Name:
E-mail: @uiuc.edu

All questions must be answered on this test form!
For each question you must show your work and (or) provide a clear argument.
All graphs must be accurate to get credit.
If you need scratch paper, use the last page or the back of the form.

Question 1

(a) Suppose that \( p_1/p_2 = 2 \) and that \( (6, 14) \) is on the budget line. Then

\[
(0, \quad ), (10, \quad ), \text{ and } (\quad , 0)
\]

are also on the budget line. (Fill in the missing numbers) \( 6 \text{ points} \)

(b) Now suppose that for a different consumer and different prices \( (10, 12) \) and \( (8, 15) \) are on the budget line, and that the person’s income is \( m = 540 \). Then

\[
p_1 = \quad , \quad p_2 = \quad
\]

You can use the grid below to help you find the answers.

\[
x_2
\]

\[
x_1
\]
Question 2 Suppose that there are two goods. The price of each unit of good 2 is 2 Dollars. The price of good 1 depends on the quantity purchased. That is, if a person buys up to 10 units, then the price of each unit is 5 Dollars. If the person buys more than 10 units, then the first 10 units are still priced at 5 Dollars per unit, while each additional unit is priced at 50 cents per unit. Suppose that the person’s income is $m = 200$.

Then the following points are on the budget line

$$(0, \_\_), (10, \_\_), (30, \_\_), \text{and } (\_\_, 0)$$

(Fill in the missing numbers) 8 points

The slope of the budget line is \_\_ when $x_1 < 10$, and \_\_ when $x_1 > 10$. 4 points
Question 3 A utility function is given by \( u(x_1, x_2) = x_1^2 x_2 \). Suppose that prices are given by \( p_1 = 1, p_2 = 2 \).

1. Compute the income offer curve and graph it in the grid below.  
   6 points

2. Now suppose that the person’s income is \( m = 18 \). Graph the budget line in the grid below.  
   3 points

3. Thus, the optimal consumption is \( x_1 = \boxed{\text{integer}}, x_2 = \boxed{\text{integer}} \)  
   3 points
**Question 4** A utility function is given by \( u(x_1, x_2) = \min\{x_1, 4x_2\} \). The price of good 2 is \( p_2 = 1 \). The price of good 1 is non-linear. In particular, if the person consumes less than 10 units, the price is 3 Dollars per unit. If the person consumes more than 10 units, then the first 10 units still cost 3 Dollars per unit, however, every additional unit costs 1 Dollar. Suppose that income is \( m = 40 \). Graph the budget line in the grid below. *Clearly indicate the budget set by shading it.* Determine graphically the optimal consumption, and graph the indifference through the optimal consumption choice. The optimal consumption is \( x_1 = \), \( x_2 = \)  

14 points
Question 5

1. A utility function is given by \( u(x_1, x_2) = x_1^3 x_2 \). Then

   \[
   \text{MRS} = \frac{\partial u}{\partial x_1} / \frac{\partial u}{\partial x_2} = \frac{3x_1^2 x_2}{x_1^3} = \frac{3}{x_1}
   \]

   \( 6 \text{ points} \)

2. Now suppose that the utility function is \( u(x_1, x_2) = (x_1^{-2} + 2x_2^{-2})^{-1/2} \). Then

   \[
   \text{MRS} = \frac{\partial u}{\partial x_1} / \frac{\partial u}{\partial x_2} = \frac{1}{2}(x_1^{-2} + 2x_2^{-2})^{-3/2} \cdot (-2x_1^{-3}) / (2x_2^{-3}) = \frac{-x_1^{-3}}{x_2^{-3}}
   \]

   \( 6 \text{ points} \)
Question 6 A consumer’s utility function \( u(x_1, x_2) \) has a MRS given by

\[
\text{MRS} = \frac{x_2^2}{x_1^2}.
\]

Suppose that prices are \( p_1 = 4, p_2 = 1 \) and that the person’s income is \( m = 90 \).

Then the optimal consumption is \( x_1 = \ldots, x_2 = \ldots \) \hspace{1cm} 14 points
Question 7 A utility function is given by \( u(x_1, x_2) = \min\{2x_1, x_1 + 2x_2\} \). Graph the indifference curve through (10, 30) in the grid below. Further, suppose that at prices \( p_1 = 1, p_2 = 4 \) and income \( m \), the optimal consumption is on this indifference curve. Then the optimal consumption is \( x_1 = \), \( x_2 = \), and income is \( m = \). 

12 points
Question 8  Joe visits an amusement park. His utility function is given by $u(x_1, x_2) = 10x_1 - x_1^2 + x_2$, where $x_1$ is the number of rides and $x_2$ the amount of money he spends on other items.

(a) Suppose the price of a ride is $p = 2$. Then he will take $x_1 =$ rides and spend $\$ $ at the park (You do not need to know income $m$ to answer this question.).

(b) Now suppose that the amusement park decides to charge a fixed entrance fee $F$ instead of a price per ride. A visitor who pays the fee can take as many rides as he/she wishes (i.e., after $F$ has been paid, the price per ride is zero). Then the person will take $x_1 =$ rides.

(c) (Difficult) Determine the maximum entry fee $F$ a person with the above preferences would be willing to pay to enter the park (if the person does not pay $F$ then he cannot enter the park and $x_1 = 0$). $F =$