$p_1 = \frac{252}{28} = 9$ and $p_2 = \frac{252}{21} = 12$. Thus,

$p_1 = 9, \ p_2 = 12$
Question 2
Question 3  Income offer curves for different price ratios are depicted below.  

1. Suppose that income is \( m = 58 \) and prices are \( p_1 = p_2 = 2 \). Then optimal consumption is \( x_1 = 24, x_2 = 8 \).

2. Now suppose that the price of good 2 decreases to \( p_2 = 1 \). Income and the price of good 1 remain unchanged. Then optimal consumption is \( x_1 = 4, x_2 = 12 \).

Question 4

1. \( \frac{\partial u(x_1, x_2)}{\partial x_1} = x_2^4 \) and \( \frac{\partial u(x_1, x_2)}{\partial x_2} = 4x_1x_2^3 \). Thus,

\[
MRS = \frac{x_2^4}{4x_1x_2^3} = \frac{x_2}{4x_1}.
\]

\[
MRS = \frac{x_2}{4x_1}, \quad \text{n}
\]
2. \( \frac{\partial u(x_1, x_2)}{\partial x_1} = (8x_1^{-1} + x_2^{-1})^{-2}8x_1^{-2} \). \( \frac{\partial u(x_1, x_2)}{\partial x_1} = (8x_1^{-1} + x_2^{-1})^{-2}x_2^{-2} \). Thus,

\[
\text{MRS} = \frac{(8x_1^{-1} + x_2^{-1})^{-2}8x_1^{-2}}{(8x_1^{-1} + x_2^{-1})^{-2}x_2^{-2}} = \frac{8x_2^2}{x_1^2}.
\]

\[
\text{MRS} = \frac{8x_2^2}{x_1^2}.
\]

Question 5

At the optimal choice \( x_1 = 5, x_2 = 15 \)
Question 6 \[ \frac{\partial u(x_1, x_2)}{\partial x_1} = 3x_1^2x_2 \text{ and } \frac{\partial u(x_1, x_2)}{\partial x_2} = x_1^3. \] Thus,
\[
\text{MRS} = \frac{3x_1^2x_2}{x_1^3} = \frac{3x_2}{x_1}.
\]
1. The equation of the income offer curve is \[ \frac{3x_2}{x_1} = 1, \] i.e.,
\[ x_2 = (1/3)x_1\]
2. The budget line equation is \[ 2x_1 + 2x_2 = 400. \] Thus, \( (8/3)x_1 = 400 \), i.e., \( x_1 = 150, \) and \( x_2 = 50 \).

Question 7 The MRS equals the price ratio 2/4 because the optimal choice is interior.

1. She will spend all her money on good 2. Her income in the base case is 300. Thus,
\[ x_1 = 0, x_2 = 75. \]
2. Now \( m = 150 \) and she will spend all income on good 1. Thus,
\[ x_1 = 50, x_2 = 0. \]

Question 8 \[ \frac{\partial u(x_1, x_2)}{\partial x_1} = \frac{100}{(x_1 + 1)^2} \text{ and } \frac{\partial u(x_1, x_2)}{\partial x_2} = 1. \] Thus,
\[
\text{MRS} = \frac{x_2}{2x_1x_2} = \frac{100}{(x_1 + 1)^2}.
\]
(a) MRS = \( p_1/p_2 \) implies \[ \frac{100}{(x_1 + 1)^2} = 4, \] i.e., \( x_1 + 1 = 5. \)

Joe’s optimal choice of \( x_1 \) is 4

The gym’s revenue (from Joe) is $16

(b) MRS = \( p_1/p_2 \) implies \[ \frac{100}{(x_1 + 1)^2} = 1, \] (x_1 + 1) = 10.

Joe’s optimal choice of \( x_1 \) is 9

The gym’s revenue (from Joe) is $19

His utility from not going is 1,000 - 100 = 900. If he goes then \( x_1 = 9. \) He spends fee \( F \) plus 9 Dollars, thus \( x_2 = 1,000 - F - 9. \) Utility is therefore, \( 990 - F - 9. \) Joe is indifferent if \( 981 - F = 900, \) i.e.,

\[ F = 81 \]