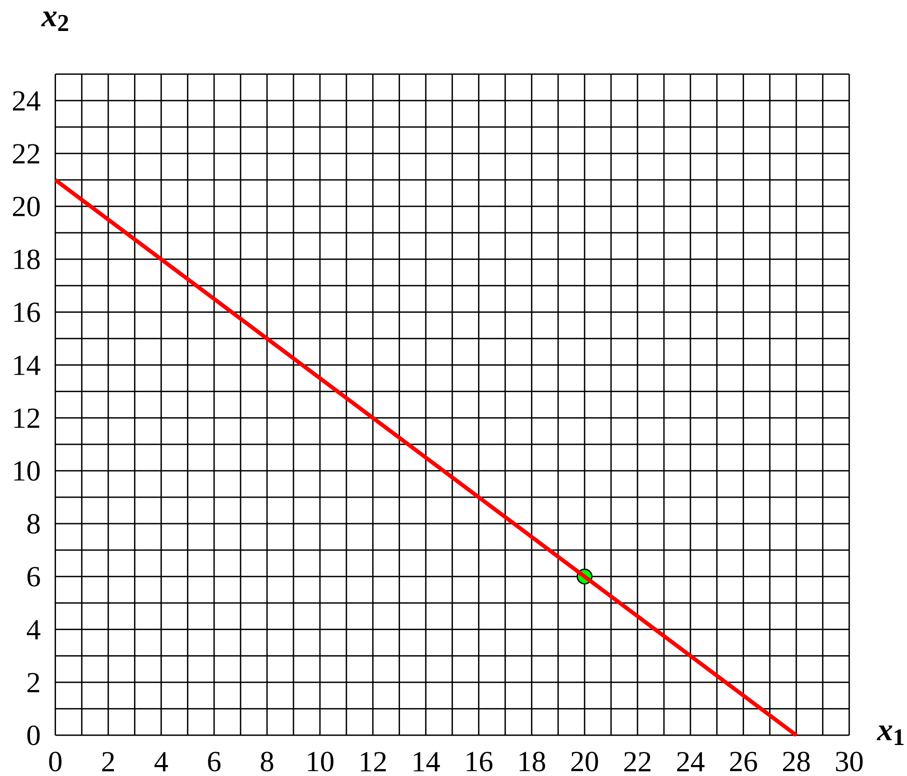
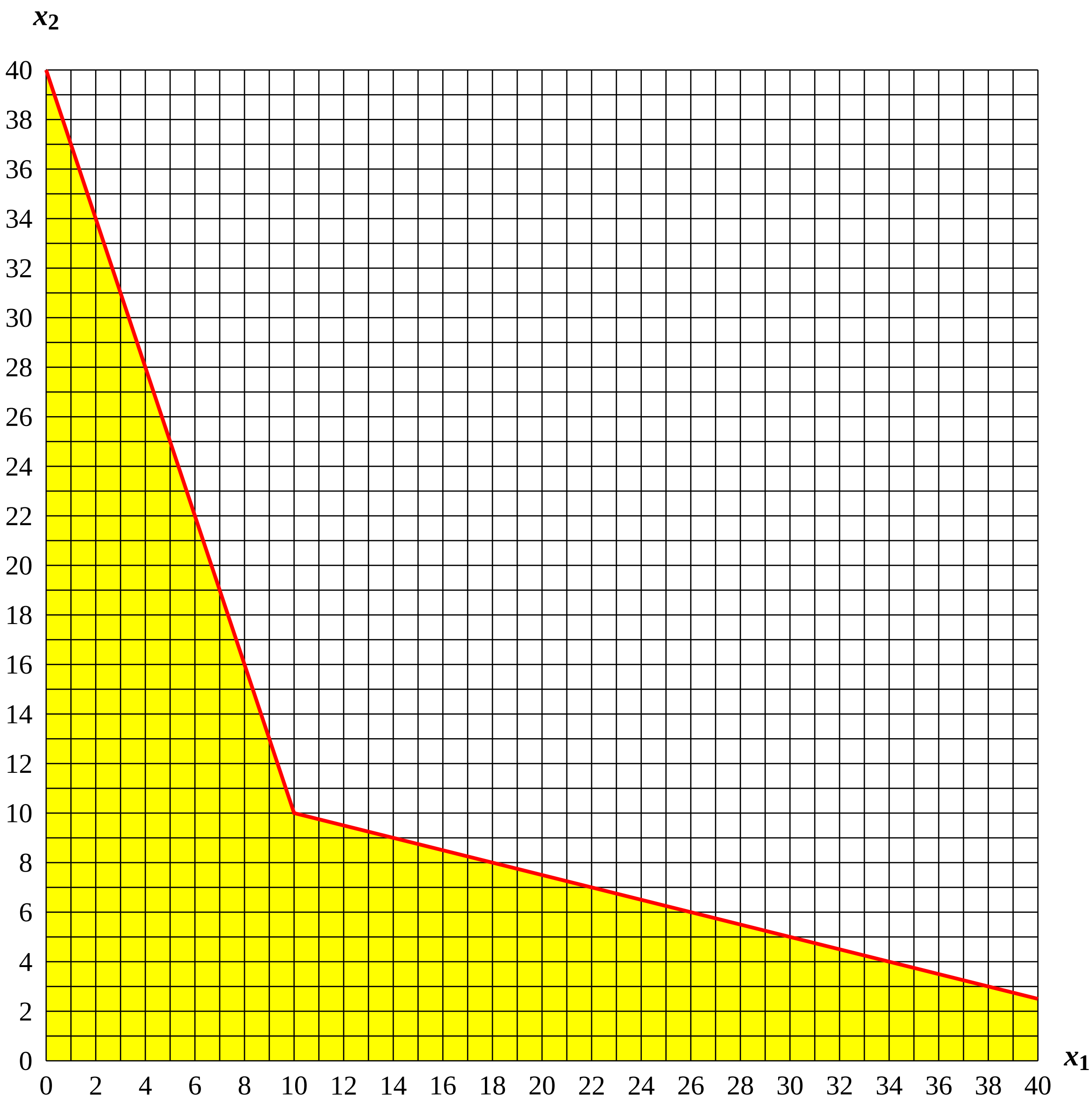
Question 1



$p_1 = 252/28 = 9$ and $p_2 = 252/21 = 12$. Thus,

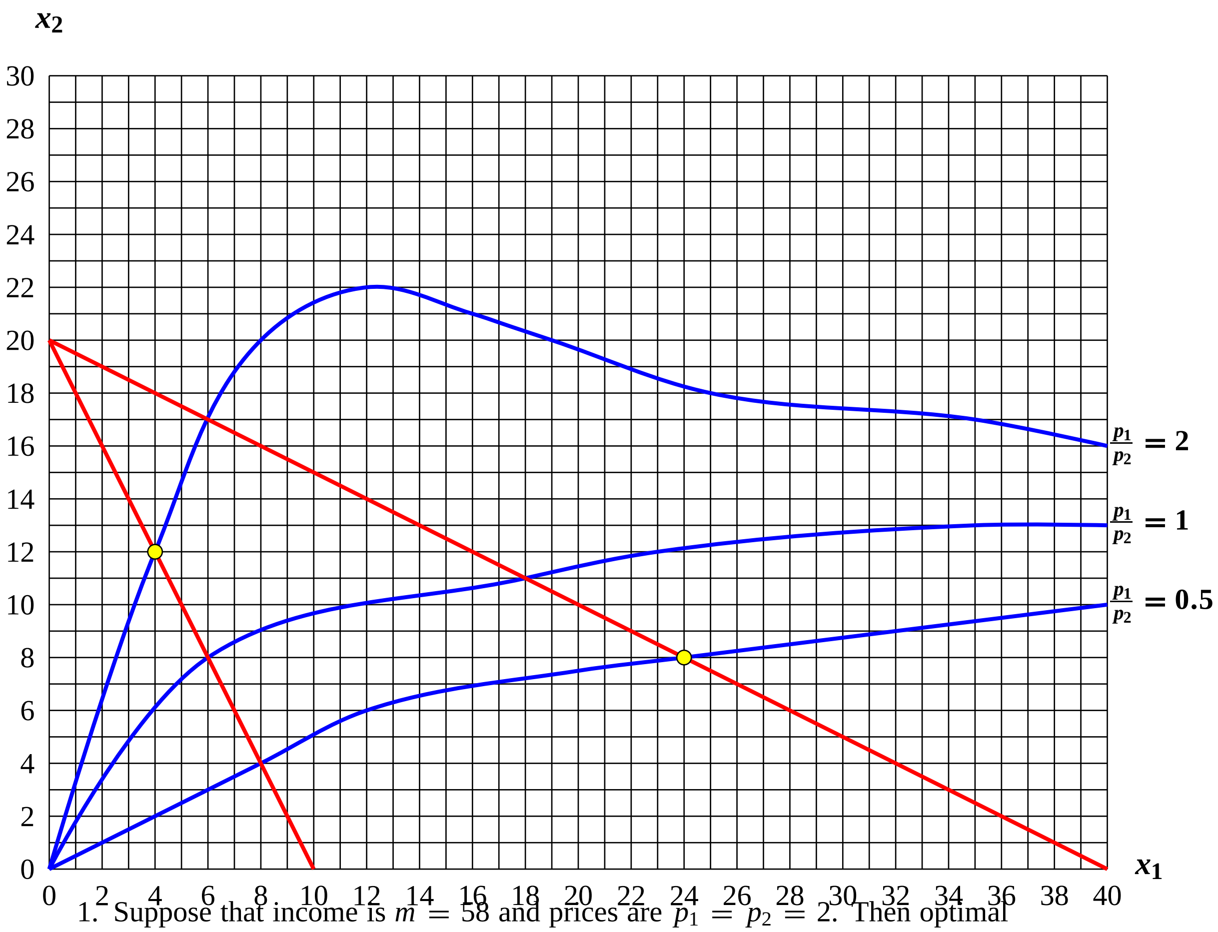
$$\boxed{p_1 = 9,\ p_2 = 12}$$

Question 2



x_2
40
38
36
34
32
30
28
26
24
22
20
18
16
14
12
10
8
6
4
2
0
0 2 4 6 8 10 12 14 16 18 20 22 24 26 28 30 32 34 36 38 40
x_1

Question 3 Income offer curves for different price ratios are depicted below. *10 points*



1. Suppose that income is $m = 58$ and prices are $p_1 = p_2 = 2$. Then optimal consumption is

   $\boxed{\mathbf{x_1 = 24,\ x_2 = 8.}}$

2. Now suppose that the price of good 2 decreases to $p_2 = 1$. Income and the price of good 1 remain unchanged. Then optimal consumption is

   $\boxed{\mathbf{x_1 = 4,\ x_2 = 12}}$

Question 4

1. $\frac{\partial u(x_1,x_2)}{\partial x_1} = x_2^4$ and $\frac{\partial u(x_1,x_2)}{\partial x_2} = 4x_1x_2^3$. Thus,

$$\text{MRS} = \frac{x_2^4}{4x_1x_2^3} = \frac{x_2}{4x_1}.$$

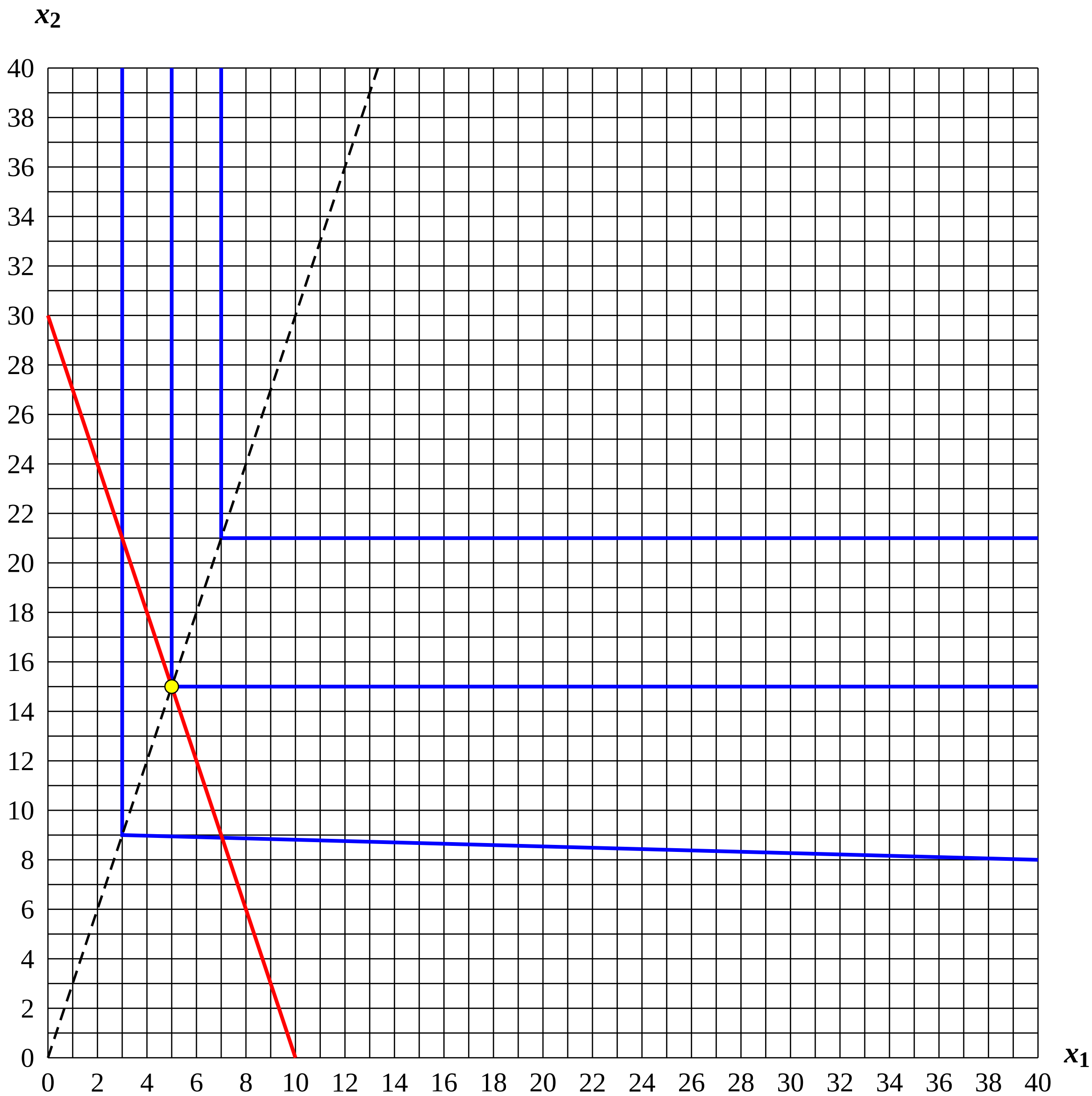
$\boxed{\mathbf{MRS = \frac{x_2}{4x_1}.}}$ n

2. $\frac{\partial u(x_1,x_2)}{\partial x_1} = (8x_1^{-1} + x_2^{-1})^{-2} 8x_1^{-2}$. $\frac{\partial u(x_1,x_2)}{\partial x_1} = (8x_1^{-1} + x_2^{-1})^{-2} x_2^{-2}$. Thus,

$$\text{MRS} = \frac{(8x_1^{-1} + x_2^{-1})^{-2} 8x_1^{-2}}{(8x_1^{-1} + x_2^{-1})^{-2} x_2^{-2}} = \frac{8x_2^2}{x_1^2}.$$

$$\boxed{\textbf{MRS} = \frac{8x_2^2}{x_1^2}.}$$

Question 5 $\boxed{\textbf{At the optimal choice } x_1 = 5, x_2 = 15}$



Question 6 $\frac{\partial u(x_1,x_2)}{\partial x_1} = 3x_1^2x_2$ and $\frac{\partial u(x_1,x_2)}{\partial x_2} = x_1^3$. Thus,

$$\text{MRS} = \frac{3x_1^2x_2}{x_1^3} = \frac{3x_2}{x_1}.$$

1. The equation of the income offer curve is $\frac{3x_2}{x_1} = 1$, i.e.,

   $\boxed{\boldsymbol{x_2 = (1/3)x_1}}$

2. The budget line equation is $2x_1 + 2x_2 = 400$. Thus, $(8/3)x_1 = 400$, i.e., $x_1 = 150$, and $x_1 = 50$.

   $\boxed{\boldsymbol{x_1 = 150,\ x_2 = 50}.}$

Question 7 The MRS equals the price ratio 2/4 because the optimal choice is interior.

1. She will spend all her money on good 2. Her income in the base case is 300. Thus,

   $\boxed{\boldsymbol{x_1 = 0, x_2 = 75}.}$

2. Now $m = 150$ and she will spend all income on good 1. Thus,

   $\boxed{\boldsymbol{x_1 = 50,\ x_2 = 0}.}$

Question 8 $\frac{\partial u(x_1,x_2)}{\partial x_1} = 100/(x_1+1)^2$ and $\frac{\partial u(x_1,x_2)}{\partial x_2} = 1$. Thus,

$$\text{MRS} = \frac{x_2^2}{2x_1x_2} = \frac{100}{(x_1+1)^2}.$$

(a) MRS $= p_1/p_2$ implies $\frac{100}{(x_1+1)^2} = 4$, i.e., $x_1 + 1 = 5$.

$\boxed{\textbf{Joe's optimal choice of } \boldsymbol{x_1} \textbf{ is 4}}$

$\boxed{\textbf{The gym's revenue (from Joe) is \$16}}$

(b) MRS $= p_1/p_2$ implies $\frac{100}{(x_1+1)^2} = 1$ $(x_1 + 1) = 10$.

$\boxed{\textbf{Joe's optimal choice of } \boldsymbol{x_1} \textbf{ is 9}}$

$\boxed{\textbf{The gym's revenue (from Joe) is \$19}}$

His utility from not going is $1,000 - 100 = 900$. If he goes then $x_1 = 9$. He spends fee F plus 9 Dollars, thus $x_2 = 1,000 - F - 9$. Utility is therefore, $990 - F - 9$. Joe is indifferent if $981 - F = 900$, i.e.,

$\boxed{\boldsymbol{F = 81}}$