All questions must be answered on this test form!
For each question you must show your work and (or) provide a clear argument.
All graphs must be accurate to get credit.
If you need scratch paper, use the last page or the back of the form.

Question 1

(a) Suppose that \( p_1/p_2 = 1/3 \) and that (6, 10) is on the budget line. Then

\[ (0, \_), (12, \_), \text{ and } (\_, 0) \]

are also on the budget line. (Fill in the missing numbers) 6 points

(b) Now suppose that for a different consumer and different prices (12, 12) and (8, 15) are on the budget line, and that the person’s income is \( m = 162 \). Then

\[ p_1 = \_, \quad p_2 = \_ \]

You can use the grid below to help you find the answers.
Question 2 Suppose that there are two goods. The price of each unit of good 2 is 2 Dollars. The price of good 1 depends on the quantity purchased. That is, if a person buys up to 10 units, then the price of each unit is 10 Dollars. If the person buys more than 10 units, then the first 10 units are still priced at 10 Dollars per unit, while each additional unit is priced at 50cents per unit. Suppose that the person’s income is \( m = 230 \).

Then the following points are on the budget line:

\[(0, \_), (10, \_), (30, \_), \text{ and } (\_, 0)\]

(Fill in the missing numbers) 8 points

The slope of the budget line is \_ when \( x_1 < 10 \), and \_ when \( x_1 > 10 \). 4 points
Question 3  A utility function is given by $u(x_1, x_2) = x_1 x_2^2$. Suppose that prices are given by $p_1 = 2$, $p_2 = 3$.

1. Compute the income offer curve and graph it in the grid below.  
   
2. Now suppose that the person’s income is $m = 36$. Graph the budget line in the grid below.  
   
3. Thus, the optimal consumption is $x_1 = $, $x_2 = $  

[Grid Diagram]
Question 4 A utility function is given by \( u(x_1, x_2) = \min\{x_1, 2x_2\} \). The price of good 2 is \( p_2 = 1 \). The price of good 1 is non-linear. In particular, if the person consumes less than 5 units, the price is 4 Dollars per unit. If the person consumes more than 5 units, then the first 5 units still cost 4 Dollars per unit, however, every additional unit costs 1 Dollar. Suppose that income is \( m = 30 \). Graph the budget line in the grid below. Clearly indicate the budget set by shading it. Determine graphically the optimal consumption, and graph the indifference through the optimal consumption choice. The optimal consumption is \( x_1 = \), \( x_2 = \) 14 points
Question 5

1. A utility function is given by \( u(x_1, x_2) = x_1^4x_2 \). Then \( \text{MRS} = \) \[ \text{6 points} \]

2. Now suppose that the utility function is \( u(x_1, x_2) = (3x_1^{-2} + x_2^{-2})^{-1/2} \). Then \( \text{MRS} = \) \[ \text{6 points} \]
Question 6  A consumer’s utility function $u(x_1, x_2)$ has a MRS given by

$$MRS = \frac{x_2}{x_1}.$$ 

Suppose that prices are $p_1 = 9$, $p_2 = 1$ and that the person’s income is $m = 240$.

Then the optimal consumption is $x_1 = \blacksquare, x_2 = \blacksquare$  

14 points
Question 7 A utility function is given by $u(x_1, x_2) = \min\{3x_1, x_1 + x_2\}$. Graph the indifference curve through (10, 30) in the grid below. Further, suppose that at prices $p_1 = 1, p_2 = 3$ and income $m$, the optimal consumption is on this indifference curve. Then the optimal consumption is $x_1 = \underline{\hspace{2cm}}, x_2 = \underline{\hspace{2cm}}$, and income is $m = \underline{\hspace{2cm}}$. 12 points
Question 8 Joe visits an amusement park. His utility function is given by \( u(x_1, x_2) = 14x_1 - x_1^2 + x_2 \), where \( x_1 \) is the number of rides and \( x_2 \) the amount of money he spends on other items.

(a) Suppose the price of a ride is \( p = 2 \). Then he will take \( x_1 = \) rides and spend \( \$ \) at the park (You do not need to know income \( m \) to answer this question.). 4 points

(b) Now suppose that the amusement park decides to charge a fixed entrance fee \( F \) instead of a price per ride. A visitor who pays the fee can take as many rides as he/she wishes (i.e., after \( F \) has been paid, the price per ride is zero). Then the person will take \( x_1 = \) rides. 4 points

(c) (Difficult) Determine the maximum entry fee \( F \) a person with the above preferences would be willing to pay to enter the park (if the person does not pay \( F \) then he cannot enter the park and \( x_1 = 0 \)). \( F = \) 4 points