The Effect of Enforcement on Firm Finance

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Abstract

This paper analyzes how an enforcement mechanism that resembles a court affects firm finance. We construct a model of enforcement which shows the following: (i) Firm default due to inability to pay and unwillingness to pay are equilibrium phenomena that arise endogenously and co-exist. (ii) The parameters that characterize enforcement have a highly non-linear influence on firm finance. (iii) Equilibrium credit rationing can occur for plausible parameter values. (iv) We identify a new type of credit rationing that is inherent in the underlying legal system. (iv) We provide complete characterizations of the effect of the enforcement parameters on the contract interest rate and the bankruptcy probability. These results help explain recent puzzles in empirical evidence on how the legal environment affects financial structure.

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1 Introduction

There is widespread agreement that the power to enforce rights and obligations in a society is essential. For simplicity, economists have focused on two extreme forms of enforcement: (i) Perfect ex-post enforcement of contracts by an exogenous, un-modeled authority – a "court," or (ii) contracts that are "self-enforcing."[1] Models that assume perfect ex-post enforcement have focused on ability to pay (e.g., the costly state verification model), where a borrower fails to repay the amount promised in a contract only when assets are below the promised amount. Otherwise, the borrower honors the promise. In contrast, models of self-enforcing contracts are common in international finance: no supra-national court exists with the power to enforce contracts among sovereign nations (cf., Obstfeld and Rogoff [26]). Eaton and Gersovitz [12] pointed out that when judicial enforcement is not possible, the problem of willingness to pay arises. The purpose of this paper is to model the intermediate case where enforcement is possible, but not all assets can be seized. In this case the twin problems of ability and willingness to pay arise.

We model enforcement as a technology with two key parameters.

1. The efficiency of enforcement is the cost paid to secure rights in court. This cost varies across countries due to different institutions (e.g., legal and accounting systems and corruption).

2. Creditor protection is the percentage of total assets that a court can seize; debtor protection is the remainder. The amount of protection is determined by factors such as the level of exemptions permitted by the bankruptcy code, inflation, the length of bankruptcy proceedings, and the debtor's ability to "hide" assets.[2]

In the model, we show that default due to *inability* to pay and *unwillingness* to pay are equilibrium phenomena that arise endogenously and co-exist. We next show, qualitatively and quantitatively, how these legal parameters affect firm finance. For some parameter values finance is not sensitive to the legal structure. For other values, after a critical threshold is reached, finance is severely compromised. We also show that equilibrium credit rationing is unlikely to occur in standard contracting models with perfect ex-post enforcement of contracts (cf., Stiglitz and Weiss [29] or Williamson [31]). Finally, we identify a new type of credit rationing that depends on the enforcement technology and lender beliefs about the return from enforcement. This credit rationing is inherent in the dynamic game of in-

[1] There are some recent exceptions to this practice: For example, theoretical work by Bond [5], [6] focuses on the incentives of judges to enforce contracts, and several empirical studies explore the link between the legal environment and firm finance (e.g., Demirgüç-Kunt and Levine [10] and La Porta et al. [24]).

[2] The creditor and debtor have opposite interests – all else equal, the debtor is better protected by high exemptions, high inflation and delay, while the creditor is better protected by the reverse. See Calomiris and Kahn [9] for a discussion of hidden assets.

complete information that we model, where agents' sequential decisions are constrained optimal (i.e., given their information and the enforcement institution).

The model helps reconcile data that are seemingly inconsistent across countries and time periods. For example, countries with strong creditor protection generally have better macroeconomic performance, but the U.S. is a notable exception (see Beim and Calomiris [4]). Similarly, inflation and corruption are negatively correlated with macroeconomic performance, but the Asian tigers in the 1990s are notable exceptions. We show that the interaction of the two key characteristics of judicial enforcement that we consider jointly determine the terms of finance, and help explain the wide variation observed in the data. For example, our model predicts that weak creditor protection in the U.S. has not been detrimental to finance because the court system is relatively efficient. However, in countries with less efficient court systems, poor creditor protection can matter greatly.[3]

The overall objective of the paper is to provide a (positive) theory that can explain the observed relationship between legal systems and firm finance. Thus, we take the legal system as given. The paper does not address the normative issue of what constitutes an "optimal" legal system. Questions of optimality would require a model that includes the policy-maker's objective, which reflects a notion of efficiency and equity. The law is deeply concerned with issues of fairness ex-post and typically reflects the fact that this may differ from what was efficient ex-ante. For example, although it may be efficient ex-ante to seize all debtor assets when bankruptcy occurs (in order to deter default), most bankruptcy codes are designed to prevent creditors from doing so ex-post.[4] Beim and Calomiris [4] note that French law explicitly recognizes and prioritizes multiple objectives. In order of priority, they are: (i) to maintain firms in operation, (ii) to preserve employment, and lastly (iii) to enforce credit contracts.

Before beginning the formal analysis we review key features of bankruptcy that are important for our model. Although institutions differ across countries (see Djankov et al. [11]), it is useful to consider an overview of a particular country's code. In the U.S. there are five types of bankruptcy, Chapters 7, 9, 11, 12 and 13. We focus on Chapter 7, often called liquidation.[5] When bankruptcy occurs under Chapter 7, the debtor gives up all non-exempt property owned at the time the bankruptcy petition is filed. If the court grants

[3]For example, Alston [1] documents that during the depression era in the U.S. a moratorium on farm foreclosures resulted in a credit crunch. Thus, even though the U.S. court system was relatively efficient, a further decrease in creditor protection had adverse affects on firm finance.

[4]*Local Loan Co. v. Hunt (1934)* notes, "one of the primary purposes of the bankruptcy act is to relieve the honest debtor from the weight of oppressive indebtedness and permit him to start afresh;" Epstein [13], p. 130.

[5]Chapter 7 is the most common type of bankruptcy for individuals; Chapter 13 is the second. Chapter 13 requires debtors to repay creditors under a court approved plan, and is used when a debtor is better off repaying but needs more time than creditors will allow. For example, if a debtor misses mortgage payments and faces foreclosure due to a temporary job loss, Chapter 13 allows the debtor up to 3 years to repay. Chapter 11 is designed for corporations seeking to reorganize debts while continuing to operate, Chapter 12 is the analog for family farms and Chapter 9 is for government bodies. Businesses can file Chapter 7 or 11, but not 13.

a discharge, the debtor is not liable for any other pre-bankruptcy debts (see pp. 134-135 of [13]) and no claims can be made against future earnings. Thus, Chapter 7 simultaneously liquidates all non-exempt assets for the benefit of creditors and protects the insolvent debtor. We model this protection via parameter η,[6] and the cost of enforcement by c.

Underlying our contract problem is a dynamic game with incomplete information and a stylized description of the enforcement technology. As is standard in much of the contracting literature, in the initial period the entrepreneur and lender share common beliefs about the possible returns from a risky investment project. The realization is the entrepreneur's private information unless costly bankruptcy occurs. We deviate from this standard framework by specifying a dynamic game that explicitly models agents' sequential decisions. First, agents write a contract. Next, the entrepreneur has the opportunity to default or to voluntarily make a contract payment. The lender then optimally chooses whether to request enforcement, given the information revealed by the entrepreneur's default decision. Finally, agents have rational expectations in the sense that all decisions are time consistent (cf., Kydland and Prescott [23]).

Our model builds on Krasa and Villamil [20], [21], [22], Khalil and Parigi [19], and Sharma [28].[7] The model is also related to recent work on limited commitment.[8] In our dynamic game with incomplete information there is limited commitment to the enforcement strategy specified in the initial contract. That is, the lender can revise the enforcement strategy after information is revealed by the entrepreneur's default decision. This is responsible for two of our results: A new type of credit rationing, and the non-linearity in firm finance with respect to enforcement costs. Neither result occurs in the standard costly state verification (CSV) model. In addition, we show that our two enforcement parameters can have a quantitatively significant impact on equilibrium credit rationing. Finally, we distinguish between default and bankruptcy. Default means that the borrower chooses not to make a payment. If default occurs, the lender then chooses whether to invoke bankruptcy proceedings to liquidate the firm. We investigate how the legal code affects agents' incentives to default and pursue bankruptcy.

[6] La Porta et al. [24] construct an index of creditor rights across countries which focuses primarily on governance (control of assets). The index measures whether (i) a country imposes restrictions such as creditor consent or minimum dividends for an entrepreneur to file for reorganization; (ii) secured creditors can take possession of the security during reorganization; (iii) secured creditors are first in line when the court distributes assets; (iv) the entrepreneur retains control of property pending reorganization. In contrast, our model focuses on asset liquidation and creditor/debtor protection (i.e., Chapter 7).

[7] We focus on pure strategy equilibria. Krasa and Villamil [21] establish conditions under which pure strategies are optimal in this environment.

[8] Our model differs from the limited commitment in Alvarez and Jermann [2] and Kehoe and Perri [18]. These models, rather than modeling a court, compare allocations relative to autarky. Thus enforcement is the ability to exclude parties from trade.

2 The Model

Consider an economy with a risk-neutral entrepreneur and lender, where agents derive utility only from consumption in the final period. The entrepreneur owns a technology that requires one unit of an input to produce an output described by the random variable X with realization $x \in [\underline{x}, \bar{x}] \subset \mathbb{R}_+$. Ex-ante the agents have a common prior $\beta(.)$ over $[\underline{x}, \bar{x}]$, where $\beta(\cdot)$ has a probability density function $h(x)$ that is differentiable and strictly positive on $[\underline{x}, \bar{x}]$. Assume that the entrepreneur has only $0 \leq 1 - \alpha < 1$ units of the input, and must therefore borrow α units from the lender.[9] The timing of events is as follows:

t=0 To produce, the entrepreneur must borrow from the lender. This is done by specifying a *legally enforceable payment schedule* $\ell(x, v)$, where x is the project realization verified by the court at $t = 2$ and v is a payment specified below.[10] If they cannot agree, no investment occurs and each receives a reservation value.

t=1 The entrepreneur, but not the lender, privately observes project realization x and selects a payment $v \geq 0$. Payment v is not enforceable by the court (though the enforceable payment $\ell(\cdot)$ depends on v). Because v is not enforceable, we refer to it as a *voluntary payment.*

t=2 The lender chooses whether to request enforcement. If no enforcement is requested, the lender's payoff is v and the entrepreneur's payoff is $x - v$. If enforcement is requested, the lender pays cost c and payment $\ell(x, v)$ is transferred to the lender. The lender's payoff is $v + \ell(x, v) - c$ and the entrepreneur's payoff is $x - [v + \ell(x, v)]$.

We focus on two parameters to describe enforcement. First, c is a deadweight loss to the contracting parties. Ceteris paribus this cost is higher if accounting standards are poor, which implies a higher cost to the court to determine the entrepreneur's assets, or corruption exists, such as bribes paid to government officials or the court.[11] Second, η determines the amount of creditor versus debtor protection. This parameter measures the percentage of total entrepreneur assets that the court cannot seize, and it is affected by factors such as the exemptions specified in the bankruptcy code, inflation, and the length of bankruptcy proceedings. The higher these factors are, the higher η, which means that creditor protection is weak (equivalently, debtor protection is strong). Thus, η determines the legal payments that the court will enforce, with the maximum enforceable payment given by $(1 - \eta)(x - v)$. [12]

[9] Because we will prove that the lender uses debt contracts, α is the debt-equity ratio.

[10] The properties of a legally enforceable payment schedule $\ell(\cdot)$ are given in Definition 1 below.

[11] See Bond [5] for an analysis of how bribes affect the judicial agency problem.

[12] For example, consider an entrepreneur whose assets include equity in a principal residence. If the entrepreneur goes bankrupt, in seven U.S. states (AR, FL, IA, KS, MN, OK, TX) the entire equity is exempt while in twenty-seven states the maximum amount of equity that can be sheltered in a principal homestead is \$20,000 or less.

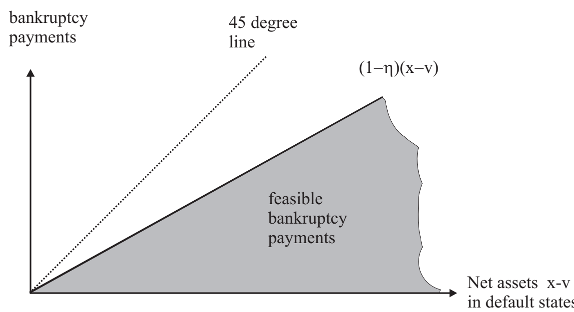

Figure 1: Feasible Bankruptcy Payments

Figure 1 illustrates the effect of the legal system on contract payments. Suppose that the entrepreneur repays nothing (i.e., $v = 0$) and the lender requests enforcement. The shaded, cone-shaped area is the set of all feasible bankruptcy payments. The court cannot seize η percent of entrepreneur assets. Thus the maximum possible payment to the lender is $(1-\eta)x$. By an appropriate choice of ℓ, any payment in the cone can be obtained. The figure illustrates an important feature of Chapter 7 bankruptcy in the U.S. – it "protects" the debtor from paying more than $(1-\eta)(x-v)$. That is, even if a contract specified a larger payment, the legally enforced payment must be in the cone.

Definition 1 *Payment schedule $\ell(x, v)$ is legally enforceable if, for all x, v with $x \geq v$, $0 \leq \ell(x, v) \leq (1-\eta)(x-v)$.*

3 The Investment Problem and Equilibrium Contract

The investment problem is a dynamic game with asymmetric information because beliefs are allowed to vary endogenously as information changes during the game. We focus on pure strategy equilibria that are Pareto efficient in the set of all perfect Bayesian Nash equilibria (PBNE) of the game. Let $v(x)$ be the entrepreneur's strategy for choosing the voluntary payment, $\ell(x, v)$ be the legally enforceable payment, $e(v)$ be the lender's enforcement strategy, and $\beta(x|v)$ be the lender's updated belief about the return at $t = 2$. If $e(v) = 1$ the lender requests the court to enforce $\ell(x, v)$, and if $e(v) = 0$ the lender does not request enforcement.

At first glance, it may seem unusual to specify beliefs as part of the contract problem. However, this natural extension of the well established Pareto approach allows for dynamic information revelation. In the contract literature, it is standard to assume ex-ante (before information is revealed) that a "planner" coordinates agents on actions and a contract that lead to an efficient allocation, subject to constraints. We also consider a planner who coordinates agents to achieve efficient outcomes, but the off-equilibrium path beliefs $\beta(x|v)$ of the lender matter in our dynamic game because different beliefs give rise to different equilibrium payoffs. Thus, the planner must now coordinate agents on the contract, the information itself, and the beliefs that could arise if the entrepreneur were to deviate from his equilibrium strategy.[13]

The choice variables in problem 1 are therefore, the payment schedule $\ell(x, v)$, the agents' payment and enforcement actions $v(x)$ and $e(v)$, respectively, and beliefs $\beta(x|v)$. Effectively, only off-equilibrium path beliefs must be chosen because constraint (5) pins down equilibrium beliefs via Bayes' rule. Note that problem 1 maximizes one agent's payoff subject to the other agent receiving a reservation utility. This is equivalent to maximizing a weighted sum of the two agents' utilities. Varying the Pareto weights (or the reservation utility) gives the entire Pareto frontier.

Problem 1 *At $t = 0$, choose $\{v(x),\ \ell(x, v),\ e(v),\ \beta(x|v)\ \}$ to maximize*

$$E_0[u_L(x)] = \int \Big[v(x) + e\big(v(x)\big)\Big(\ell\big(x, v(x)\big) - c\Big)\Big]\, d\beta(x) \tag{1}$$

subject to

$$E_0[u_E(x)] = \int \Big[x - v(x) - e\big(v(x)\big)\ell\big(x, v(x)\big)\Big]\, d\beta(x) \geq \bar{u}_E \tag{2}$$

$$v(x) \in \arg\max_{v \geq 0} \Big[x - v - e\big(v(x)\big)\ell\big(x, v(x)\big)\Big] \tag{3}$$

$$e(v) = 1 \textit{ if and only if} \int \big[\ell(x, v) - c\big]\, d\beta(x|v) \geq 0 \tag{4}$$

$$\beta(x|v) \textit{ is derived from } \beta(x) \textit{ using Bayes' rule whenever possible} \tag{5}$$

$$\ell(x, v) \textit{ is enforceable} \tag{6}$$

The objective of problem 1 is the lender's payoff. Constraint (2) assures that the entrepreneur gets at least reservation utility $\bar{u}_E$. Constraint (3) requires the entrepreneur's payment choice v at $t = 1$ to be optimal, i.e., maximize utility given realization x. Constraint (4) requires the lender's enforcement choice to be optimal, and (5) requires beliefs to be consistent. Thus, (3)–(5) require the strategies v, e and beliefs $\beta(\cdot|v)$ to be a perfect Bayesian

[13] Many different off equilibrium path beliefs $\beta(x|v)$ support efficient outcomes. Each equilibrium consists of a strategy profile and a belief system, but agents' payoffs in all of these equilibria are the same. Our approach differs from the refinements literature in game theory, which imposes restrictions on beliefs (e.g., intuitive, divine, etc.). Instead, we admit any belief that supports an allocation on the Pareto frontier (where payoffs are maximized) of equilibria. Some refinement criteria may provide equilibria where the lender gets a lower payoff; this will not occur under our approach.

Nash equilibrium (PBNE). Finally, (6) requires payment $\ell(x, v)$ to be enforceable as specified in Definition 1.

We now characterize the solutions of problem 1. Let $\bar{v}$ denote the face value of the contract (principal and interest).

Lemma 1 *Without loss of generality, we restrict attention to strategies $\mathscr{v}(x)$ which assume at most two values, $\bar{v}$ and 0, a payment function $\ell(x, v)$ and enforcement strategy $\mathscr{e}(v)$ with the following properties:*

1. *If $v \geq \bar{v}$ then $\ell(x, v) = \mathscr{e}(v) = 0$;*
2. *If $0 < v < \bar{v}$ then $\ell(x, v) = (1 - \eta)(x - v)$.*
3. *If $0 \leq v < \bar{v}$ then $\mathscr{e}(v) = 1$.*
4. *If $v \notin \{0, \bar{v}\}$, then the off-equilibrium path belief $\beta(x|v) = 1$ for $x = \bar{x}$.*

The intuition for the proof of the Lemma is as follows. First, consider two states x, x' in which no enforcement occurs. The resulting total payments to the lender are then $\mathscr{v}(x) = v$ and $\mathscr{v}(x') = v'$. If $v > v'$, the entrepreneur would always choose the lower payment. Hence, all non-bankruptcy payments are the same, and equal to $\bar{v}$. Now consider two states x, x' for which enforcement occurs. The total payments are $T = v + \ell(x, v)$ and $T' = v' + \ell(x', v')$. Define an alternative contract with $\ell(x, 0) = T$ and $\ell(x', 0) = T'$; the total payment is the same if the entrepreneur chooses zero repayment at $t = 1$. One can then show that this alternative contract fulfills all constraints of problem 1.

Lemma 1 has four key implications. First, we can restrict attention to payments that are either 0 or $\bar{v}$ on the equilibrium path. Second, Lemma 1 implies that default occurs if and only if $v = 0$, where payment 0 is default and payment $\bar{v}$ is no default. Third, it is possible to exclude renegotiation. This is optimal from an ex-ante perspective because it increases commitment to the ex-ante contract. Renegotiation in our model would correspond to an off-equilibrium path payment $0 < v < \bar{v}$ by the entrepreneur.[14] If the entrepreneur were to make such a payment, the investor would seek enforcement rather than renegotiation, thereby claiming the remaining non-exempt assets $(1 - \eta)(x - v)$. Finally, Lemma 1 also determines a set of beliefs that support Pareto efficient solutions to problem 1. These beliefs are optimistic in the sense that the lender believes that the highest possible realization $\bar{x}$ occurred.

[14] See Krasa and Villamil [21], Sharma [28] and Hvide and Leite [17] for CSV models where the entrepreneur can make a renegotiation offer. In Krasa and Villamil and Sharma, the renegotiation proofness constraint does not bind when the optimal contract involves deterministic verification, which corresponds to debt. In contrast to our model where the court enforced payment $\ell(x, v)$ can depend on offer v, Hvide and Leite consider a smaller universe of contracts where the contract payment is independent of v. They obtain strategic default that corresponds to debt forgiveness (renegotiation).

In Theorem 1 below we show that a simple debt contract solves problem 1. The key characteristic of a simple debt contract is that the maximum possible amount of assets, $(1-\eta)(x-v)$, is transferred in the case of default up to the amount owed, $\bar{v}$. Let x^* be the lowest non-bankruptcy state.

Definition 2 *$\{\ell(x,v), v(x)\}$ is a simple debt contract if there exists $\bar{v}$ and $x^* \in [\underline{x}, \bar{x}]$ with $x^* \geq \bar{v}$ such that*

$$\ell(x,v) = \begin{cases} \min\{(1-\eta)x, \bar{v}\} & \textit{if } x < x^*,\ v = 0; \\ 0 & \textit{if } v \geq \bar{v}; \\ (1-\eta)(x-v) & \textit{otherwise;} \end{cases} \qquad v(x) = \begin{cases} \bar{v} & \textit{if } x \geq x^*; \\ 0 & \textit{if } x < x^*; \end{cases}$$

We now prove that the optimal contract is simple debt. The main difference between our argument and that for the standard CSV model is generated by sequential rationality – the investor must be willing to enforce when the entrepreneur defaults. In the standard CSV model the sole concern is to minimize expected bankruptcy costs; there is no need to provide an incentive to enforce.

Theorem 1 *Simple debt contracts solve problem 1.*

Before we provide the intuition for the proof of theorem 1, we explain the difference between our model and definition of simple debt relative to the classic CSV model (Gale and Hellwig [15], Townsend [30] or Williamson [31]). In the CSV model default occurs if and only if the entrepreneur is unable to pay. In contrast, our analysis accommodates both inability to pay and willful default. An example illustrates the intuition. Assume that a debtor owes $\bar{v} = \$100,000$, has home equity of $\$50,000$, private property of $\$80,000$, and retirement savings of $\$100,000$. The total value of the debtor's assets is $x = \$230,000$, which exceeds $\bar{v}$. If the debtor files for bankruptcy in Texas, under state law all equity in a homestead and pension/retirement accounts are exempt, as is personal property up to \$60,000. Chapter 7 specifies that exempt assets cannot be used to satisfy creditor claims. As a consequence, the court can only seize $(1-\eta)x = \$20,000$. This amount is transferred to the creditors (net of c) and the case is discharged by the court. The debtor is "protected" from paying the remaining $\$80,000$. Given a particular bankruptcy code, it may therefore be optimal for a debtor to default. In our model default is "willful" whenever total assets x exceed the amount owed $\bar{v}$, but the bankruptcy code protects the debtor from judgments against the exempt portion of assets.

Figure 2 illustrates the default regions in a simple debt contract when $\underline{x} = 0$. Inability to pay occurs when debtor assets are less than the amount owed, $x < \bar{v}$. This corresponds to area A. Willful default occurs when the debtor defaults but total assets, x, are sufficient to pay $\bar{v}$. This occurs when $x > \bar{v}$ for two distinct reasons. First, because η percent of the

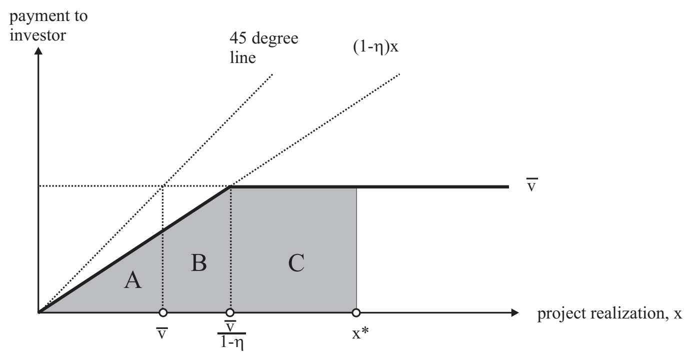

Figure 2: Simple Debt Contracts and Bankruptcy Regions

debtor's assets are not seized, the debtor will not repay if $(1-\eta)x \le \bar{v}$, i.e., if $x \le \frac{\bar{v}}{1-\eta}$. This is area B. Second, constraint (4) may generate an additional willful default region given by area C. Constraint (4) gives the lender the incentive to request enforcement if the entrepreneur does not pay $\bar{v}$ and requires the lender's enforcement decision to be sequentially rational, i.e., the lender's expected enforcement payoff must cover cost c. Region C will arise if regions A and B are not sufficient to cover cost c.[15] In the efficient equilibria characterized by problem 1, the entrepreneur will announce default in just enough states to induce the lender to enforce whenever the entrepreneur defaults, i.e., (4) must hold. This adds region C with bankruptcy states $\frac{\bar{v}}{1-\eta} \le x < x^*$ when (4) binds. Finally, constraint (3) ensures that the entrepreneur is willing to default when the lender expects default to occur, i.e., $\ell(x, v) \le \bar{v}$. In a simple debt contract it will be optimal to make the bankruptcy payment as large as possible. Therefore, $\ell(x, v) = \bar{v}$ in region C.

Figure 3 illustrates the intuition for Theorem 1. We consider a simple debt contract with face value $\bar{v}_D$ and an arbitrary debt contract with face value $\bar{v}_A$—in the proof this latter contract need not be debt. The lender's expected payment under contract $\bar{v}_A$ is given by the area $b+c+d+e$. We find a simple debt contract with face value $\bar{v}_D$ such that the lender's expected payment is the same as under the original contract, i.e., $a+b+e = b+c+d+e$. This implies that $a+b > b+c$, where $b+c$ is the bankruptcy area under the alternative contract and $a+b$ is the bankruptcy area under the simple debt contract. Therefore, if in both contracts bankruptcy occurs for all states $x < x_A^*$ then the lender's expected bankruptcy payment is strictly higher under the simple debt contract $\bar{v}_D$. This implies that constraint (4) is slack. As a consequence, we can reduce the size of the bankruptcy set for the simple debt contract to x_D^*, thereby decreasing expected enforcement

[15] If the lender expects the entrepreneur to default only if $x < \frac{\bar{v}}{1-\eta}$, the lender will never enforce. This implies that the entrepreneur has the incentive to default in additional states.

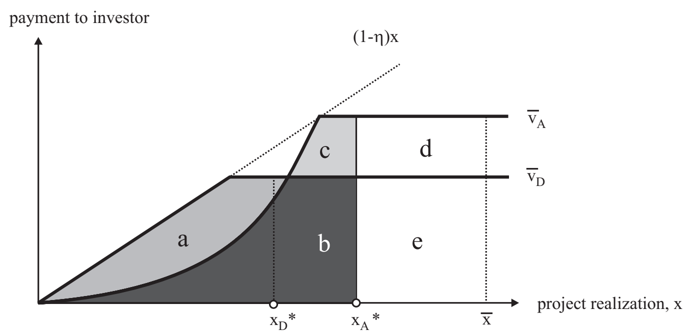

Figure 3: Optimal Simple Debt Contracts

costs, which increases the lender's payoff.

We can now use Lemma 1 and theorem 1 to simplify problem 1 as follows.

Problem 2 *At $t = 0$, choose $\bar{v}$ and x^* to maximize*

$$E_0[u_L(x)] = \int_{\underline{x}}^{\frac{\bar{v}}{1-\eta}} (1-\eta)x \, d\beta(x) + \int_{\frac{\bar{v}}{1-\eta}}^{\bar{x}} \bar{v} \, d\beta(x) - \int_{\underline{x}}^{x^*} c \, d\beta(x) \tag{7}$$

subject to

$$E_0[u_E(x)] = \int_{\underline{x}}^{\frac{\bar{v}}{1-\eta}} \eta x \, d\beta(x) + \int_{\frac{\bar{v}}{1-\eta}}^{\bar{x}} (x - \bar{v}) \, d\beta(x) \geq \bar{u}_E \tag{8}$$

$$\frac{\bar{v}}{1-\eta} \leq x^* \tag{9}$$

$$\int_{\underline{x}}^{\frac{\bar{v}}{1-\eta}} (1-\eta)x \, d\beta(x|x < x^*) + \int_{\frac{\bar{v}}{1-\eta}}^{x^*} \bar{v} \, d\beta(x|x < x^*) - c \geq 0 \tag{10}$$

$$(1-\eta)(\bar{x} - \bar{v}) - c \geq 0 \tag{11}$$

Objective (7) and constraint (8) correspond to (1) and (2) of problem 1. Constraint (9) specifies that default must occur at least in all states x with $x < \frac{\bar{v}}{1-\eta}$, which implies $\frac{\bar{v}}{1-\eta} \leq x^*$ (see figure 2). This follows from (3). Finally, constraints (10) and (11) are implied by (4). Specifically, (10) considers the case where payment occurs on the equilibrium path and (11) considers off-equilibrium path payments v. Because beliefs off the equilibrium path are optimistic as explained above, (4) implies $(1-\eta)(\bar{x} - v) - c \geq 0$ for all $v < \bar{v}$, which in turn implies (11). Finally, (5) and (6) of problem 1 are satisfied by construction. Note that the existence of a solution follows from standard compactness and continuity arguments.

We now point out several differences between our model and two important literatures. First, our dynamic enforcement game is fundamentally different than the CSV model with full commitment because, as explained above, sequential rationality may require area C. Further, Figure 2 shows that η generates a "change in slope," and hence payoffs, in regions A and B. We will show that this can lead to important quantitative differences relative to the CSV model. We note that this latter effect does not require a dynamic game. Second, there is a sizable literature on strategic default in incomplete contract models (cf., Anderson and Sundaresan [3] or Mella-Barral and Perraudin [25]). These models consider dynamic games with renegotiation, but unlike our model with a stylized description of bankruptcy liquidation (Chapter 7), they assume an exogenous legal authority that solely assigns ownership rights. Bankruptcy is interpreted as a situation where "control" is transferred from the firm to creditors, and strategic default corresponds to debt forgiveness. The debtor is not "forced into costly bankruptcy" because it is Pareto improving for both parties to renegotiate to avoid costly liquidation. In contrast, we model the liquidation process and show how parameters η and c affect the debtor's incentive to seek bankruptcy protection. In our model agents may choose to enter bankruptcy even if they could pay, which is consistent with empirical observation.[16]

4 Enforcement and Entrepreneur Finance

The point of our analysis is to construct a model of enforcement where the legal system is described by parameters, η and c, and the lender has an incentive to request enforcement (which is ensured by constraints (10) and (11)). In this section we analyze the solution to the contract problem, both qualitatively and quantitatively. Theorems 2 and 3 provide complete characterizations of the effect of enforcement parameters η and c, respectively, on the bankruptcy probability and the interest rate.

4.1 The Effect of Exemptions and Inflation: η

Parameter η determines the percent of total assets that the court cannot seize due to exemptions in the legal code or because inflation lowers the real value of creditor claims. Theorem 2 investigates the impact of η on the optimal contract. The result follows because increasing η decreases the cone of feasible payments (see figure 1). Recall that the face value and interest rate are related by $\bar{v} = \alpha(1 + r)$ and that the bankruptcy probability is $\beta[\underline{x}, x^*]$.

[16]Testimony before the House Judiciary Committee in 2002 indicated that in the U.S. "about 25% of Chapter 7 debtors could have repaid at least 30% of their non-housing debts over a 5-year repayment plan, after accounting for monthly expenses and housing payments" and that "about 5% of Chapter 7 filers appeared capable of repaying all of their non-housing debt over a 5-year plan." Under Chapter 7, this debt is extinguished and need never be repaid.

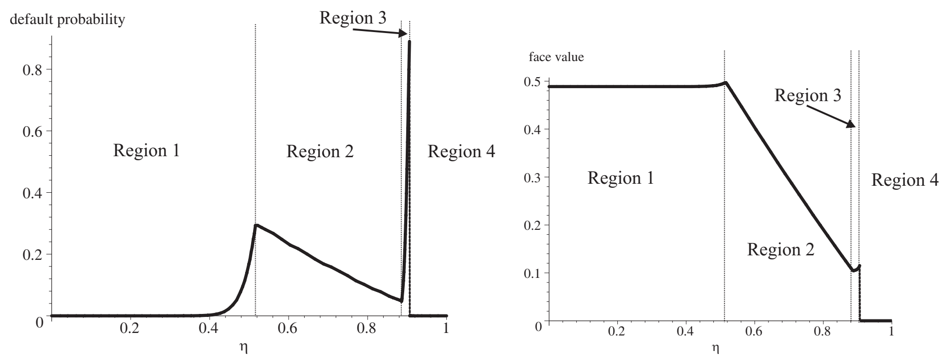

Figure 4: The Four Regions of Theorem 2

Theorem 2

1. *Assume that η is increased. Then the lender's expected payoff decreases. The decrease is strict if $c > 0$ and if bankruptcy occurs with positive probability.*

2. *When η changes, the effect on the interest rate and the bankruptcy probability is characterized by four distinct parameter regions.*

   Region 1 *If (8) binds but (10) and (11) do not, which occurs if η and c are not too large, the interest rate and bankruptcy probability are increasing in η.*

   Region 2 *If (8), (10) and (11) do not bind, which occurs for intermediate values of η, the interest rate and bankruptcy probability are decreasing in η.*

   Region 3 *If (10) binds, which occurs for larger values of η, the bankruptcy probability is increasing in η. The interest rate is increasing in η if (8) also binds.*

   Region 4 *If η is sufficiently close to 1, the bankruptcy probability is 0. The interest rate is constant unless (11) binds, in which case it decreases.*

Figure 4 illustrates Theorem 2. In the graphs we show how the bankruptcy probability and the contract's face value vary with η for a lognormal distribution with parameters $E[\ln X] = 0.08$ and $0.10 = \sqrt{\mathrm{Var}(\ln X)}$ (this corresponds to a mean of about 1.09 and a standard deviation of 0.11), $c = 0.1$ and $\bar{u}_E = 0.6$. The face value is $\bar{v} = 0.5(1 + r)$ because we assume that the lender provides 50% of project finance ($\alpha = 0.5$). The intuition for each region is as follows.

In region 1 as η increases, the entrepreneur retains more assets in bankruptcy. In order to make up for this, the lender raises the face value. In the graph the increase in the face value is small until η is close to region 2. In contrast, the increase in the bankruptcy probability is more rapid because increasing the face value has direct and indirect effects on bankruptcy – as η increases the entrepreneur enjoys more bankruptcy protection and this increases the entrepreneur's incentive to default.

In region 2 an increase in η, ceteris paribus, would lead to a further increase in the bankruptcy probability. However, at the end of region 1 it is inefficient to increase the bankruptcy probability further because expected bankruptcy costs are large. In order to keep the bankruptcy probability at least constant, the face value must be decreased.[17] However, as η gets larger it becomes optimal to actually decrease the bankruptcy probability. Recall figure 2. In region 2, $x^* = \frac{\bar{v}}{1-\eta}$. Moreover, at the optimum the marginal loss to the lender of lowering the face value by $\Delta\bar{v}$ must equal the marginal gain of a decreased bankruptcy probability. If $\bar{v}$ is decreased by $\Delta\bar{v}$, then x^* decreases by $\Delta\bar{v}/(1-\eta)$, which is the lender's gain from less bankruptcy. This benefit increases as η increases. Therefore, a larger η results in a lower x^* and hence a lower bankruptcy probability. This decrease of x^* accelerates the drop in the face value because to keep the bankruptcy probability constant, we must lower $\bar{v}$. Hence, to lower the bankruptcy probability, $\bar{v}$ must decline at an even faster rate.

Region 3 occurs when η is relatively large and (10) binds. In figure 2 this means that x^* is increased. The bankruptcy probability quickly increases to a level where it is no longer optimal to provide finance, which leads to region 4.

We now perform a quantitative experiment in order to illustrate Theorem 2. As in the previous figure, we assume a debt equity ratio of $\alpha = 0.5$ and that bankruptcy costs are $c = 0.1$. We consider three different distributions:

1. *Empirical:* Boyd and Smith [8] computed this distribution using COMPUSTAT data for the U.S. mining industry. They constructed a discrete distribution with 12 realizations with mean 1.069 and standard deviation 0.056 (cf., Appendix 6.2).

2. *Normal:* A normal distribution with the same mean and variance as the empirical distribution.

3. *Lognormal:* A lognormal distribution with the same mean and variance.[18]

We choose $\bar{u}_E = 0.5594$ for the normal and lognormal distribution and $\bar{u}_E = 0.5571$ for the empirical distribution. Given these values the lender's payoff is the same for all three distributions when $\eta = 0$.

Figure 5 shows the interest rate and the bankruptcy probability for the three different distributions in our quantitative experiment. Two features of the figures are striking. First, for low η both the bankruptcy probability and the interest rate do not change significantly

[17] Recall the argument for region 1 that increasing η, keeping the face value constant, increases the bankruptcy probability.

[18] Thus, $E[\ln X] = 0.0657$ and $0.0526 = \sqrt{\mathrm{Var}(\ln X)}$.

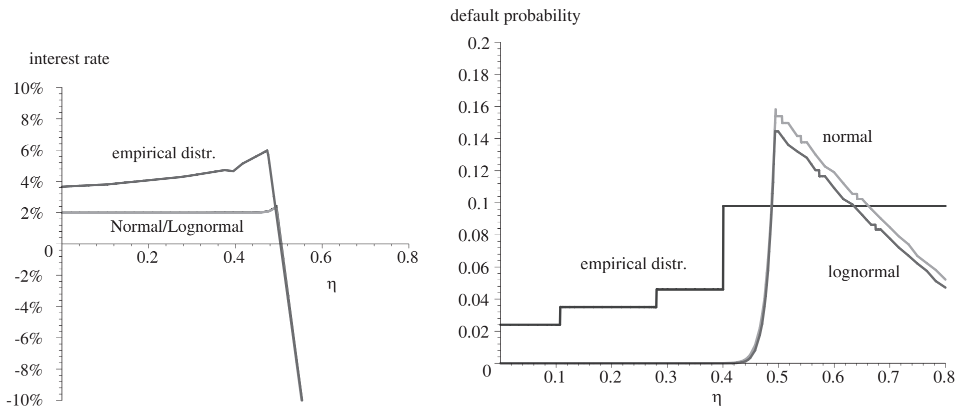

Figure 5: The effect of η on the interest rate and the bankruptcy probability

for the normal or lognormal distribution. In contrast, in the same region the empirical distribution is somewhat more responsive to changes in η. The first two moments of all three distributions are the same, thus the differences in the graphs indicate that higher order moments matter. Second, figure 5 displays a rapid drop off in the interest rate after a critical value is reached. The critical value for the normal and lognormal distributions is $\eta = 49\%$ and for the empirical distribution is 47%. Our model does not explicitly contain a constraint describing the lender's opportunity cost of providing finance. However, it is clear that lenders will be reluctant to provide finance once the interest rate becomes very low. Therefore an increase of η can quickly lead to a breakdown in finance.

At first glance, the value for η of 47–49% may appear large. However, two examples illustrate that it can be quite reasonable. First, recall that under bankruptcy law in Texas all equity in a homestead and pension/retirement accounts are exempt. Suppose, for example, that a court assesses that a debtor has \$1 million in assets. If homestead equity is \$200,000 and retirement savings are \$300,000, then the value for η is 50%.[19] The second example applies to countries with inflation and bankruptcy delays where contracts cannot be indexed for inflation (as in Mexico before 1996). If a bankruptcy case is expected to take 6 years (as was the case in Mexico), then a steady inflation rate of 11% compounded over 6 years lowers the value of creditor claims by about 50%. The actual average inflation rate of 16.6% reported by the Banco de México for the last 10 years clearly indicates that the drop-off is a legitimate concern in many economies.[20]

[19] If assets can be concealed, then η can be even higher.

[20] Boyd, Levine and Smith [7] find empirical evidence of an inflation threshold of 15% (for economies with inflation rates exceeding 15%, there is a discrete drop in financial sector performance). This corresponds to the non-linearity in our theoretical model at a critical η in figure 5. The value of η implied by an inflation rate

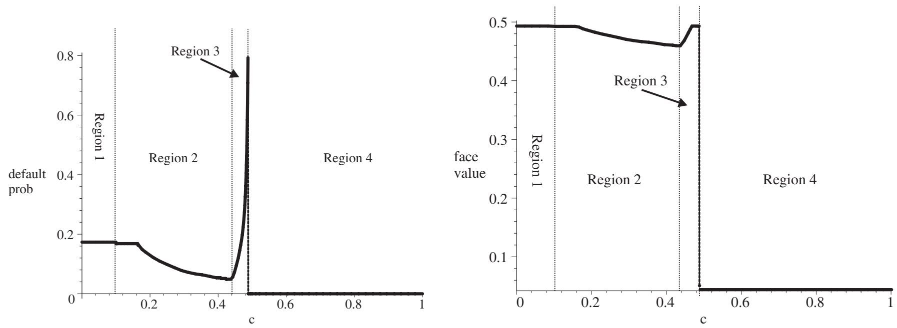

Figure 6: The Four Regions of Theorem 3

4.2 Efficiency of the Court: c

Theorem 3 analyzes the effect of c on finance. The size of c measures the efficiency of bankruptcy procedures. Assume that $\beta(x)$ has a density function $h(x)$ that is differentiable.

Theorem 3

1. *Assume that c is increased. Then the lender's expected payoff is decreased. The decrease is strict if the bankruptcy probability is strictly positive.*

2. *When c changes, the effect on the interest rate and the bankruptcy probability is characterized by four distinct parameter regions.*

Region 1 *If (8) binds, but (10) and (11) do not bind, which occurs for small c, the interest rate and the bankruptcy probability do not depend on c.*

Region 2 *If (8), (10) and (11) do not bind, which may occur for intermediate values of c, the bankruptcy probability and the interest rate are decreasing in c.*

Region 3 *If (10) binds but (11) does not bind, which occurs for larger values of c, the interest rate and bankruptcy probability increase. If (8) holds with equality, the interest rate is constant.*

Region 4 *If c is sufficiently large, the bankruptcy probability is zero. The interest rate is constant, unless (11) binds, in which case it decreases.*

Figure 6 illustrates Theorem 3. We choose the same lognormal distribution as in figure 4, i.e., $E[\ln X] = 0.08$ and $0.1 = \sqrt{\mathrm{Var}(\ln X)}$. We also choose $\eta = 0.5$ and $\bar{u}_E = 0.6$. Again, the face value is $\bar{v} = 0.5(1 + r)$ because we assume that $\alpha = 0.5$.

of π over n years is given by $1 - \eta = (1 - \pi)^n$.

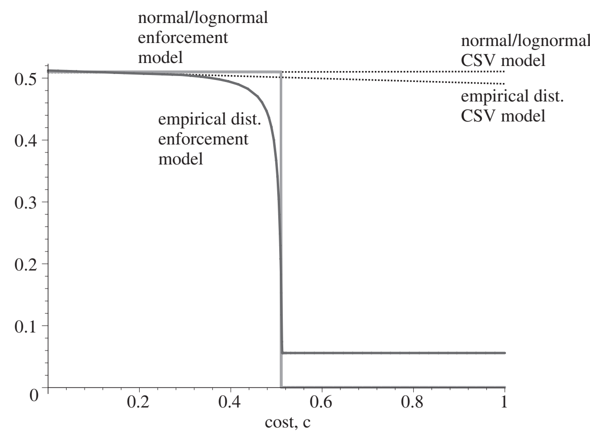

Figure 7: The Effect of c on the Lender's Expected Payoff

In region 1, the entrepreneur's reservation utility constraint binds. Therefore, the face value does not change with c. This, in turn, means that the bankruptcy probability remains constant. In region 2, costs c are sufficiently high that it becomes optimal to reduce the face value $\bar{v}$. Reducing $\bar{v}$ reduces the bankruptcy probability and saves expected bankruptcy costs. For the lender, this saving compensates for the lower face value. In region 3, (10) binds. The argument is similar to that for Theorem 2, i.e., x^* must be increased in order to provide the lender an incentive to request enforcement. Once c is sufficiently large it becomes optimal not to provide finance, or to invest solely in projects that are fully collateralized, i.e., where $\underline{x} >> 0$. The inability of entrepreneurs to obtain finance is a significant problem in many emerging markets. Our result indicates that high enforcement costs can easily be a source of credit market failure. In practice, cost c includes payments to accountants, lawyers, and the court to establish the size of the entrepreneur's assets, x, and bribes to expedite the case or influence the outcome. The government can play an important role in determining the size of c by requiring a high level of disclosure and routine accounting practices, and by policies to deter corruption.

We now investigate the quantitative implications of changes in c on finance. We assume that $\eta = 0$. Otherwise, we use the same three distributions and parameter values for u_E considered previously. Figure 7 shows how the lender's expected payoff varies with enforcement cost c. The most striking result is that there are two regions over which changes in c have almost no effect (these correspond to regions 1 and 4 in Theorem 3). Region 2 from Theorem 3 does not occur for these parameter values. Moreover, the intermediate region 3 is very small and highly sensitive to small changes in c. The transition between region 1, where finance occurs, and region 4 where finance is severely compromised is especially rapid for the normal and the lognormal distributions. The intuition for this is as follows. In the computations, the face value is approximately 0.510. Let $h(x)$ be the density of the lognormal distribution considered. For $\eta = 0$ the left hand-side of (10), the lender's expected payoff from enforcement, is given by

$$\frac{\int_0^{0.510} x h(x)\, dx}{\int_0^{0.510} h(x)\, dx} - c = 0.508 - c.$$

Constraint (10) does not bind if $c \leq 0.508$, which characterizes region 1. Next, note that (10) cannot hold if c is strictly greater than the face value $\bar{v}$. Thus all $c > 0.510$ are in region 4, and the transition between regions 1 and 4 consists of all c between 0.508 and 0.510. Now compare this to the empirical distribution, where the face value is 0.518. Then

$$\frac{\int_0^{0.518} x\, d\beta(x)}{\int_0^{0.518} d\beta(x)} - c = 0.291 - c.$$

Thus, region 1 extends to all $c \leq 0.291$. Next, note that (10) cannot hold if $c > 0.518$. The transition between regions 1 and 4 consists of all c between 0.291 and 0.518. Because the empirical distribution is both discrete and has more mass in the lower tail, (10) starts to bind earlier, leading to a bigger transition region. However, the transition is still steep.

Theorem 3 and Figure 7 are interesting because they indicate that countries with poor institutions may experience rapid and severe "financial crises" due to a small change in fundamentals, c, such as a bribery or accounting scandal. Theorem 3 predicts that this phenomenon would not be observed in low cost countries (e.g., the U.S.), but would be observed in intermediate cost countries. Countries with high costs (e.g., sub-Saharan Africa) would have low expected returns, and therefore would receive little private investment unless c was lowered substantially. Finally, Figure 7 illustrates that the predictions of our dynamic enforcement game can differ markedly from those of the CSV model.

4.3 Types of Equilibrium Credit Rationing

4.3.1 Backward Bending Loan Supply

When bankruptcy is costly, it is well known that the supply curve for loans can bend backward (see Williamson [31]).[21] The reason for this result is that increasing the interest rate has two opposing effects. On the one hand it increases the lender's expected revenue. On the other hand it increases the lender's expected bankruptcy costs. As a result the lender's expected payoff will decrease when the interest rate reaches a critical level r_h^* as shown in the first panel of Figure 8. In our model, this point r_h^* is reached when the entrepreneur's

[21] The possibility of a backward bending loan supply curve was identified by Hodgman [16] and developed by Stiglitz and Weiss [29] for specific assumptions on the return distribution. Williamson [31] derived a backward bending supply curve endogenously in the CSV model, cf., Freixas and Rochet [14].

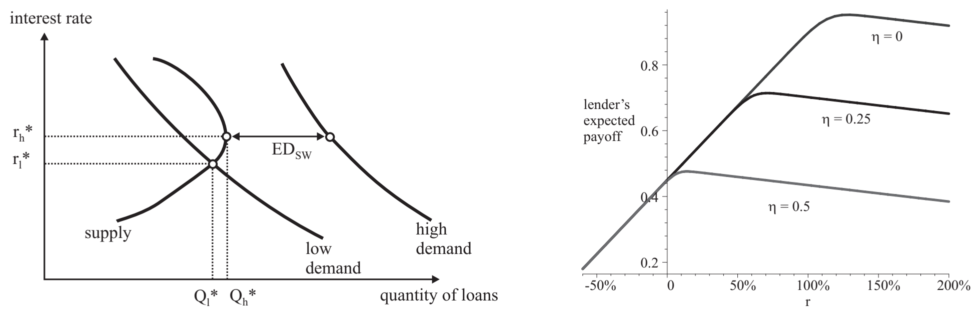

Figure 8: Equilibrium Credit Rationing

reservation utility constraint (8) becomes slack. The reason for this is that the lender would need to raise the interest rate in order for (8) to hold with equality.[22] Thus, the equilibrium market interest rate cannot exceed r_h^*.

The first panel of figure 8 shows that whether or not equilibrium credit rationing occurs depends on the level of demand.

- When demand is low, demand equals supply and the equilibrium interest rate and loan quantity are r_l^* and Q_l^*. There is no credit rationing.

- When demand is high, the supply and demand curves do not intersect and equilibrium credit rationing is given by amount $\mathrm{ED_{SW}}$. Borrowers are willing to pay a higher face value to receive a higher loan quantity, but lenders refuse because lender return is maximal at r_h^* and Q_h^*.

Riley [27] claimed that credit rationing caused by a backward bending loan supply curve was empirically implausible, but his challenge was purely theoretical. In contrast, a useful feature of our model is that it can be used to assess quantitatively the parameters at which credit rationing may arise, i.e., the critical value r_h^*. Thus, we now compare the r_h^* for the CSV and enforcement models for the same three distributions used throughout the quantitative analysis. In all cases we assume that $c = 0.1$. In the enforcement model we assume that $\eta = 0.49$, which is the critical value computed previously. In both models we assume a debt-equity ratio of $\alpha = 0.5$.

Table 1 reports the results. For the enforcement model, we use the optimization routine described in Appendix 6.2. To determine the interest rate that yields the maximum lender payoff we set $\bar{u}_E = 0$. For the CSV model, we set $\eta = 0$ and ignore constraints (10)

[22] If we embed our analysis in a general equilibrium model with many lenders and projects that have the same distribution, we would generate a backward bending aggregate supply curve as depicted in the second panel of figure 8.

and (11). In both models, credit rationing arises because raising the interest rate increases the bankruptcy probability. However, table 1 shows that credit rationing occurs in the enforcement model at a lower real interest rate and a lower bankruptcy probability. The intuition for this result is shown in the second panel of Figure 8. The exemptions shift the lender's expected payoff function down and to the left. Hence the maximum is obtained at a lower r^*. This occurs because the marginal benefit of an increase in the interest rate (or face value) is more quickly outweighed by the increased expected bankruptcy costs in the enforcement model than in the CSV model.[23]

Table 1: Comparison of the CSV and Enforcement Models

CSV Model	*Empirical Dist.*	*Lognormal Dist.*	*Normal Dist.*
r^*	130.2%	108.9%	109.5%
Bankruptcy Probability	49.7%	33.3%	34.5%
Enforcement Model	*Empirical Dist.*	*Lognormal Dist.*	*Normal Dist.*
r^*	2.8%	3.17%	3.37%
Bankruptcy Probability	9.8%	14.9%	15.8%

4.3.2 Equilibrium Quantity Rationing: Inelastic Supply and Demand

We now focus on constraint (11), and show that it produces a new type of credit rationing. This constraint ensures that the debtor could not "shave down" payments to the lender in a renegotiation. For example, assume that the debtor owes $\bar{v} = \$100,000$, but offers to pay only $\$80,000$. If the debtor knows that the creditor will never go to court, the debtor would always shave the payment to $\$80,000$. Whether or not the creditor chooses to go to court depends on the cost of the court, c, the exemption, η, and what the creditor believes the realization to be (recall that x is the entrepreneur's private information unless enforcement occurs). The lender's expected payoff is the left-hand side of (11). Because the interest rate, r, and the face value, $\bar{v}$, are related by $\bar{v} = \alpha(1+r)$, we can rewrite (11) as

$$r \leq \frac{(1-\eta)(\bar{x}-\alpha)-c}{(1-\eta)\alpha}. \tag{12}$$

[23]Let x^*_{csv} be the lowest non-bankruptcy state in the CSV model. For bankruptcy to occur if and only if $x < x^*$, the contract's face value must be $\bar{v} = x^*$. The interest rate is implicitly given by $\bar{v} = \alpha(1+r)$, where $\alpha = 0.49$. Let x^*_e be the lowest non-bankruptcy state in the enforcement model. Then the face value is $\bar{v} = x^*(1-\eta)$, resulting, ceteris paribus, in a lower interest rate. In addition $x^*_e < x^*_{csv}$. This follows because raising the face value by some amount ϵ raises the lowest non-bankruptcy state by ϵ in the CSV model and by only $\epsilon/(1-\eta)$ in our model.

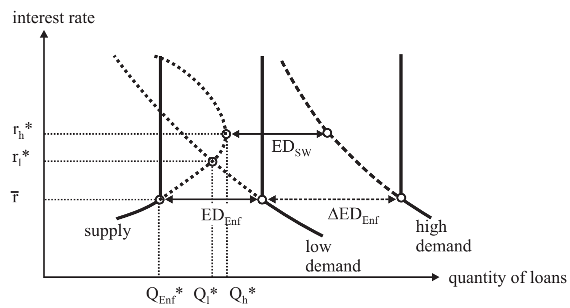

Figure 9: Loan Market Equilibria

Constraint (12) has interesting implications for credit rationing. In figure 9, the solid lines are the supply and demand curves when (12) binds. As in figure 8, the dotted continuations of these curves are the standard supply and demand curve absent (12).

- If demand is low and (12) does not bind, then demand equals supply, the equilibrium is r_l^*, Q_l^*, and there is no credit rationing.
- If demand is low and (12) binds, then the posted r_l^* is shaved down to $\bar{r}$, as is any other posted interest rate $r > \bar{r}$. As a consequence, both demand and supply curves are inelastic for $r \geq \bar{r}$, as the solid demand and supply curves in figure 9 show. Thus, supply and demand do not intersect, the effective interest rate is $\bar{r}$, and the equilibrium amount of credit rationing is $\mathrm{ED}_{\mathrm{Enf}}$. This shows that in the enforcement model, credit rationing can occur even when the supply curve does not bend back.
- If demand is high and (12) does not bind, the standard (dotted) demand and supply curves do not intersect. The resulting equilibrium interest rate is r_h^* and credit rationing is $\mathrm{ED}_{\mathrm{SW}}$.
- If demand is high and (12) binds, then credit rationing is $\mathrm{ED}_{\mathrm{Enf}} + \Delta\mathrm{ED}_{\mathrm{Enf}}$, which is strictly larger than the standard Stiglitz and Weiss or Williamson amount, $\mathrm{ED}_{\mathrm{SW}}$.

The credit rationing that we describe is not caused by a backward sloping supply schedule as in Stiglitz and Weiss [29] or Williamson [31]. Rather, credit rationing due to constraint (12) is caused by the legal system. Further, this friction affects both loan supply and demand because agents have rational expectations (i.e., they know how beliefs are formed both on and off the equilibrium path). Another difference with [29] and [31] is that in these models an optimal and uniquely determined interest rate arises. In contrast, in our model only the equilibrium quantity is determined but not the interest rate. This has two important implications. First, our framework provides an equilibrium explanation for “short-side” quantity rationing constraints. Figure 9 shows that lenders are not willing to supply loans in excess of Q_{Enf} when $c > 0$ and $\eta > 0$ because if (12) is violated the lender will not have an incentive to request enforcement. When this occurs, finance breaks down. Similarly, borrower demand is given by the bold-faced demand curve, even when the stated rate is very high. In this sense, the dynamic game with asymmetric information provides a theory of quantity constraints that is summarized by (12). Second, because stated and effective interest rates may differ substantially when enforcement is imperfect, this presents an important challenge for empirical analysis as only stated interest rates are directly observed.

5 Concluding Remarks

This paper proposes a model of judicial enforcement that considers both ability and willingness to pay. The model also distinguishes between and models the borrower’s default decision and the decision to pursue costly bankruptcy. The analysis helps explain empirical observations that are difficult to reconcile with existing models. It also sheds light on why firms, especially in emerging financial markets with poor institutions, experience difficulty raising finance. In future work, it will be interesting to more broadly explore the following quantitative implications of the model.

i. Different sectors of the economy (e.g., farming, manufacturing, retail) have different underlying return distributions. Which sectors are more sensitive to changes in the enforcement parameters?

ii. In many emerging markets, courts are “unreliable” in the sense that there is significant uncertainty about exemptions, inflation, court fees, and court behavior. Formally, this can be modeled by assuming that η is a random variable rather than a fixed percentage. How does uncertainty about this parameter affect entrepreneur finance?

iii. Most legal codes contain a normative assessment of social justice. Our model can be used to understand the tradeoffs between social justice and economic efficiency. For example, the recent revisions of the Mexican bankruptcy code explicitly treat low income debtors more favorably than high income debtors, which corresponds to a higher η. Our model can be used to show quantitatively how generous such a policy can be without depressing debt finance.

6 Appendix

6.1 Proofs

Proof of Lemma 1. Consider any solution to problem 1. Let $X_N = \{x|e(v(x)) = 0\}$ and $X_D = \{x|e(v(x)) = 1\}$. Then X_N and X_D partition the set of all possible realizations $[\underline{x}, \bar{x}]$ into a set of non-bankruptcy and a set of bankruptcy states. Note that $v(x)$ is constant on X_N. Assume by contradiction that there exist x, x' such that $v = v(x) < v(x') = v'$. Then the entrepreneur's payoff could be increased in state x' by switching from payment v' to payment v, a contradiction to (3). Let $\bar{v}$ be the entrepreneur's payment on X_N.

Consider the following alternative contract.

$$v_A(x) = \begin{cases} \bar{v} & \text{if } x \in X_N \\ 0 & \text{if } x \in X_D. \end{cases} \qquad e_A(v) = \begin{cases} 0 & \text{if } v \geq \bar{v} \\ 1 & \text{if } v < \bar{v}. \end{cases}$$

$$\ell_A(x, v) = \begin{cases} 0 & \text{if } v \geq \bar{v} \\ (1-\eta)(x - v) & \text{if } 0 < v < \bar{v} \\ v(x) + \ell(x, v(x)) & \text{if } v = 0. \end{cases}$$

It follows immediately that the payoffs to both parties under the alternative contract are the same as under the original contract. In particular, if $x \in X_N$ then payment $\bar{v}$ occurs under both contracts. If $x \in X_D$ then under the original contract payment $v(x)$ was made and the court enforced payment $\ell(x, v(x))$. The total payment was $v(x) + \ell(x, v(x))$, which is the same if the debtor were to pay 0 in all states in X_D under the alternative contract. Hence, it is optimal for the debtor to choose $v(x) = 0$ for all $x \in X_D$.

It remains to show that the constraints are all satisfied. (2) holds because the payments are the same under both contracts. (3) is automatically satisfied for $v = \{0, \bar{v}\}$. Next, it is not optimal for the entrepreneur to choose a payment $v > \bar{v}$. Now assume that the entrepreneur chooses $0 < v < \bar{v}$. Let $\beta(x|v)$ be the belief in the original equilibrium. Recall from the previous paragraph that under the proposed new solution $v(x) = 0$ or $\bar{v}$ for all $x \in X_D \cup X_N$. Because a v with $0 < v < \bar{v}$ is never paid in our new solution, the PBNE allows the creditor to have any belief if v were to be paid. Let such beliefs be the same as those under the original solution, i.e., $\beta(x|v)$. Then $\ell_A(x, v) \geq \ell(x, v)$ implies

$$\int \left[\ell_A(x, v) - c\right] d\beta(x|v) \geq \int \left[\ell(x, v) - c\right] d\beta(x|v) \geq 0,$$

The last inequality follows from (4). Therefore, it is optimal for the lender to enforce.

Finally, when the entrepreneur selects $v = 0$, then

$$\int \left[\ell_A(x, 0) - c\right] d\beta_A(x|0) \geq \int \left[\ell(x, v(x)) - c\right] d\beta_A(x|0)$$
$$= \int \int \left[\ell(x, v) - c\right] d\beta(x|v(y))\, d\beta(y|v(y) < \bar{v}) \geq 0.$$

The last inequality follows from (4) and the fact that $v(x) = 0$ for all $x \in X_D$ in our candidate solution. Hence (4) is satisfied for the alternative contract.

No enforcement occurs if the entrepreneur pays $\bar{v}$. Therefore payment $\ell_A(x, \bar{v})$ can be assumed to be 0. Finally, under the alternative contract payments $v \notin \{0, \bar{v}\}$ do not occur in equilibrium. Therefore, (4) is less tight if we assume the most optimistic beliefs, that $\beta(\bar{x}|v) = 1$ off the equilibrium path. ■

Proof of Theorem 1. Assume by way of contradiction that an arbitrary contract, which is not simple debt, $\{v(x), \ell(x, v), e\}$, solves problem 1. Because of lemma 1 we can assume that $v(x)$ is either 0 or $\bar{v}$. Choose x_D^* such that

$$\beta\big([\underline{x}, x_D^*]\big) = \beta\big(\{x|v(x) = 0\}\big). \tag{13}$$

Constraint (6) implies $\ell(x, 0) \leq (1 - \eta)x$. Constraint (3) implies that $\ell(x, 0) \leq \bar{v}$ for all x with $v(x) = 0$. Therefore, $\ell(x, 0) \leq \min\{(1 - \eta)x, \bar{v}\}$. Thus,

$$\int_{\{x|v(x)=0\}} \ell(x, 0)\, d\beta(x) + \int_{\{x|v(x)=\bar{v}\}} \bar{v}\, d\beta(x) \leq \int_{\underline{x}}^{\bar{x}} \min\{(1 - \eta)x, \bar{v}\}\, d\beta(x).$$

Therefore there exist $\bar{v}^* \leq \bar{v}$ such that

$$\int_{\{x|v(x)=0\}} \ell(x, 0)\, d\beta(x) + \int_{\{x|v(x)=\bar{v}\}} \bar{v}\, d\beta(x) = \int_{\underline{x}}^{\bar{x}} \min\{(1 - \eta)x, \bar{v}^*\}\, d\beta(x). \tag{14}$$

Let

$$\ell^*(x, 0) = \min\{(1 - \eta)x, \bar{v}^*\}, \text{ and } v'(x) = \begin{cases} 0 & \text{if } x < x_D^*; \\ \bar{v}^* & \text{if } x \geq x_D^*. \end{cases}$$

Let $e^*(v) = 1$ if and only if $v < \bar{v}^*$. By construction the lender's total payment and hence the entrepreneur's expected utility under $\{\ell^*, v', e^*\}$ is same as under $\{\ell, v, e\}$. Moreover, because the bankruptcy probability does not change, the lender's payoff is unchanged. We show that (4) is slack, if $\bar{v}^* < \bar{v}$.

Note that $\bar{v}^* < \bar{v}$ and (13) imply $\int_{x_D^*}^{\bar{x}} \bar{v}^* \, d\beta(x) < \int_{\{x|v(x)=\bar{v}\}} \bar{v} \, d\beta(x)$. Therefore, (14) implies $\int_{\underline{x}}^{x_D^*} \ell^*(x, 0) \, d\beta(x) > \int_{\{x|v(x)=0\}} \ell(x, 0) \, d\beta(x)$. (13) gives $\int_{\underline{x}}^{x_D^*} \big[\ell^*(x, 0) - c\big] \, d\beta(x) > \int_{\{x|v(x)=0\}} \big[\ell(x, 0) - c\big] \, d\beta(x)$. Thus,

$$
\begin{aligned}
\int \big[\ell^*(x,0) - c\big] \, d\beta(x|v'(x) = 0) &= \frac{1}{\beta([\underline{x}, x_D^*])} \int_{\underline{x}}^{x_D^*} \big[\ell^*(x,0) - c\big] \, d\beta(x) \\
&> \frac{1}{\beta([\underline{x}, x_D^*])} \int_{\{x|v(x)=0\}} \big[\ell(x,0) - c\big] \, d\beta(x) \\
&= \frac{1}{\beta(\{x|v(x) = 0\})} \int_{\{x|v(x)=0\}} \big[\ell(x,0) - c\big] \, d\beta(x) \\
&= \int \big[\ell(x,0) - c\big] \, d\beta(x|v(x) = 0) \geq 0,
\end{aligned}
$$

which implies that (4) is slack.

Note that $(1 - \eta)x_D^* > \bar{v}^*$. Otherwise, if $(1 - \eta)x_D^* \leq \bar{v}^*$ then $x_D^* < \bar{v}/(1 - \eta)$. This would imply $v(x) = 0$ for all x with $0 < x < \bar{v}/(1 - \eta)$, which contradicts (13). Now decrease x_D^* marginally to x^* and define

$$
v^*(x) = \begin{cases} \bar{v}^* & \text{if } x \geq x^*; \\ 0 & \text{if } x < x^*. \end{cases}
$$

The lender's expected payment is unchanged, therefore the entrepreneur's payoff is unaffected. Because $\beta([\underline{x}, x^*]) < \beta(\{x|v(x) = 0\})$, there are less bankruptcies under $\{\ell^*, v^*, e^*\}$, thereby strictly increasing the lender's payoff. Next, (3) holds by definition. (4) is satisfied because it was shown to be slack for contract $\{\ell^*, v', e^*(v)\}$ and because x^* is only marginally smaller than x_D^*. (5) holds because $\bar{v}^* < \bar{v}$. (6) holds by construction. As a consequence, $\{\ell^*, v^*, e^*\}$, fulfills all constraints of problem 1 and increases the investor's payoff. This contradicts the proposed optimality of $\{\ell, v, e\}$. ■

Proof of Theorem 2.

Statement 1. It follows immediately that the lender's payoff is non-increasing in η as the constraint set becomes smaller when η is increased. We now show that the decrease is strict if bankruptcy occurs with positive probability. First, assume that constraints (10) and (11) are slack. Then (9) binds. As a consequence, increasing η increases x^* and the expected bankruptcy costs, which strictly decreases the lender's expected payoff. Next, assume that (10) binds. Then increasing η again increases x^*, making the lender strictly worse off. Finally, if (11) binds, then the lender is worse off because the face value is lowered.

Statement 2, Region 1. Constraint (8) holds with equality, i.e.,

$$\int_{\underline{x}}^{\frac{\bar{v}(\eta)}{1-\eta}} \eta x \, d\beta(x) + \int_{\frac{\bar{v}(\eta)}{1-\eta}}^{\bar{x}} (x - \bar{v}(\eta)) \, d\beta(x) = u_E \tag{15}$$

Taking the derivative of (15) with respect to η and solving for $\frac{d\bar{v}(\eta)}{d\eta}$ yields

$$\frac{d\bar{v}(\eta)}{d\eta} = \frac{\int_{\underline{x}}^{\frac{\bar{v}}{1-\eta}} x \, d\beta(x)}{\beta\left(\left[\frac{\bar{v}}{1-\eta}, \bar{x}\right]\right)} > 0. \tag{16}$$

Recall that the interest rate is given implicitly by $\bar{v} = (1+r)\alpha$. Therefore, (16) implies that the face value and the interest rate are strictly increasing in η.

If (10) and (11) are slack then constraint (9) must bind. Taking the derivative of (9) with respect to η and solving for $\frac{d\bar{v}(\eta)}{d\eta}$ yields

$$\frac{d\bar{v}(\eta)}{d\eta} = \frac{dx^*(\eta)}{d\eta}(1-\eta) - x^*(\eta), \tag{17}$$

This and (16) imply that $\frac{dx^*(\eta)}{d\eta} > 0$, i.e., the lowest bankruptcy state and therefore the bankruptcy probability are increasing in η.

Statement 2, Region 2. If (8), (10) and (11) do not bind, then (9) binds. The first order condition is

$$(1-\eta) \int_{x^*}^{\bar{x}} h(x) \, dx - ch(x^*) = 0. \tag{18}$$

The second order condition is

$$-(1-\eta)h(x^*) - ch'(x^*) \leq 0. \tag{19}$$

Taking the derivative in (18) with respect to η, and solving for $\frac{dx^*(\eta)}{d\eta}$ yields

$$\frac{dx^*(\eta)}{d\eta} = -\frac{\beta(x \geq x^*(\eta))}{(1-\eta)h(x^*(\eta)) + ch'(x^*(\eta))}. \tag{20}$$

Therefore, (19) implies that the bankruptcy probability is decreasing. Finally, (17) implies that $\frac{d\bar{v}(\eta)}{d\eta} < 0$ if $\frac{dx^*(\eta)}{d\eta} \leq 0$, i.e., the implied interest rate is decreasing.

Statement 2, Region 3. Assume that (10) binds. First, assume that $\bar{v} < (1-\eta)x^*$, i.e., (9) is slack. It follows immediately that (8) binds. Assume by contradiction that (8) is slack. Now raise $\bar{v}$. We can lower x^* because the lender's expected payment in bankruptcy states is increased. Therefore, the bankruptcy probability is decreased. This and the increase in $\bar{v}$ makes the lender strictly better off, a contradiction.

Because (8) binds, we get (15). Therefore, (16) implies that the face value $\bar{v}(\eta)$ is increasing in η. Let η' be marginally larger than η. Then $\frac{\bar{v}(\eta')}{1-\eta'} < x^*(\eta)$. Let ℓ and ℓ' be the optimal contracts given η and η' respectively. Because (8) binds and $\bar{v}(\eta') < \bar{v}(\eta)$, we get $\int_{\underline{x}}^{x^*(\eta)} \ell(x,0)\, d\beta(x) > \int_{\underline{x}}^{x^*(\eta)} \ell'(x,0)\, d\beta(x)$. In order for (10) to be satisfied, $x^*(\eta') > x^*(\eta)$, i.e., the bankruptcy probability increases.

Finally, assume that $\bar{v}(\eta) = (1-\eta)x^*(\eta)$. Then (10) implies

$$\int_{\underline{x}}^{x^*} \big(1-\eta)x \, d\beta(x) = c\beta(x < x^*). \tag{21}$$

Taking the derivative with respect to x^* and solving for $\frac{dx^*(\eta)}{d\eta}$ yields

$$\frac{dx^*(\eta)}{d\eta} = \frac{1}{h(x^*)\big((1-\eta)x^* - c\big)} \int_{\underline{x}}^{x^*} x \, d\beta(x),$$

which is strictly positive, because (21) implies $c < (1-\eta)x^*$.

Statement 2, Region 4. If η is close to 1, (10) cannot hold if bankruptcy occurs with positive probability. The face value $\bar{v} = (1-\eta)\bar{x}$, i.e., it decreases with η. Finally, if (11) binds in this case, then $\bar{v} = (1-\eta)\bar{x} - c$, i.e., the face value and the interest decrease. ■

Proof of Theorem 3.

Statement 1. Assume by contradiction that the lender's payoff increases if costs are increased from c to c'. Let $\bar{v}'$, $x^{*\prime}$ be the solution to problem 2 when costs are c'. Then $\bar{v}'$, $x^{*\prime}$ fulfills all constraints of problem 2 when costs are $c < c'$. However, the expected bankruptcy costs decrease. Therefore, $\bar{v}'$, $x^{*\prime}$ dominates the contract that is optimal when costs are c, a contradiction.

Statement 2, Region 1. (8) determines face value $\bar{v}$. If (10) does not bind, then $x^*(1-\eta) = \bar{v}$. Therefore, the face value and bankruptcy probability do not change in this region.

Statement 2, Region 2. If (8), (10) and (11) are slack, the first and second order conditions are given again by (18) and (19). Taking the derivative with respect to c yields

$$\frac{dx^*(c)}{dc} = -\frac{h(x^*(c))}{(1-\eta)h(x^*(c)) + ch'(x^*(c))}. \tag{22}$$

Also note that $x^*(c)(1-\eta) = \bar{v}(c)$. Therefore (19) implies that the bankruptcy set and the face value decrease.

Statement 2, Region 3. If (8) binds, $\bar{v}$ is independent of c and the face value is constant. As c is increased, (10) becomes tighter. As a consequence, x^* must be increased, thereby increasing the bankruptcy probability. Now assume that (8) is slack. Then (9) must bind. Otherwise, we could increase the lender's payoff by increasing $\bar{v}$. Therefore, (10) implies $\frac{1}{\beta(x<x^*)}\int_{\underline{x}}^{x^*}(1-\eta)x\,d\beta(x) = c$. Thus if c is increased, x^* must be increased, thereby increasing the bankruptcy probability. Since $x^*(c)(1-\eta) = \bar{v}(c)$, the face value and interest rate increase as well.

Statement 2, Region 4. If c is sufficiently large, then (10) is only satisfied if bankruptcy never occurs. The face value remains constant as long as (11) does not bind. If (11) binds then $\bar{v} = (1-\eta)\bar{x} - c$ and increasing c decreases $\bar{v}$ and the interest rate. ■

6.2 Explanation of the Quantitative Analysis

Table 2 reports data that was constructed by Boyd and Smith [8] for the U.S. mining industry. They find the highest overall return realization in the data, $\bar{x}$, the lowest, $\underline{x}$, and then divide the distance between $\underline{x}$ and $\bar{x}$ into ten intervals of equal length. They assign a probability to each discrete state according to its empirical frequency. The data contain no observations for bankrupt entrepreneurs. Thus, they add two equi-probable bankruptcy states to the distribution, where the probability of the bankruptcy range is chosen to match the national average annual failure rate for nonfinancial entrepreneurs over the period 1972-91. The probabilities of the non-bankruptcy states are adjusted so that all sum to one. Boyd and Smith [8] estimate that this industry raises about half of total funds via equity.[24]

Table 2: Project Return Data for the U.S. Mining Industry (in real units)

state	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}	x_{11}	x_{12}
return	.056	.167	.295	.437	.580	.723	.865	1.008	1.151	1.294	1.436	1.579
prob.	.003	.003	.011	.007	.011	.011	.052	.399	0.380	.089	.026	.007

Consider a discrete distribution with realizations x_i, $i = 1, \dots, n$. First, assume the entrepreneur's reservation utility constraint binds and compute $\bar{v}$. Next, check whether the enforcement constraint (10) is satisfied. Then check whether (11) holds. If both constraints hold then we have a candidate optimum. If (11) is violated then replace the previous $\bar{v}$ by the value at which (11) binds. If (10) holds for this new value of $\bar{v}$ this is a candidate optimum. If (10) is violated, increase the number of bankruptcy states until (10) binds.[25]

Next, consider the case where the reservation utility constraint (8) is slack. In such a case it is optimal to choose $\bar{v} = (1-\eta)x_i$ for one of the realizations x_i. If (10) is violated, again increase the number of bankruptcy states. Each realization x_i, $i = 1, .., n$ gives a

[24] The precise computation for the industry is 58.3% of total funds are equity and $\alpha = 41.7\%$ are debt.

[25] We cannot simply raise $\bar{v}$ to get (10) to hold as this would violate either (8) or (11).

candidate optimum. The true optimum provides the highest payoff to the lender among all candidate optima.[26]

[26]The code is available upon request. We approximate the lognormal and normal distributions by discrete distributions with sufficiently many realizations.

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