All questions must be answered on this test form!
For each question you must show your work and (or) provide a clear argument.
All graphs must be accurate to get credit.
If you need scratch paper, use the last page or the back of the form.

Question 1

1. Suppose there are only two goods. If Joe spends all of his income then he can afford 12 units of good 1 and 6 units of good 2. Furthermore, if he wants to purchase 2 units of good 1 and still remain on his budget line he must give up 3 units of good 2. Graph the budget line in the grid above.

2. Suppose that Joe’s income is \( m = 96 \). Then

\[
p_1 = \quad , \quad p_2 =
\]

5 points
Question 2 Suppose that there are two goods. The price of each unit of good 2 is 2 Dollars. The price of good 1 depends on the quantity purchased. That is, if a person buys up to 5 units, then the price of each unit is 4 Dollars. If the person buys more than 5 units, then the first 5 units are still priced at 4 Dollars per unit, while each additional unit is priced at 1 Dollar per unit. Suppose that the person’s income is \( m = 50 \).

10 points

1. Graph the budget line using the grid below.
2. Clearly indicate the budget set by shading it.
Question 3  Income offer curves for different price ratios are depicted below.  

1. Suppose that income is $m = 58$ and prices are $p_1 = p_2 = 2$. Then optimal consumption is

\[ x_1 = \quad x_2 = \]

2. Now suppose that the price of good 2 decreases to $p_2 = 1$. Income and the price of good 1 remain unchanged. Then optimal consumption is

\[ x_1 = \quad x_2 = \]
Question 4

1. A utility function is given by \( u(x_1, x_2) = x_1^3 x_2 \). Then

\[
\text{MRS} = \frac{\partial u}{\partial x_1} / \frac{\partial u}{\partial x_2} = \frac{3x_1^2}{x_2}.
\]

2. Now suppose that the utility function is \( u(x_1, x_2) = (x_1^{-1} + 2x_2^{-1})^{-1} \). Then

\[
\text{MRS} = \frac{\partial u}{\partial x_1} / \frac{\partial u}{\partial x_2} = \frac{2}{x_1^2} / \frac{1}{x_2^2} \cdot 2.
\]
Question 5 A consumer’s utility function is given by \( u(x_1, x_2) = \min\{2x_1, 3x_2\} \). Assume that prices are \( p_1 = 1 \), \( p_2 = 3 \) and income is \( I = 27 \).

(a) Graph the budget line in the grid below.  
(b) Graph at least three indifference curves.  
(c) Graphically solve for the optimal consumption choice.  

At the optimal choice \( x_1 = \), \( x_2 = \)
Question 6  A utility function is given by $u(x_1, x_2) = x_1 x_2^2$. Prices are $p_1 = 2$, $p_2 = 2$.

1. The equation of the income offer curve is \[ \frac{x_2}{x_1} = \frac{2}{600}. \] \[ x_2 = \] 

2. Suppose that income is $m = 600$. Then optimal consumption is \[ x_1 = \quad , x_2 = \quad . \]
**Question 7** Mary consumes only two goods and she has perfect substitutes preferences for them. Currently prices are $p_1 = 2$ and $p_2 = 4$, and she consumes 10 units of each good. We refer to this as the base case.

1. Suppose that the price of good 2 increases to $p_2 = 5$ everything else remains the same as in the base case. Then her optimal consumption is

\[
\begin{align*}
    x_1 = & \quad , x_2 = \\
\end{align*}
\]

Note: There is enough information to solve this question.

2. Now suppose that $p_1$ increases to $p_1 = 4$, $p_2$ increases to $p_2 = 6$ and income increases by 50% compared to the base case.

Then her optimal consumption is

\[
\begin{align*}
    x_1 = & \quad , x_2 = \\
\end{align*}
\]

Note: There is enough information to solve this question.
Question 8 Joe’s utility function is given by \( u(x_1, x_2) = x_2 - 16(x_1 + 1)^{-1} \), where \( x_1 \) is the number of hours he spends in a gym and \( x_2 \) is money he spends on everything else. His income is \( m = 1,000 \). The price of good 2 is \( p_2 = 1 \)

(a) Suppose that the gym charges 4 Dollars per hour, i.e., \( p_1 = 4 \). Then

- Joe’s optimal choice of \( x_1 \) is
- The gym’s revenue (from Joe) is

(b) Now suppose that the gym charges a membership fee of 4 Dollars (this membership reduces income \( m \) by 4 Dollars), but with the membership the hourly price is now \( p_1 = 1 \). Then

- Joe’s optimal choice of \( x_1 \) is
- The gym’s revenue (from Joe) is $

The maximum membership fee \( F \) the gym can charge, at which Joe is just indifferent between going to the gym and not going to gym (not going means that \( x_1 = 0 \) and Joe does not pay the fee) is given by

\[ F = \]