3$** *3 points*



Question 4 The optimal consumption is $\boxed{\mathbf{x_1 = 16, x_2 = 4.}}$ *14 points*



Question 5

1. $\frac{\partial u(x_1,x_2)}{\partial x_1} = 3x_1^2 x_2$ and $\frac{\partial u(x_1,x_2)}{\partial x_2} = x_1^3$.
   Thus, *6 points*

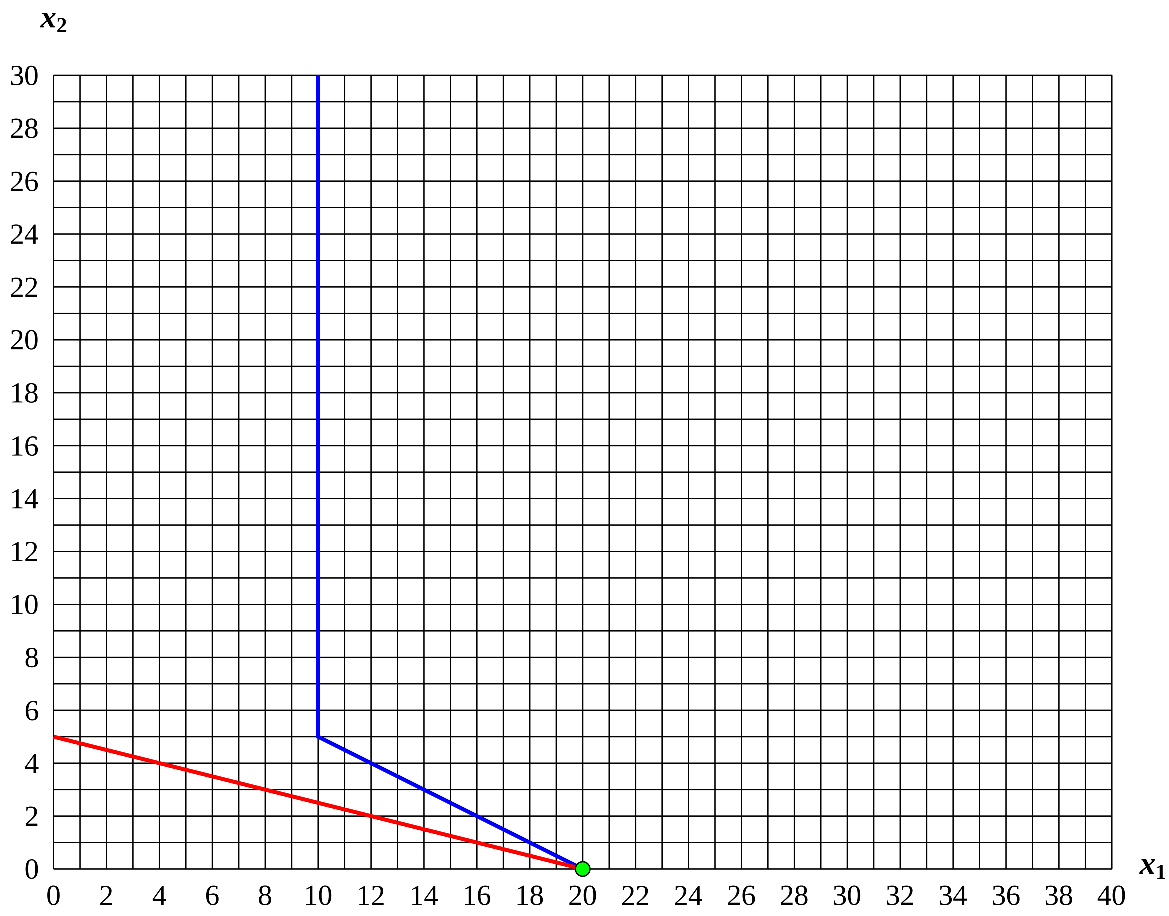
   $$\boxed{\mathbf{MRS = \frac{3x_2}{x_1}}}$$

2. $\frac{\partial u(x_1,x_2)}{\partial x_1} = (-1/2)(x_1^{-2} + 2x_2^{-2})^{-3/2}(-2)(x_1)^{-3}$ and $\frac{\partial u(x_1,x_2)}{\partial x_2} = (-1/2)(x_1^{-2} + 2x_2^{-2})^{-3/2}(-2)(2)(x_2)^{-3}$. Then *6 points*

$$\boxed{\mathbf{MRS} = \frac{x_2^3}{2x_1^3}}.$$

Question 6 The equation of the income offer curve is MRS $= \frac{x_2^2}{x_1^2} = 4$. Thus, $x_2 = 2x_1$. The budget line equation is $4x_1 + x_2 = 90$. Thus, $6x_1 = 90$ and hence $x_1 = 15$ and $x_2 = 30$. Then the optimal consumption is $\boxed{\mathbf{x_1 = 15, x_2 = 30.}}$ *14 points*

Question 7 At prices $p_1 = 1$, $p_2 = 4$ and income m, the optimal consumption is on this indifference curve. Then the optimal consumption is $\boxed{\mathbf{x_1 = 20\ , x_2 = 0}}$, and income is $\boxed{\mathbf{m = 20}}$. *12 points*



Question 8 His utility function is given by $u(x_1, x_2) = 10x_1 - x_1^2 + x_2$. Thus, MRS $= 10 - 2x_1$.

(a) Suppose the price of a ride is $p = 2$. Then $10 - 2x_1 = 2$. Thus, he will take $\boxed{\boldsymbol{x_1 = 4}}$ rides and spend $\boxed{\textbf{\$ 8}}$ at the park. *4 points*

(b) Since $p = 0$, we get $0 = 10 - 2x_1$. Thus, the person will take $\boxed{\boldsymbol{x_1 = 5}}$ rides. *4 points*

(c) (*Difficult*) The utility from not going to the park is m, where m is the person's income. Given the result from (b), the utility of going to the park is $50 - 26 + m - F$. Thus, at the maximum F we have $25 + m - F = m$. Hence,

Determine the maximum entry fee F a person with the above preferences would be willing to pay to enter the park (if the person does not pay F then he cannot enter the park and $x_1 = 0$). Thus, $\boxed{\boldsymbol{F = 25}.}$ *4 points*