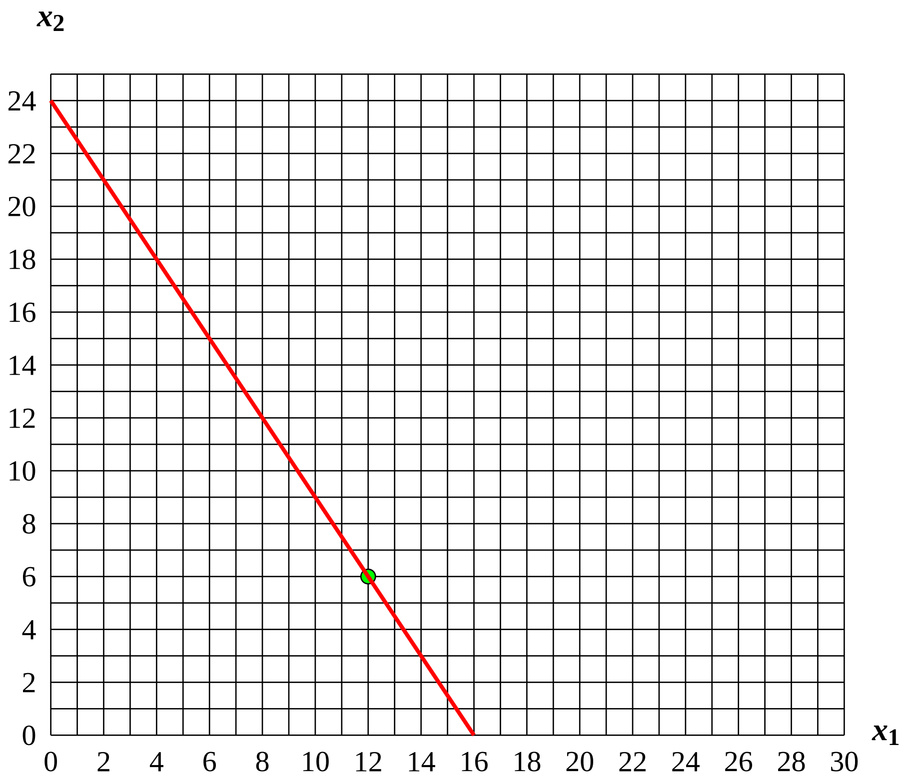
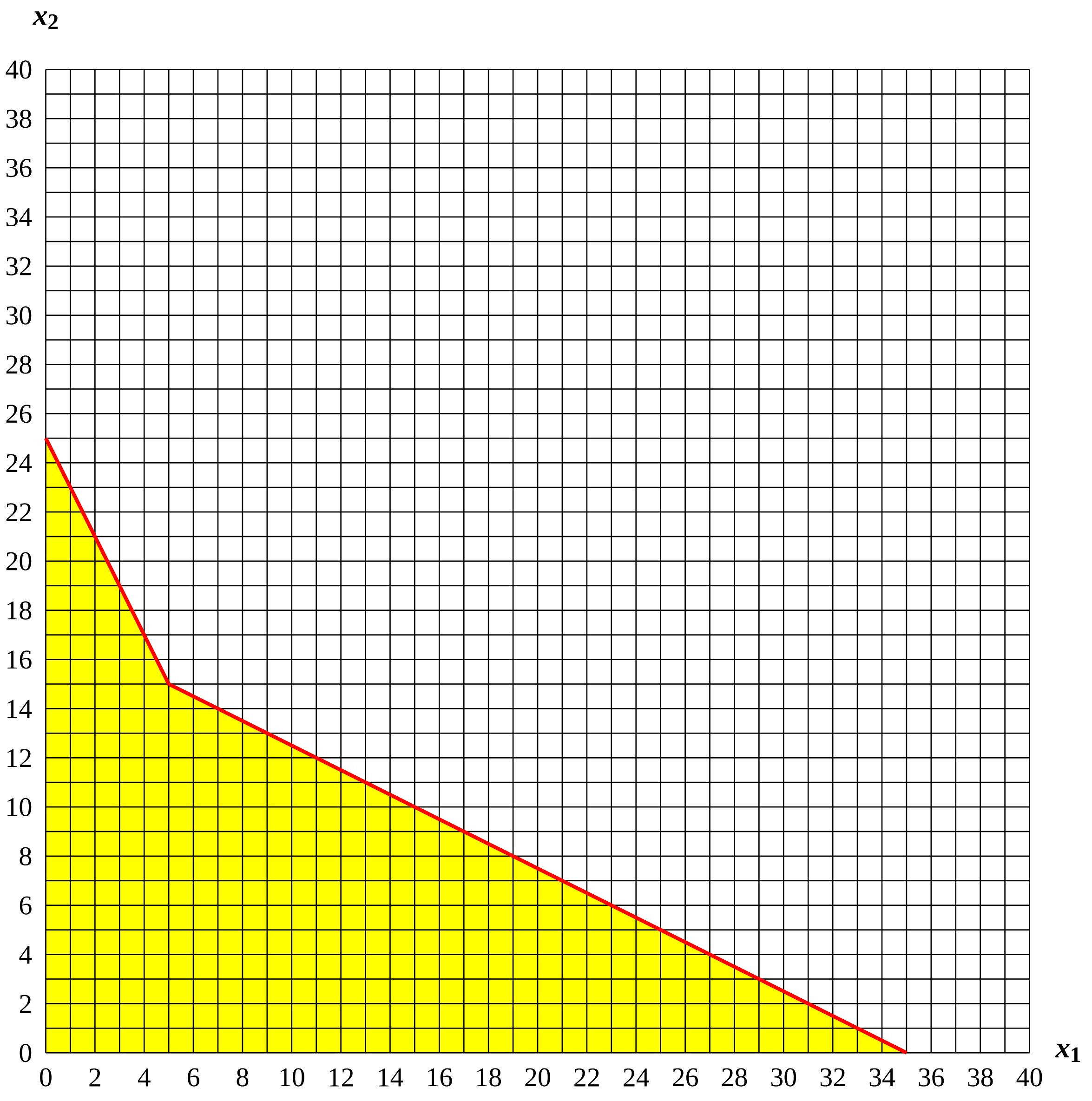
Question 1



$p_1 = 96/16 = 6$ and $p_2 = 96/24 = 4$. Thus,

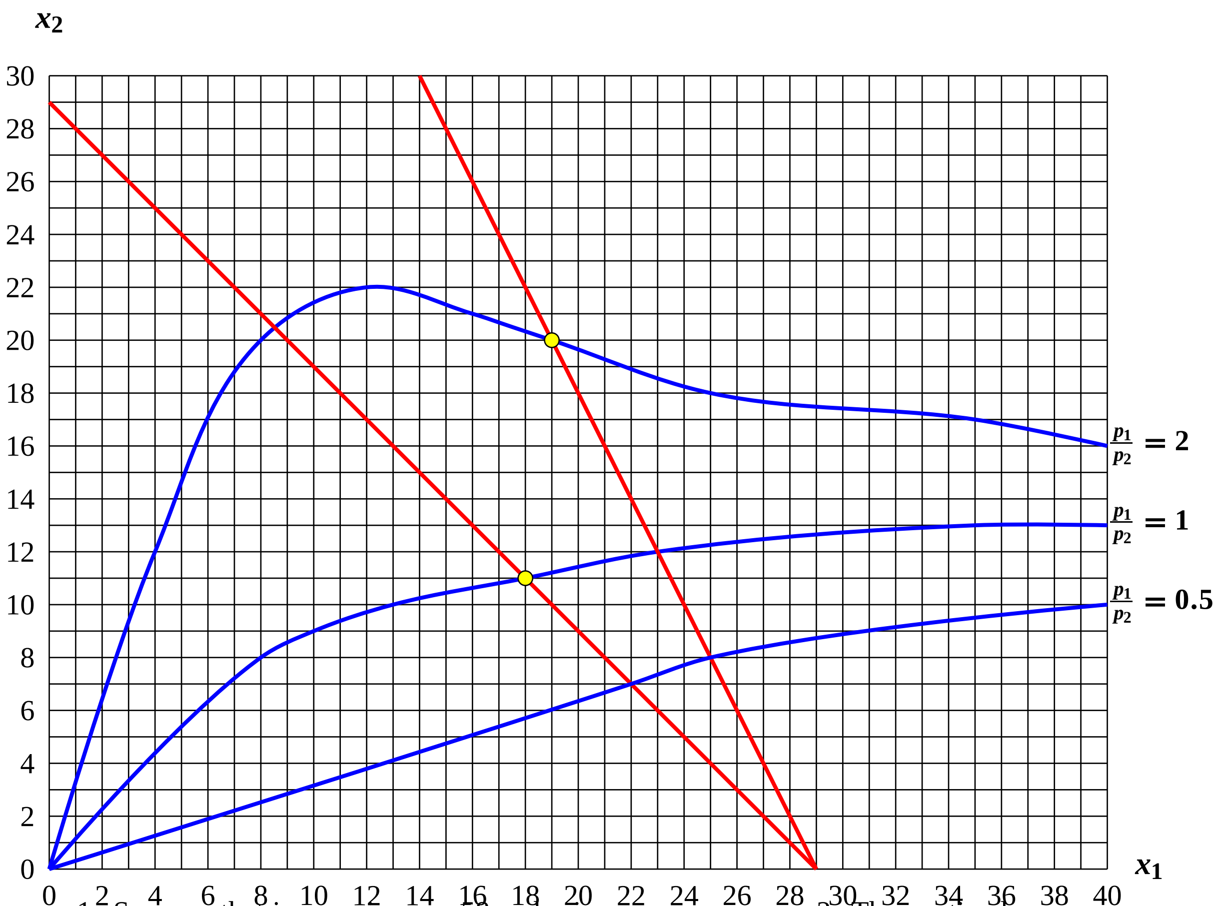
$$\boxed{p_1 = 6,\ p_2 = 4}$$

Question 2



x_2
40
38
36
34
32
30
28
26
24
22
20
18
16
14
12
10
8
6
4
2
0
0 2 4 6 8 10 12 14 16 18 20 22 24 26 28 30 32 34 36 38 40
x_1

Question 3 Income offer curves for different price ratios are depicted below. *10 points*



1. Suppose that income is $m = 58$ and prices are $p_1 = p_2 = 2$. Then optimal consumption is

   $\boxed{x_1 = 18, x_2 = 11.}$

2. Now suppose that the price of good 2 decreases to $p_2 = 1$. Income and the price of good 1 remain unchanged. Then optimal consumption is

   $\boxed{x_1 = 19, x_2 = 20}$

Question 4

1. $\frac{\partial u(x_1,x_2)}{\partial x_1} = 3x_1^2x_2$ and $\frac{\partial u(x_1,x_2)}{\partial x_2} = x_1^3x_2$. Thus,

$$\text{MRS} = \frac{3x_1^2x_2}{x_1^3x_2} = \frac{3x_2}{x_1}.$$

   $\boxed{\text{MRS} = \frac{3x_2}{x_1}.}$

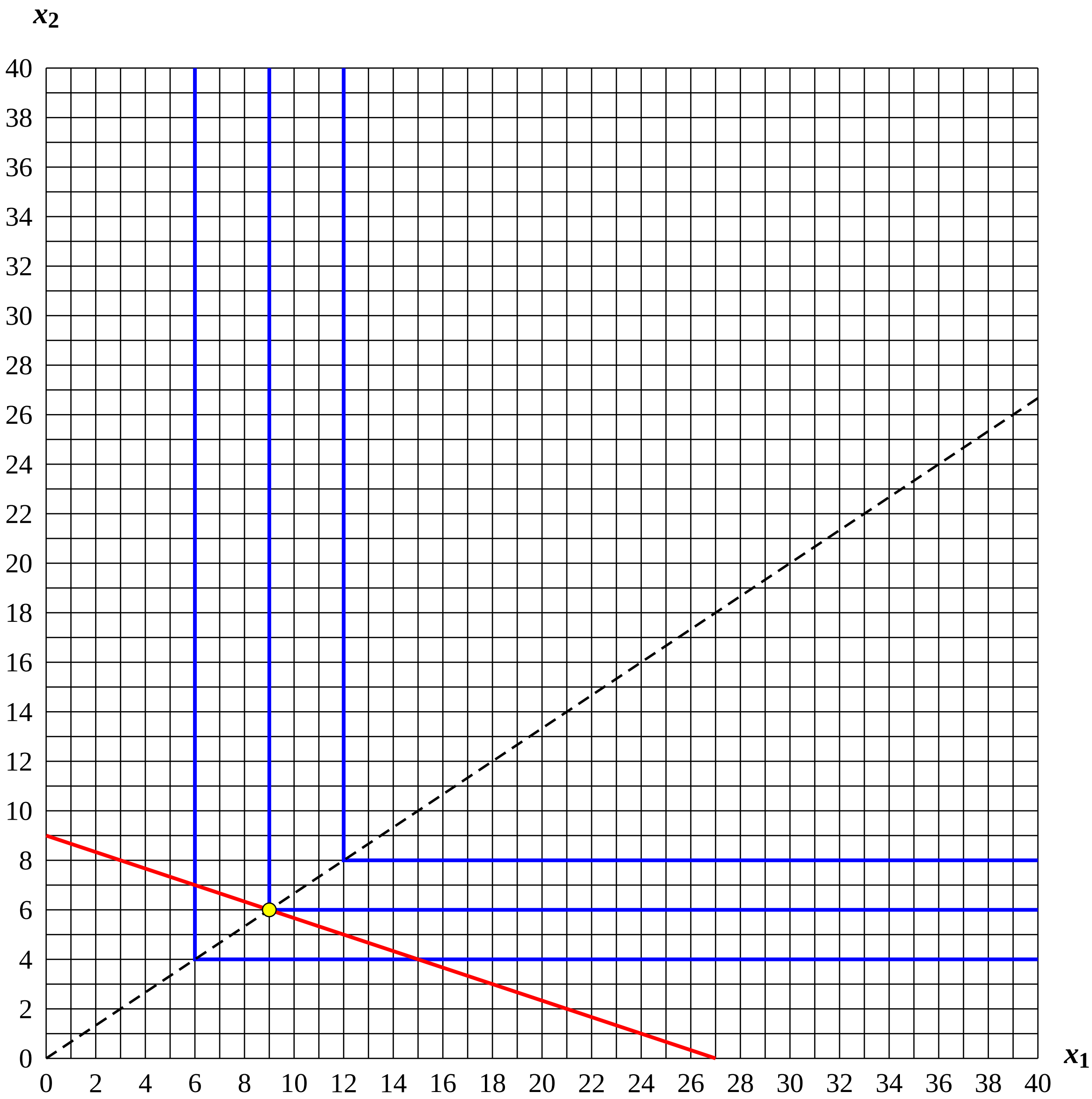
2. $\frac{\partial u(x_1,x_2)}{\partial x_1} = (x_1^{-1} + 2x_2^{-1})^{-2}x_1^{-2}$. $\frac{\partial u(x_1,x_2)}{\partial x_1} = (x_1^{-1} + 2x_2^{-1})^{-2}2x_2^{-2}$. Thus,

$$\text{MRS} = \frac{(x_1^{-1} + 2x_2^{-1})^{-2}x_1^{-2}}{(x_1^{-1} + 2x_2^{-1})^{-2}2x_2^{-2}} = \frac{x_2^2}{2x_1^2}.$$

$$\boxed{\textbf{MRS} = \frac{x_2^2}{2x_1^2}.}$$

Question 5

$$\boxed{\textbf{At the optimal choice } x_1 = 9,\ x_2 = 6}$$



Question 6 $\frac{\partial u(x_1,x_2)}{\partial x_1} = x_2^2$ and $\frac{\partial u(x_1,x_2)}{\partial x_2} = 2x_1x_2$. Thus,

$$\text{MRS} = \frac{x_2^2}{2x_1x_2} = \frac{x_2}{2x_1}.$$

1. The equation of the income offer curve is $\frac{x_2}{2x_1} = 1$, i.e.,

   $\boxed{\boldsymbol{x_2 = 2x_1}}$

2. The budget line equation is $2x_1+2x_2 = 600$. Thus, $3x_2 = 600$, i.e., $x_2 = 200$, and $x_1 = 100$.

   $\boxed{\boldsymbol{x_1 = 100,\ x_2 = 200.}}$

Question 7 The MRS equals the price ratio 2/4 because the optimal choice is interior.

1. She will spend all her money on good 1. Her income in the base case is 60. Thus,

   $\boxed{\boldsymbol{x_1 = 30, x_2 = 0.}}$

2. Now $m = 90$ and she will spend all income on good 2. Thus,

   $\boxed{\boldsymbol{x_1 = 0,\ x_2 = 15.}}$

Question 8 $\frac{\partial u(x_1,x_2)}{\partial x_1} = 16/(x_1+1)^2$ and $\frac{\partial u(x_1,x_2)}{\partial x_2} = 1$. Thus,

$$\text{MRS} = \frac{x_2^2}{2x_1x_2} = \frac{16}{(x_1+1)^2}.$$

(a) MRS $= p_1/p_2$ implies $\frac{16}{(x_1+1)^2} = 4$, i.e., $x_1 + 1 = 2$.

Joe's optimal choice of x_1 is 1

The gym's revenue (from Joe) is $4

(b) MRS $= p_1/p_2$ implies $\frac{16}{(x_1+1)^2} = 1$ $(x_1+1) = 4$.

Joe's optimal choice of x_1 is 3

The gym's revenue (from Joe) is $7

His utility from not going is $1,000 - 16 = 984$. If he goes then $x_1 = 3$. He spends fee F plus 3 Dollars, thus $x_2 = 1,000 - F - 3$. Utility is therefore, $993 - F$. Joe is indifferent if $993 - F = 984$, i.e.,

$\boxed{\boldsymbol{F = 9}}$