Question 1

(a) Suppose that \( \frac{p_1}{p_2} = 2 \) and that \((6, 14)\) is on the budget line. Then

\[
(0, 26), (10, 6), \text{ and } (13, 0)
\]

are also on the budget line. \textit{(Fill in the missing numbers)} \hspace{1cm} 6 \text{ points}

(b) See the green budget line below. The intercepts are \((18, 0)\) and \((0, 27)\). Since income is 540, this implies \(p_1 = \frac{540}{18}\) and \(p_2 = \frac{540}{27}\), i.e.,

\[
p_1 = 30, \ p_2 = 20
\]

You can use the grid below to help you find the answers.

![Graph](image)

Question 2

The following points are on the budget line

\[
(0, 100), (10, 75), (30, 70), \text{ and } (310, 0)
\]

\textit{(Fill in the missing numbers)} \hspace{1cm} 8 \text{ points}

The slope of the budget line is \([-2.5]\) when \(x_1 < 10\), and \([-1/4]\) when \(x_1 > 10\). \hspace{1cm} 4 \text{ points}
Question 3  Note that $\frac{\partial u(x_1,x_2)}{\partial x_1} = 2x_1x_2$ and $\frac{\partial u(x_1,x_2)}{\partial x_2} = x_1^2$. Thus, $MRS = 2x_2/x_1$. The equation of the income offer curve is therefore $2x_2/x_1 = 1/2$, i.e., $x_1 = 4x_2$.

1. Compute the income offer curve and graph it in the grid below.  
2. Now suppose that the person’s income is $m = 18$. Graph the budget line in the grid below.  
3. Thus, the optimal consumption is $x_1 = 12, x_2 = 3$.
Question 4  The optimal consumption is $x_1 = 16, x_2 = 4$.  

Question 5  

1. \[
\frac{\partial u(x_1, x_2)}{\partial x_1} = 3x_1^2x_2 \text{ and } \frac{\partial u(x_1, x_2)}{\partial x_2} = x_2^3.
\]

Thus, \[
\text{MRS} = \frac{3x_2}{x_1}
\]
2. \[ \frac{\partial u(x_1, x_2)}{\partial x_1} = (-1/2)(x_1^{-2} + 2x_2^{-2})^{-3/2}(-2)(x_1)^{-3} \] and \[ \frac{\partial u(x_1, x_2)}{\partial x_2} = (-1/2)(x_1^{-2} + 2x_2^{-2})^{-3/2}(-2)(x_2)^{-3}. \] Then

\[ \text{MRS} = \frac{x_2^2}{2x_1^3}. \]

**Question 6** The equation of the income offer curve is \( \text{MRS} = \frac{x_2^2}{2x_1^3} = 4. \) Thus, \( x_2 = 2x_1. \)

The budget line equation is \( 4x_1 + x_2 = 90. \) Thus, \( 6x_1 = 90 \) and hence \( x_1 = 15 \) and \( x_2 = 30. \) Then the optimal consumption is \( x_1 = 15, \ x_2 = 30. \)  

**Question 7** At prices \( p_1 = 1, \ p_2 = 4 \) and income \( m, \) the optimal consumption is on this indifference curve. Then the optimal consumption is \( x_1 = 20, \ x_2 = 0. \) and income is \( m = 20. \)
Question 8  His utility function is given by \( u(x_1, x_2) = 10x_1 - x_1^2 + x_2 \). Thus, MRS = 
10 − 2x_1.

(a) Suppose the price of a ride is \( p = 2 \). Then 10 − 2x_1 = 2. Thus, he will take 
\[ x_1 = 4 \] rides and spend \$8 at the park. \[ \text{4 points} \]

(b) Since \( p = 0 \), we get 0 = 10 − 2x_1. Thus, the person will take 
\[ x_1 = 5 \] rides. \[ \text{4 points} \]

(c) (Difficult) The utility from not going to the park is \( m \), where \( m \) is the person’s income. Given the result from (b), the utility of going to the park is 50 − 26 + 
\( m - F \). Thus, at the maximum \( F \) we have 25 + \( m - F = m \). Hence,
Determine the maximum entry fee \( F \) a person with the above preferences 
would be willing to pay to enter the park (if the person does not pay \( F \) then 
he cannot enter the park and \( x_1 = 0 \)). Thus, \[ F = 25. \] \[ \text{4 points} \]