Social ideology and taxes in a differentiated candidates framework

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Abstract

Many observers have diagnosed an increasing ideological polarization, particularly on non-economic matters, in the United States. How does this influence the political competition between candidates on economic issues, in particular, size of government? We analyze this question using a differentiated candidates framework in which two office-motivated candidates differ in their fixed ideological position and choose a level of government spending and implied taxes to maximize their vote share. We provide conditions under which a unique equilibrium exists. In equilibrium, candidates propose different tax rates, and the extent of economic differentiation is influenced by the distribution and intensity of non-economic preferences in the electorate. In turn, the extent of economic differentiation influences whether parties divide the electorate primarily along economic or social lines.

Keywords: Differentiated candidates, policy divergence, ideology.

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# Introduction

Many observers have diagnosed an increasing polarization between the two major parties in the United States. This is reflected both at the elite level, in particular in Congress (Poole and Rosenthal 1984, 1985, 2000) and among Democratic and Republican activists and voters (Abramowitz and Saunders 2008, Harbridge and Malhotra 2011). In the American National Election Survey, respondents report their own ideological position on a scale from 1 (very liberal) to 7 (very conservative). The number of respondents who report one of the extreme positions (1, 2, 6 or 7) has grown from 20 and 21 percent in 1972 and 1976 to 31 and 30 percent in the 2004 and 2008, respectively. Moreover, liberal and conservative voters have become considerably more reliable supporters of Democrats and Republicans, respectively, over the last generation.\(^1\)

If we accept the widespread view that Reagan’s “conservative revolution” has created a cultural wedge between the parties that has only widened in the 1990s and 2000s, what consequences for the parties’ economic policies would we expect? Our objective in this paper is to develop a theory of candidate competition and voting that accounts for a strong influence of both economic and non-economic issues on individual voting behavior, and helps us understand how ideological polarization (i.e., an intensification of the voters’ party preferences based on cultural or other non-economic issues) influences the positions that candidates take on economic issues.

The standard models in political economy are ill-equipped to analyze how economic and ideological factors interact in political competition. If the simple one-dimensional policy model is interpreted as one of economic policy, there is, by definition, no ideological dimension, and voters split according to their economic preferences even if there is only slight differentiation between the economic platforms proposed by the candidates. The probabilistic voting model accommodates both an “economic” dimension on which candidates choose a policy and an “ideological” dimension which is an additive shock to the utility of voters and can be thought of as arising from cultural issues on which the candidates’ positions differ. However, in the equilibrium of the standard probabilistic voting model, both candidates always propose the same economic policy, and thus the voters’ preference for one of the candidates is only determined by their ideological posi-

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\(^1\)For example, whether a voter regularly goes to church (a proxy for attitudes towards social issues) has become a strong predictor of voting intentions. According to the exit polls of the 2008 U.S. presidential election, voters who attended church weekly voted for McCain 55-43, while occasional church-goers voted for Obama 57-42, and those who never go to church voted for Obama 67-30. This indicates that non-economic issues have a strong effect on voting behavior.
tion and not by their economic preferences and characteristics. However, there is ample evidence that economic positions play a substantial role in determining voter behavior.

In our model of political competition, economic issues are the main flexible position for candidates, while their ideological positions are fixed. This is plausible: It may be very difficult for a candidate to credibly change a position on a position issue such as abortion, the death penalty or gun control, there are no comparable constraints that prevent a politician who favored a 5 percent sales tax in a previous campaign to credibly advocate a 6 percent or a 4 percent rate in the current campaign. Thus, we assume that candidates are exogenously committed to their ideological positions, and different voters prefer one or the other candidate because of these fixed positions. However, voters also care about the candidates’ economic positions, and the candidates can choose these economic positions freely to maximize their electoral success.

Our framework has two advantages. First, it delivers an equilibrium in which voter behavior is determined by both economic and ideological preferences, because both the candidates’ immutable positions on social issues and their equilibrium platforms on economic issues differ: Social conservatives who happen to be sufficiently keen on government spending may vote for the Democrat, and social liberals who are sufficiently opposed to high taxation may vote for the Republican. Second, we can use this framework to analyze the influence of ideological polarization on the candidates’ equilibrium choice of an economic platform. Within our framework we can think of ideological polarization as a measure of preference intensity on the ideological component of utility. In addition, we can consider the effects of shifts in the ideological composition of the electorate (say, an increase in the number of social conservatives).

Our main results are as follows. We first show that an equilibrium is characterized by a cutoff for each ideological type. Cutoff voters are indifferent between candidates and therefore must strictly prefer the economic platform of the candidate whose ideological position they dislike. Thus, a socially liberal cutoff voter is in favor of less government spending than a socially conservative cutoff voter. In equilibrium, candidates propose tax rates that are higher than the rates preferred by the most socially liberal cutoff voter and smaller than the one preferred by the most socially conservative cutoff voter. A candidate who marginally increases his proposed tax rate gains votes among social conservatives, but loses some liberals, and those gains and losses exactly

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2 A reason for this difference is also that the optimal economic policy (for any preference type) depends on the state of the economy and thus naturally changes over time, while one’s view of the desirability of gay marriage or abortion restrictions is more likely to be fairly constant over time.
balance in equilibrium for each candidate. Note that the statement that more government spending increases the set of conservatives who vote for the candidate does not imply that higher tax rates are on average popular with social conservatives as a group. Clearly, at least some social conservatives (and quite possibly a majority of them) dislike higher taxes, but those are not the swing voters that the candidates focus on.

Initially, in Section 4 and 5, we focus on the case of two groups of liberals and conservatives (we show in Section 6 that our model can accommodate an arbitrary number of ideological groups). In this case, taking the opponent’s tax rate as given, varying a candidate’s tax rate generates a curve of cutoff voter pairs, and a candidate chooses the best cutoff voter pair from this curve. We show that, in equilibrium, the two candidates’ curves are tangent to each other at the equilibrium-induced cutoff voter pair. They are also tangent to a curve that connects all cutoff voter combinations with the same vote share for the Democrat.

We provide sufficient conditions for an equilibrium to exist and to be unique. The graphical characterization of the equilibrium described above can be used to study the comparative statics properties of the equilibrium, because it is relatively easy to characterize how parameter changes affect iso-voteshare curves.

We show that any parameter change induces the candidates to change their respective platforms in the same direction. That is, changes in cultural polarization (either in the number of liberals or conservatives, or in the intensity with which they care about non-economic issues) either lead to an increase of both the Democratic and the Republican tax rate, or to a decrease of both of them. This appears consistent with the observed recent movement of both parties’ economic platforms to the right.\footnote{For example, the health care plan of the Republican party in the 1990s involved subsidies for low income households and an individual mandate to buy health insurance. A similar plan was eventually passed in 2010 by a Democratic Congress, and against strong Republican opposition.}

If there are more socially conservative voters, or if socially conservative voters’ emphasis on cultural issues increases, then both candidates propose more government spending, but the small-government candidate’s vote share increases. Intuitively, a larger share of social conservatives in the electorate has the effect that candidates care more about socially-conservative swing voters, and since these voters are economically quite liberal (otherwise, they would be hardcore Republican supporters, not swing voters), candidates cater to these voters by proposing higher government spending. A similar effect arises when social conservatives care more intensely about non-
economic issues, because this implies that the socially-conservative cutoff voter shifts to a more economically-liberal type (who likes higher government spending). The opposite conclusions hold when there are more socially-liberal voters, or if they care more about cultural issues.

We also analyze under which conditions increased ideological polarization leads to increased or decreased economic political polarization, in the sense that the two party platforms move farther apart from each other, or closer together. We show that this depends on whether liberals and/or conservatives become more partisan, as well as on the shape of voter utility functions. Because of its effect on equilibrium economic policies, increasing ideological polarization does not necessarily lead to the electorate’s voting behavior being more divided along ideological lines – what matters for whether this is the case is whether economic policies relatively diverge by less than ideological platforms.

In Section 2, we discuss the related literature. Section 3 sets up the model. Section 4 derives conditions for equilibrium existence and uniqueness, and Section 5 contains the main comparative static analysis for the case of two ideological groups (“liberals” and “conservatives”). Section 6 shows that our analysis can be generalized to an arbitrary number of ideological groups (i.e., voter groups who differ in how intensely they feel about non-economic issues). Section 7 concludes.

2 Related literature

Our model is based on the general differentiated candidates framework developed in Krasa and Polborn (2009a, 2010a, 2010b), in which the two competing candidates have some characteristics that cannot be changed, but choose a position (or “policy”) in order to maximize their respective probability of winning. Voters’ utility depends on both fixed characteristics and flexible policies in a general, not necessarily separable way. This is the main difference to classical valence or probabilistic voting models in which the fixed characteristic and the utility from policy are additively separable. Krasa and Polborn (2009a) derive general necessary and sufficient conditions on voter preferences under which political competition by two office-motivated candidates will lead to policy convergence. In particular, they show that voter preferences that reflect some complementarity between a candidate’s competence in a particular policy area and the policy that he proposes to implement will generically induce policy divergence, provided that candidates differ in competence.

\footnote{One implication of our results is that observation of ideological voter intensity and candidate behavior can provide insights about the shape of voter utility functions.}
Krasa and Polborn (2010b) is a particular application of this effect, in which voters care about two public goods provided by the office holder, and each candidate has an advantage in providing one of these goods.

Our model focuses on the interaction between ideological and economic voter preferences in determining candidate platforms, and is thus related to the probabilistic voting model (PVM), in which voters also have ideological and economic preferences. However, because candidates are assumed in the standard PVM to have exactly the same ability to implement any economic policy, both candidates choose the same economic policy in equilibrium, and thus, voting behavior is determined only by the voters’ position on the “ideological” dimension in which candidates are exogenously fixed. In contrast, there are economic differences between candidates’ equilibrium positions in our model, and thus voting behavior depends on both economic and ideological preferences, and this dual dependence generates the most interesting effects in our model.

Specifically, in the PVM, equilibrium platforms are a weighted maximum of the economic utility of voter groups, where the weight of each group is proportional to the value of its ideological density at zero (i.e., to the number of “swing voters” in the respective group who are ideologically indifferent between the candidates. Thus, a proportional increase of the importance of ideology for all voters, or even of just liberals or conservatives, has no effect whatsoever on equilibrium platforms in the PVM because it neither changes the preferences of swing voters nor their numbers. In contrast, intensification of ideological preferences does affect the identity of swing voters in our model, and this channel is what drives changes in equilibrium economic platforms.

The advantage of our model relative to a citizen-candidate model in which candidates are fixed to their “ideal position” in every policy area is that there is a unique equilibrium in our model, and that we can relate changes in ideological polarization of the electorate to changes in the economic policies proposed by the candidates.5

In a standard one-dimensional spatial model, equilibrium policy depends only on the ideal policy position of the median voter, but is independent of the higher-order moments of the distribution of voter preferences.6 Lindqvist and Östling (2010) find empirical evidence that a larger degree of

5The citizen-candidate framework (Besley and Coate 1997) can handle multidimensional policy spaces without fundamental difficulties. However, there are generally very many equilibria that only share the property that both candidates always receive the same number of votes. Just like in the one-dimensional setup, the citizen candidate model imposes few restrictions on which policies can arise in equilibrium. Thus, no useful comparative static analysis with respect to social polarization is possible in that framework.

6In Meltzer and Richard (1981), a classical political-economy model of redistribution, the income distribution in
preference polarization is associated with a smaller size of government, but the theoretical basis
for this effect remains a bit unclear.

There are a number of papers that use different variations on the spatial model to analyze how
increasing diversity of voter preferences affects the size of government. Austen-Smith and Waller-
stein (2006) analyze how, in a legislative bargaining model, general redistribution is affected by the
existence of racial preferences. Lizzeri and Persico (2001, 2004) analyze the incentives of politi-
cians for redistribution under different electoral systems and show that expanding the set of citizens
who are eligible to vote may induce candidates to change their equilibrium platforms from patron-
age policies towards policies that have more general benefits. Somewhat relatedly, Fernández and
Levy (2008) develop a model in which all poor voters prefer general redistributive taxation, but
have conflicting interests regarding a number of local public goods that are beneficial only for a
subset of them. They show that this setup leads to a non-monotonic relationship between prefer-
ence fragmentation and redistribution. Preference diversity in all of these models is “economic”,
i.e., politicians have different types of economic policies (such as general and targeted redistri-
bution) at their disposal, voters are interested in both general interest and (some) special interest
policies, and they only care about their total economic benefit from the bundle of policies that are
enacted by the election winner. In contrast, our model has a simpler economic policy (as it con-
tains only the choice of one parameter, the tax rate), but it analyzes how this choice is affected by
preference diversity in non-economic dimensions, which are non-existent in these models.

Roemer (1998) analyzes a model in which, like in our model, voters care about economic
policy and about government policy along a non-economic dimension. Parties are considerably
more complex in Roemer’s model: They consist of some members who want to maximize the
probability of winning the election and some who want to maximize the expected (policy) utility
of particular party members. Since party positions on both dimensions are flexible, it is necessary
to introduce this modeling of parties in order to ensure the existence of an equilibrium. Under
certain conditions, Roemer finds that a higher weight on non-economic policy in the voters’ utility
function decreases the optimal tax rate for the party that prefers more redistribution.
society matters, but only to the extent that it influences the median voter’s preferences for redistribution.
3 Model

3.1 Description of the model

Two candidates, \( j = D, R \), compete in an election. There are two major components of policy, which Stokes (1963) calls “position issues” and “valence issues”. Position issues are ideological issues such as abortion or gun control, and candidates are exogenously committed to differentiated positions; due to their own history or their party label, they cannot credibly change this position. Let \( \delta_D, \delta_R \in \Delta \subset \mathbb{R} \) be the candidate’s ideological position (i.e., on position issues). Without loss of generality we can assume that \( \delta_D < \delta_R \) and \( \delta_D + \delta_R = 0 \).

In contrast to position issues, valence issues are related to the management of public good provision by the office holder. Each candidate proposes a level \( g \) of public goods that is supported by a tax rate \( t \). All voters prefer higher \( g \) and lower \( t \), but the rate at which they trade these off differs between individual voters.

A voter’s type is described by two parameters, \( \theta \) and \( \delta \), where \( \theta \in [0, \bar{\theta}] \) is an economic preference parameter that measures the preference for public good provision, and \( \delta \in \Delta \) is the voter’s ideological position. Ideological utility depends on the voter’s position \( p \) and the candidate’s position \( q \in \Delta \). A voter’s total utility is given by

\[
 u_{\theta,\delta}(x, g, q) = x + \theta w(g) - (\delta - q)^2, \tag{1}
\]

where \( x \) is the voter’s (private) consumption; \( g \) is the amount of public good provided, and \( q \) is the candidate’s ideological position.

Candidates differ in their ability to provide public goods. Each candidate \( j \) proposes a tax rate \( t_j \), which is applies to each agent. We assume that all agents have the same income, which is normalized to 1.\footnote{Since the focus of this paper is to understand the impact of social ideology on economic policy, we simplify the income distribution by assuming that there is no heterogeneity.} Thus, tax revenue if candidate \( j \) is elected is \( t_j \) and is used to pay for fixed costs of running the government and for the provision of a public good \( g \). The ability to provide the public good differs among candidates, and is given by an affine linear production function, \( g_j = f_j(t_j) = a_j t_j - b_j \). We analyze situations in which candidate \( R \) has an advantage with respect to fixed cost \( b \), while his opponent \( D \) has a higher marginal product in public good provision. In addition, fixed costs \( a_D \) should not be so large that candidate \( D \)’s provision of public goods is always less than that of candidate \( R \) for any tax level. Formally,
**Assumption 1.** There exists $0 < t_1 < t_2 < 1$ such that $0 < f_D(t_1) < f_R(t_1)$ and $0 < f_R(t_2) < f_D(t_2)$.

Note that this assumption immediately implies that the Republican candidates has lower fixed costs but higher marginal costs of providing public services, i.e., $a_R < a_D, b_R < b_D$.

**Assumption 2.** The voter’s utility function for public goods $w$ satisfies the following conditions:

1. $w' > 0, w'' < 0$.
2. $w'(0) = \infty$, and $\lim_{x \to 0} w(x)/w'(x) = 0$ if $w(0) = -\infty$.  

The main focus of the paper is to understand the impact of changes in voter ideology on economic policy. Therefore, we want to allow for arbitrary distribution of ideology types $\delta \in \Delta$. To keep the model tractable, we assume that $\theta$ is uniformly distributed on $[0, 1]$.

The timing of events is as follows:

**Stage 1** Candidates $j = D, R$ simultaneously announce tax rates $t_j \in [0, 1]$.

**Stage 2** Each citizen votes for his preferred candidate, taking into account the proposed policy platform. Each candidate’s payoff is given by his vote share.

### 3.2 Discussion of modeling choices

**Differential candidate capabilities.** A key assumption of the differentiated candidates model is that candidates have differential abilities, with one candidate better at providing limited government, while the other candidate is better than his competitor for large expenditures. As argued in Krasa and Polborn (2009a, 2010b, 2010a), this assumption appears eminently reasonable. Economists agree that workers or firms differ in their productivities, and this fact is evident as output can easily be measured in many private sector occupations. In contrast, the “output” of politicians in terms of public good production is significantly more difficult to measure, and thus it is tempting to use expenditures on inputs as a proxy measure for the quantity of the public good supplied. However, in reality, citizens derive utility, for example, from the quality of education in state schools and not *per se* from the money spent on education. Thus, when two competing candidates announce tax rates...

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8If $w(0) > -\infty$ then the last condition automatically follows from $w'(0) = \infty$. For standard utility functions with $w(0) = -\infty$, such as the logarithmic utility function, the condition is also satisfied.
candidates propose to spend the same amount of money on schools, this does not mean that both of them would produce the same quality of service for citizens if elected. Our model formalizes this notion.

Alternatively, the advantage may also be based on interactions with other political institutions. For example, a Democratic president may have a comparative advantage to convince a majority of legislators in Congress to pass a “big government” bill because he has more leverage over the future career of the potential supporters of such a bill (predominantly Democratic legislators). Relative to a Republican president, a Democratic president can use this leverage to convince these legislators to vote for the bill without delay and without having to give them too much in terms of pork projects. Hence, a Democratic president may be the more efficient provider of big government bills, and vice versa for a Republican.

**Ricardian equivalence.** In our model, all government expenditures have to be financed by contemporaneously raised taxes, and we therefore use “higher taxes” and “more government spending” as synonymous. When the government can run a deficit, taxes and spending need not be the same in any given year, but Ricardian equivalence suggests that current government spending is the appropriate measure for the taxes that have to be raised either today or in the future to finance today’s government spending. We would therefore interpret periods in which government spending increased as a percentage of GDP (such as Reagan’s or George W. Bush’s presidency) as periods of “higher taxes”, even if nominal tax rates remained constant or even declined, while the shortfall was made up by a deficit.

**Exogenous participation.** We assume that all citizens vote for their preferred candidate, independent of the strength of their preference. This implies that candidates will focus exclusively on “swing voters” who are (almost) indifferent between them, while taking the votes of their core supporters for granted. This is a standard assumption in most candidate competition models in particular in the standard Downsian model and in the PVM. If, instead, each voter type’s participation probability is an increasing function of the extent of its preference, then candidates have a considerably more complicated problem to solve, because they have to consider how policy changes would

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Note that, in the Downsian model, competition for moderate swing voters has the effect that all voters are indifferent between the two candidates in equilibrium. In contrast, in the equilibrium of our model, almost all voters have a strict preferences for their candidate, so the assumption that participation is exogenous is considerably milder in our model.
affect the participation rates of all voter types.

**Uniform distribution of \( \theta \).** We assume that the economic preference parameter \( \theta \) is uniformly distributed. This increases tractability considerably because it implies that iso-voteshare curves are linear, which provides for simplified equilibrium existence conditions and facilitates a graphical comparative statics analysis. It also enables us to allow for general distributions of ideological preferences.\(^{10}\) We have analyzed the model for non-uniform distributions of \( \theta \) in a previous version of this paper that is available from the authors upon request, and it should be clear that all results remain qualitatively robust as long as the distribution of \( \theta \) does not deviate too much from a uniform distribution.

4  Equilibrium

4.1 Determining the Cutoff Voters

We first show that for fixed ideology \( \delta \) a candidate’s supporters consist of \( \theta \)-types below or above a cutoff, where the location of the cutoff depends on the ideological type. Formally, the sets of a candidate’s supporters are of the form \( \{L\} \times (0, \theta_L] \cup \{C\} \times (0, \theta_C] \), or \( \{L\} \times [\theta_L, 1] \cup \{C\} \times [\theta_C, 1) \). In the following, denote the amount of public goods provided by the two candidates by \( g_D = f_D(t_D) \) and \( g_R = f_R(t_R) \), respectively.

A voter of type \((\delta, \theta)\) prefers candidate \(D\) over candidate \(R\) if and only if

\[
(1 - t_D) + \theta w(g_D) - (\delta - \delta_D)^2 \geq (1 - t_R) + \theta w(g_R) - (\delta - \delta_R)^2.
\]

Solving (2) for the voter of ideology type \( \delta \) who is indifferent between the candidates and using the fact that \( \delta_D = -\delta_R \) yields

\[
\theta^*_\delta = \frac{(t_D - t_R) + 2\delta(\delta_R - \delta_D)}{w(g_D) - w(g_R)}.
\]

Voter types with a higher \( \theta \) than this vote for the candidate who offers more public goods, while lower types support the other candidate. If \( g_D = g_R \), then (2) simplifies to \((1 - t_D) - (\delta - \delta_D)^2 \geq (1 - t_R) - (\delta - \delta_R)^2\), i.e., the equation is independent of \( \theta \). As a consequence, all voters with ideology \( \delta \) either vote for the same candidate, or are indifferent between candidates.

\(^{10}\)We start in Section 4 and 5 by analyzing the case of two groups of liberals and conservatives, respectively, but we show in Section 6 that our model can accommodate an arbitrary number of ideological groups.
We will show that, in equilibrium, \( g_D > g_R \), so that \( w(g_D) > w(g_R) \) and candidate \( R \) receives the votes of all voters with \( \theta \leq \theta^*_R \) and candidate \( D \) receives the votes of all voters with \( \theta \geq \theta^*_D \). This is the case that the left panel of Figure 1 illustrates. Note that (3) implies that the cutoff line has a positive slope. Intuitively, for more socially conservative voters on the cutoff line their preference for the Democrat’s economic platform must just counterbalance their cultural preferences for the Republican. As a consequence, a socially conservative cutoff voters must be more fiscally liberal, while a culturally liberal cutoff voter must be more economically conservative.

![Figure 1: The effect of tax changes on cutoff voters](image)

For our analysis, we need the derivatives of \( \theta^*_\delta \) and \( \theta^*_D \) with respect to \( t_D \) and \( t_R \), taking into account that \( g_D \) and \( g_R \) are functions of \( t_D \) and \( t_R \), respectively.

\[
\frac{\partial \theta^*_\delta}{\partial t_D} = \frac{w(g_D) - w(g_R) - [(t_D - t_R) + 2\delta(\delta_R - \delta_D)]a_Dw'(g_D)}{(w(g_D) - w(g_R))^2}, \quad (4)
\]

\[
\frac{\partial \theta^*_\delta}{\partial t_R} = \frac{w(g_D) - w(g_R) - [(t_D - t_R) + 2\delta(\delta_R - \delta_D)]a_Rw'(g_R)}{(w(g_D) - w(g_R))^2}, \quad (5)
\]

Clearly, it cannot be the case in equilibrium that \( \frac{\partial \theta^*_\delta}{\partial t_D} > 0 \) for all types, because in this case, candidate \( D \) could lower taxes and move all cutoffs to the left and thus increase his vote share. Similarly, it cannot be true that \( \frac{\partial \theta^*_\delta}{\partial t_R} < 0 \) for all \( \delta \).

Equation (4) implies that \( \frac{\partial \theta^*_\delta}{\partial t_D} \) is decreasing in \( \delta \). Thus, in an equilibrium \( \frac{\partial \theta^*_\delta}{\partial t_D} > 0 \) for social liberals (low \( \delta \)) while \( \frac{\partial \theta^*_\delta}{\partial t_D} < 0 \) for social conservatives (high \( \delta \)). Intuitively, raising taxes creates a trade-off between losing some socially liberal but fiscally conservative voters and gaining some social conservatives who prefer a higher consumption of public goods. Similarly, we must have \( \frac{\partial \theta^*_\delta}{\partial t_R} < 0 \) for social liberals and \( \frac{\partial \theta^*_\delta}{\partial t_R} > 0 \) for social conservatives. Thus, if candidate \( R \) raises taxes, he loses social conservatives but gains some social liberals.
The right panel of Figure 1 illustrates the effect of an increase in spending (taxes) by the Democrat, or equivalently a decrease in spending by the Republican, keeping spending by the other party fixed. Both of these changes imply larger economic differences between the two candidates, starting for a situation where \( t_D - t_R > 0 \) (which will be the case in equilibrium). Consequently, voters now split more along economic lines and less along ideological preferences. In other words, the separation lines becomes flatter. For example, voter A, a social conservative but economic liberal now switches his vote from the Republican to the Democrat. In contrast, voter B, a social liberal now votes for the Republican. More generally, the socially liberal cutoff voter moves up, while the social conservative cutoff voters moves down.

4.2 Necessary and Sufficient Conditions for the Case of Two Ideology types

We start by analyzing our model for two ideological types \( \delta_1 < 0 < \delta_2 \) which we can interpret as “liberals” (i.e., voters who get an ideological benefit if the Democratic candidate wins) and “conservatives,” respectively. In Section 6 we will show that the general case can be mapped into this special one, by redefining \( \delta_1 \) and \( \delta_2 \) as the “average” liberal and conservative type, respectively.

Define
\[
d = -2\delta_1(\delta_R - \delta_D), \quad r = 2\delta_2(\delta_R - \delta_D), \quad \theta_L^* = \theta_{\delta_1}^\ast, \quad \text{and} \quad \theta_C^* = \theta_{\delta_2}^\ast. \tag{6}
\]

Thus, \( \theta_L^* \) and \( \theta_C^* \) are the cutoffs for the liberal and conservative type, respectively, while \( d \) and \( r \) is their net-benefit from their ideologically preferred candidate.

In order to determine the equilibrium cutoffs, \( \theta_L^* \) and \( \theta_C^* \), it is useful to investigate how the candidates’ tax rates affect the cutoff types in a \( \theta_L - \theta_C \) diagram. We first define functions \( k_D \) and \( k_R \) that map the respective candidate’s tax rate into a curve of the cutoff points \( \theta_L^* \) and \( \theta_C^* \), taking as given the tax rate of the opponent (which we suppress in the notation). Thus, \( k_D \) describes the feasible set of cutoff voter combinations that the Democratic candidate can implement for any tax rate between 0 and 1, and \( k_R \) is the same curve for the Republican.

An important characteristic of these curves is their signed curvature. In general, the signed curvature of a two-dimensional curve \((x_1(t), x_2(t))\) is defined as
\[
\kappa = \frac{x_1 x_2'' - x_1' x_2'}{(x_1'^2 + x_2'^2)^{3/2}}. \tag{7}
\]

The absolute value of \( \kappa \) at a particular point is the inverse of the radius of the circle that approximates the curve in this point; thus, a small value of \( \kappa \) corresponds to an almost linear curve, while
a large value of $\kappa$ is a strongly bent curve. A positive value of $\kappa$ indicates that, as $t$ increases, the
cutoff point moves through the curve in a clockwise direction (and vice versa).\footnote{For example, consider $t \mapsto (r \sin(t), r \cos(t))$. This is a circle with radius $r$, and has curvature $\kappa = -1/r$. The negative sign indicates that as we raise $t$, the curve is drawn clockwise.}

The following Lemma 1 characterizes the curves $k_D$ and $k_R$, drawn in Figure 2.

**Lemma 1.**

1. The function $k_R : [0, 1] \to \mathbb{R}^2$ defined by $t_R \mapsto (\theta_L^*(t_R), \theta_C^*(t_R))$ has signed curvature of

$$k_R = -\frac{(r + d) a^2 R w''(g_R)}{w(g_D) - w(g_R)} \left( \left( \frac{\partial \theta_L^*}{\partial t_R} \right)^2 + \left( \frac{\partial \theta_C^*}{\partial t_R} \right)^2 \right)^{-3/2}.$$  \hspace{1cm} (8)

2. The function $k_D : [0, 1] \to \mathbb{R}^2$ defined by $t_D \mapsto (\theta_L^*(t_D), \theta_C^*(t_D))$ has signed curvature of

$$k_D = -\frac{(r + d) a^2 D w''(g_D)}{w(g_D) - w(g_R)} \left( \left( \frac{\partial \theta_L^*}{\partial t_D} \right)^2 + \left( \frac{\partial \theta_C^*}{\partial t_D} \right)^2 \right)^{-3/2}.$$  \hspace{1cm} (9)

Lemma 1 implies that the signs of $k_D$ and $k_R$ equal the sign of the term in the denominator
(because $w'' < 0$, and all the other terms are positive). Thus, if $w(g_D) > w(g_R)$, both curves rotate in
a counterclockwise direction, and vice versa if instead $w(g_D) < w(g_R)$. Lemma A.1 in the Appendix
shows that the shapes of $k_R$ and $k_D$ are those drawn in Figure 2.

In equilibrium each candidate chooses a tax rate that maximizes his vote share, taking the
opponent’s tax rate as given. Note that vote shares are constant for all $(\theta_L, \theta_C)$ that satisfy an
equation of the form $G(\theta_L, \theta_C) = \bar{k}$, where $\bar{k}$ is a constant. These are straight lines with slope $-\pi_L/\pi_C$.

In the left panel of Figure 3, consider point $(\theta_L, \theta_C)$, the cutoffs implied by some tax rates $(t_D, t_R)$. Is $(t_D, t_R)$ an equilibrium? Note that candidate $D$, who can move along the convex curve $k_D$ by changing $t_D$, could increase his vote share only if he gets to a point below the constant vote share line, which is not possible. In contrast, candidate $R$ can increase his vote share by moving to any point above the constant vote share line, for example to $(\tilde{\theta}_L, \tilde{\theta}_C)$, which is, in fact, his optimal deviation. As indicated in Figure 2, curve $k_R$ rotates in a counter-clockwise direction as $t_R$ increases, so to reach $(\tilde{\theta}_L, \tilde{\theta}_C)$ requires a decrease in $t_R$. Since candidate $R$ can improve by deviating, $(\theta_L, \theta_C)$ is not an equilibrium.

For $(\theta'_L, \theta'_C)$ to be an equilibrium, we must have a situation as drawn in the right panel where both $k_D$ and $k_R$ are tangent to the constant vote share line at some point $(\theta'_L, \theta'_C)$. The slope at $(\theta'_L, \theta'_C)$ must therefore be $-\pi_L/\pi_C$, and any “small” deviation makes the deviating candidate worse off. By a small deviation, we mean one that does not change the structure of voter support in the sense that, for both ideological groups, it is still the case that low $\theta$-types vote Republican and high $\theta$-types vote Democrat (i.e., cutoff types $(\tilde{\theta}_L, \tilde{\theta}_C)$ remain above the 45-degree line).

Now suppose that some equilibrium cutoff type is on the boundary, i.e., either $\theta'_L \in \{0, \bar{\theta}\}$ or $\theta'_C \in \{0, \bar{\theta}\}$. Consider for example the case where $\theta'_L = 0$ so that the Democrat receives the support of all liberal voters. Since small changes of $t_D$ and $t_R$ do not affect liberals, the two candidates

\[\text{Figure 3: Necessary conditions for an equilibrium.}\]
will focus exclusively on the conservative cutoff voter $\theta_C^*$. The Democrat will then try to choose $t_D$ to minimize and the Republican choose $t_R$ to maximize $\theta_C^*$. Lemma 2 formally states the necessary equilibrium conditions both for the interior and the boundary case.

**Lemma 2.** Let $(t_D^*, t_R^*)$ be an equilibrium with $0 < t_D^*, t_R^* < 1$. Then either all liberals vote for candidate D and all conservatives for candidate R, or the following holds:

$$a_Dw'(g_D^*) = a_Rw'(g_R^*). \quad (10)$$

If the equilibrium is interior, i.e., if $0 < \theta_L^*$ and $\theta_C^* < \bar{\theta}$ then curves $k_R$ and $k_D$ have slope $-\pi_L/\pi_C$ at the equilibrium cutoffs $(\theta_L^*, \theta_C^*)$, i.e.,

$$\frac{(w(g_D^*) - w(g_R^*)) - [(t_D^* - t_R^*) + v]a_Dw'(g_D)}{w(g_D^*) - w(g_R^*) - [(t_D^* - t_R^*) - d]a_Dw'(g_D)} = \frac{\pi_L}{\pi_C}. \quad (11)$$

If $\theta_L^* = 0$, then

$$w(g_D^*) - w(g_R^*) = ((t_D^* - t_R^*) + v)a_Dw'(g_D^*); \quad (12)$$

If $\theta_C^* = 1$, then

$$w(g_D^*) - w(g_R^*) = ((t_D^* - t_R^*) - d)a_Dw'(g_D^*). \quad (13)$$

Lemma 2 distinguishes three possible equilibrium configurations. First, it may be that ideological payoffs are so large that they dominate economic considerations for all voters, so that all liberals vote for the Democrat and all conservatives vote for the Republican. In this case, government spending and taxes are (generically) not uniquely pinned down because they do not have a marginal effect on the candidates’ set of supporters. In the following, we disregard this rather uninteresting case.

In case that at least one ideological group is competitive (in the sense that some of its members vote for the Democrat and some vote for the Republican), (10) must hold in equilibrium. Note that, because $a_D > a_R$ and $w'' < 0$, (10) implies that $g_D^* > g_R^*$. As a consequence, voter types below the cutoff $\theta_p^*$ of their respective ideology type $P$ vote for the Republican, and those above the cutoff vote for the Democrat. Graphically, all equilibria must be above the 45 degree line in Figure 3.

The equilibrium in a competitive case falls into two subcases. If both ideological groups are competitive for the candidates (i.e., both an interior liberal cutoff voter and an interior conservative

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**Footnote:** We will prove must be the case in equilibrium.
cutoff voter exists), then (11) must hold, which basically says that a marginal increase in a candidate’s tax rate wins the same number of ideologically-conservative, economically-liberal voters as it loses ideologically-liberal, economically-conservat ive voters.

In contrast, (12) and (13) deal with the cases that only one group is competitive. For example, if only social conservatives are competitive (while all liberals vote for the Democrat), then (12) holds. To understand this equation, consider the conservative cutoff voter who is indifferent between the Democratic and Republican candidates’ equilibrium platforms,

$$\theta^*_C w(g^*_D) - t^*_D = \theta^*_C w(g^*_R) - t^*_R + r. \tag{14}$$

For $$(t^*_D, t^*_R)$$ to be an equilibrium, it is necessary that $t^*_D$ maximizes the utility of the cutoff voter $\theta^*_C$ — if $t^*_D$ did not maximize the utility of the cutoff voter $\theta^*_C$, the Democrat could choose a policy that the cutoff voter strictly prefers and that therefore increases the set of voters who vote for him; but this cannot be true in an equilibrium. Differentiating the left hand side of (14) with respect to $t_D$ yields $\theta^*_C w'(g^*_D)a_D - 1 = 0$, which we can solve for $1/\theta^*_C = a_D w'(g^*_D)$. Rearranging (14) gives

$$w(g^*_D) - w(g^*_R) = [(t^*_D - t^*_R) + r]/\theta^*_C = [(t^*_D - t^*_R) + r]a_D w'(g^*_D),$$

as in (12). Analogously, (13) follows from equilibrium conditions for the liberal cutoff voter in the case that only liberals are competitive between the candidates while all conservatives vote Republican.

The following Corollary 1 summarizes some immediate implications of Lemma 2 for equilibrium behavior of candidates in an interior equilibrium. It shows that the Democrat proposes higher taxes than the Republican, and that this yields a higher public good provision under a Democrat (the latter point is not immediately obvious because the two candidates’ production functions differ, and is proved in the Appendix). As a consequence, all voters above an (ideology-specific) cutoff point vote for the Democrat, and all below vote for the Republican, as we have already claimed.

**Corollary 1.** In any interior equilibrium (i.e., $0 < t^*_D, t^*_R < 1$ and $0 < \theta^*_C, \theta^*_L < \bar{\theta}$) the following holds:

1. Candidate D proposes higher spending and taxes than candidate R, i.e., $t^*_D > t^*_R$.

2. Candidate D proposes a higher level of public goods than candidate R, i.e., $g^*_D > g^*_R$. 

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3. The social conservative cutoff voter is economically more liberal than the socially liberal cutoff voter, i.e., $\theta_C^* > \theta_L^*$.

Condition (10) in Lemma 2 implies a general comparative static result. Since $w'' < 0$ it follows that $w'$ is monotone, and consequently, any changes in the preference distribution of voters or their ideological intensity must either increase or decrease public good provision for both candidates. Since an increase in public good provision can only occur when taxes are increased, this implies that no change in electoral preferences leads to an increase in the equilibrium value of $t_R$ and, at the same time, to a decrease in $t_D$; whenever preference change affects equilibrium platforms, the change goes in the same direction. However, in contrast to the classical median voter model, or the probabilistic voting model in which candidate platforms move exactly in parallel, a parameter change may lead to more or less differentiation in economic platforms in our model, i.e., the policy difference $t_D - t_R$ may decrease or an increase. We will return to this issue in more detail in Section 5.

We now turn to sufficient conditions for existence of an equilibrium. Lemma 2 shows that there are no interior equilibria below the 45 degree line in Figure 3, so we can focus on points above the 45 degree line that satisfy the necessary conditions outlined in Lemma 2. It turns out that it is relatively easy to check the robustness of a strategy profile against deviations that do not change the structure of equilibrium, in the sense that also after the deviation, $R$ still gets the support of sufficiently low types, and $D$ those of sufficiently high types. Substantively, the following definition of a “local” equilibrium captures the idea that while candidates’ commitment to tax rates are only credible if they do not deviate too much from the direction that voters expect. For example, a Democrat may have problems to credibly change his policy proposal in a way that suddenly attracts all fiscal conservatives. A local equilibrium just needs to be robust against deviations that are credible in this sense:

**Definition 1.** $(t_R^*, t_D^*)$ is a local equilibrium if and only if $(t_R^*, t_D^*)$ is a Nash equilibrium when candidate strategies are restricted to the sets $\{t_R|f_R(t_R) < f_D(t_D^*)\}$ and $\{t_D|f_D(t_D) > f_R(t_R^*)\}$, respectively.\(^{13}\)

Consider again the right panel of Figure 3. The necessary conditions for an equilibrium are satisfied at $(\theta_L^*,\theta_C^*)$. It is also clear that at least any local deviation (in the sense of Definition 1)\(^{13}\)
cannot increase the vote share of a candidate: Given the results of Lemma 1, any point on \( k_R \) is below, and any point on \( k_D \) is above the isoprobability curve. Thus, we have the following result.

**Theorem 1.** Suppose the necessary conditions of Lemma 2 are satisfied for \( 0 < t_D^*, t_R^* < 1 \). Then \( t_D^*, t_R^* \) is a local equilibrium.

**4.3 Existence and Uniqueness of Equilibria**

In an interior equilibrium, \( k_D \) and \( k_R \) are tangent to each other (see Figure 3). This property is equivalent to (10) in Lemma 2, which we can solve to write \( t_D \) as a function of \( t_R \) (this is possible because \( w' \) is strictly monotone). The set of tangency points of \( k_D \) and \( k_R \) defines a curve \( E(t_R) = (\theta^*_L(t_R, t_D(t_R)), \theta^*_C(t_R, t_D(t_R))) \) has a signed curvature of

\[
\hat{k} = \frac{k_R}{|t_D'(t_R) - 1|}.
\]  

(15)

2. \( D_{t_R} k(t_R) = [1 - t_D'(t_R)]D_{t_R} k(t_R) \)

From Lemma 1, we know that \( \kappa_R > 0 \) (in the relevant range of points above the 45 degree line, i.e. whenever \( \theta_C > \theta_L \)). Thus, the first point of Lemma 3 implies that the signed curvature of \( E(t_R) \), is strictly positive, and therefore \( E \) turns counter-clockwise. The second point determines whether the curve is convex or concave toward the origin. If \( t_D'(t_R) < 1 \), the derivative of \( E \) must point in the same direction as the derivative of \( k_R \) and thus, \( E \) is concave toward the origin, just like \( k_R \). In contrast, if \( t_D'(t_R) > 1 \), \( E \) is convex toward the origin. Depending on whether \( t_D'(t_R) \) is smaller or greater than 2, the curvature of \( E \) is larger or smaller than that of \( k_R \).

The sign of \( 1 - t_D'(t_R) \) is determined by the curvature of the utility function \( w \) and the sign of the third derivative. In particular, the following holds:

**Lemma 4.** Suppose that \( w'(x)w'''(x) > 2(w''(x))^2 \) for all \( x > 0 \) then \( t_D'(t_R) > 1 \) for all \( t_R > \frac{b_R}{a_R} \). If the inequality is reversed, i.e., \( w'(x)w'''(x) < 2(w''(x))^2 \) for all \( x > 0 \) then \( t_D'(t_R) < 1 \) for all \( t_R > \frac{b_R}{a_R} \).
Lemma 4 immediately implies that if the third derivative of utility is negative then \( t'_D(t_R) \) is always less than 1. If the third derivative is strictly positive, then the sign of \( t'_R(t_R) < 1 \) as long as the utility function has enough curvature, i.e., \((-w'')/w' \) and \( w'' \) are sufficiently large. For example, for utility \( w(x) = x^{1-s}/(1-s), s > 0 \) we get \( w'(x)w'''(x) - 2(w''(x))^2 = s(1-s)/x^{2(1+s)} \). Thus, \( t'_D > 1 \) for \( s < 1 \) and \( t'_D < 1 \) for \( s > 1 \). For \( s = 1 \), which corresponds to the case \( w(x) = \ln(x) \) we get \( t'_D \equiv 1 \).

Lemma 4 shows how characteristics of the utility function influence whether some change in fundamentals leads to increasing or decreasing economic policy divergence between parties. Of course, third derivatives of utility functions are not a very intuitive concept and may be difficult to measure directly. However, it may be useful to think about analogous problems in the analysis of choice under uncertainty; for example, whether individuals respond to increased uncertainty about future income by saving more today depends on conditions involving the third derivative of the individual’s expected utility function (Kimball 1990). Rather than saying that expected utility theory does not tell us very much about precautionary saving (because conditions depend on third derivatives that are difficult to measure), the standard interpretation of this result in expected utility theory is that, if we believe that individuals would react to increased uncertainty by increasing precautionary saving, then this tells us something about the shape of the individuals’ utility functions. Analogously, if we, for example, observe empirically that both candidates propose to increase spending and that their proposals are farther apart, then this tells us something about the shape of the voters’ utility function \( w(x) \).

To get existence of interior equilibria, we must show that curve \( E \) assumes slope \(-\pi_L/\pi_C\) at some point \((\theta_L, \theta_C)\) that is strictly inside the set of voter types, \([0, \bar{\theta}]^2\). To ensure that equilibrium tax rates are less than 1, we assume that the marginal benefit to consumers of increasing \( t_D \) close to \( t_D = 1 \) is smaller than the marginal benefit of increasing \( t_R \) at the point \( \bar{t} \) where \( f_D \) and \( f_R \) intersect. This condition appears quite reasonable and is formally stated in Theorem 2, item 2.

Further, it is clear that an interior equilibrium does not exist for large \( d \) or \( r \). For example, if \( d \) is sufficiently large, then candidate \( D \) receives the support of all liberals, and only cutoff \( \theta_C^* \) matters. If both \( d \) and \( r \) are large, then in equilibrium all liberals vote for \( D \), all conservatives for \( R \), and the tax rates become irrelevant. Evidently, this case is not particularly interesting. In contrast, if \( r \) and \( d \) are small, then both ideological groups will be competitive, and small changes of the candidates’ tax rates affect the two cutoff values. We now show that for, in this case, an interior equilibrium exists and is unique. Note that the upper bound for \( r \) is not explicitly provided in the statement of
Theorem 2. Suppose that

1. Assumptions 1 and 2 hold.

2. Let \( \bar{t} = \frac{b_D - b_R}{a_D - a_R} \) be the tax level at which both candidates would provide the same level of public goods, i.e., \( \bar{g} = a_D \bar{t} - b_D = a_R \bar{t} - b_R \). Then the marginal utility of a change of \( \bar{t} \) by candidate \( R \) is larger than if candidate \( D \) changes the tax rate at \( t = 1 \), i.e., \( a_R w'(\bar{g}) > a_D w'(a_D - b_D) \).

3. \( w'(x)w'''(x) - 2(w''(x))^2 \) has the same sign for all \( x > 0 \).

Then there exists \( \bar{r} > 0 \) and \( \hat{\theta} > 0 \) such that for any \( d < \frac{b_R}{a_D} r < \bar{r} \), and \( \hat{\theta} > \hat{\theta} \), there exists a unique local equilibrium. Moreover, this equilibrium is interior, i.e., \( 0 < \theta^*_C, \theta^*_L < 1 \) and \( 0 < t^*_D, t^*_R < 1 \).

In the proof the theorem, we first show that there exists a solution to the first order condition (11) from Lemma 2, which can be rewritten as

\[
\frac{w(g^*_D) - w(g^*_R)}{t^*_D - t^*_R + \pi_C \theta - \pi_L d} = a_D w'(g_D) = a_R w'(g_R). \tag{16}
\]

First, suppose that \( d = r = 0 \). Then, existence of a solution to (16) follows from the geometric argument depicted in the left panel of figure 4.\(^{14}\)

Figure 4: Existence of equilibrium policies \( t^*_R, t^*_D \)

\(^{14}\)The argument for the case without ideology, i.e., \( d = r = 0 \) can be found in Krasa and Polborn (2009b) that analyzes this special case only, and is superseded by this paper.
Absent the ideology term, preferences are given by \((1 - t) + \theta w(g)\). If taxes are on the horizontal and \(w(g)\) is depicted on the vertical axis, then indifference curves are straight lines with slope \(1/\theta\). The solid straight line depicts the indifference curve of a voter who is indifferent between the two candidates — this is the cutoff voter. No candidate can make the cutoff voter strictly better off, since this would entail finding a point \((t, w(g))\) that is above the above the indifference curve and is also feasible. Tangency of this line at \(w(g_R')\) and \(w(g_R^*)\) also means that (16) is satisfied, i.e., the slope at \(w(g_R^*)\) and \(w(g_D^*)\) are equal, and also correspond to the slope of the line connecting \(w(g_R^*)\) with \(w(g_D^*)\).

Condition 1 of the Theorem, and in particular the Inada condition specified in assumption 2 ensures that public good provision \(g_D^*, g_R^* > 0\). Condition 2 ensures that taxes are less than 1. In particular, as depicted in the left panel of the figure, the slope at \(w(f_R(t_R))\), indicated by the dashed line, is less than that of \(w(f_D(t_D))\) at \(t\). This ensures that any line that is tangent to both \(w(f_D(t_D))\) and to \(w(f_R(t_R))\) has a slope that is steeper than that of \(w(f_D(t_D))\). As a consequence, taxes are bounded away from 1. Also note that unlike in the standard median voter model, all voters except for the cutoff voter strictly prefer one of the two candidates. In particular, all voters whose indifference curves are steeper than the solid line strictly prefer the Republican’s equilibrium platform, and vice versa for the Democrat. Thus, even though both candidates here choose equilibrium policies that cater to the same voter type like in the standard median voter model, unlike in that model, this does not result in universal indifference among all voters in equilibrium.

Now suppose that \(d, r > 0\). The right panel depicts a situation where \(\pi_C r - \pi_L d < 0\). The first order condition still requires that the slope at \(w(g_R^*)\) equals that at \(w(g_D^*)\). However, now the line running through both points (dashed in the graph) is more flat, as the horizontal move in the slope is reduced by \(\pi_L d - \pi_C r\). It is clear that such tangent lines exist, and that \(t_R^*\) and \(t_D^*\) are still bounded away from 1, unless \(|\pi_L d - \pi_C r|\) becomes to large, Moreover, to ensure interiority of the liberal and conservative cutoffs, \(d\) and \(r\) cannot be too large, else votes would solely be based on ideology.

5 Comparative Statics

We now investigate the impact of changes in ideological polarization on the candidates’ economic platforms. Ideological polarization includes both changes in the composition of the electorate from social conservatives and social liberals, and in the intensity with which these groups care about the ideological differences between candidates.
Consider first an increase of the proportion of social conservatives in the electorate (i.e., $\pi_C$ increases). In this case, the constant vote share line becomes flatter. In Figure 5, the original equilibrium is the tangency point of $E$ and the solid constant vote share line. The new, flatter lines are dashed, and the white circle marks the new tangency point. In both cases (the left panel with $t_D'(t_R) < 1$ and the right panel with $t_D'(t_R) > 1$), the new equilibrium moves in the direction of the rotation of the curves, i.e., both $t_R$ and $t_D$ increase. Furthermore, as one would expect, increasing the number of conservatives $\pi_C$ increase candidate $R$’s vote share. All these effects would be exactly reversed if the number of social liberals $\pi_L$ increases, since in this case the constant votes share lines would become steeper.

What is the intuition for the somewhat surprising result that an increase in the number of social conservatives (i.e., of voters with a cultural bias for the small government party) leads both candidates to increase their proposed tax rate? Remember that candidates compete for the support of cutoff voters, and that cutoff voters are torn between their economic and cultural-ideological preferences in that they like the economic position of one candidate and the cultural position of the other. In particular, the socially-conservative cutoff voter prefers a higher level of government spending than is provided by both candidates, while the socially liberal cutoff voter prefers a smaller level of government spending. An increase in the number of social conservatives makes it attractive for both candidates to put more weight on the economic preferences of the conservative cutoff voter, and thus to increase the provision of public goods.

Moreover, this effect is likely to be very robust, in the sense that it does not depend on the spe-
specific setup. In our model, the productivity difference between candidates is what drives candidate differentiation. Alternatively, one could, for example, imagine a model in which the Republican candidate has a more conservative ideal position on both the cultural and the economic dimension than the Democrat, and both candidates are policy motivated and use their economic position to maximize their expected utility from the implemented policy. Whether in this or any other model in which candidates have at least some incentive to care about winning the election or about their vote share, they will consider the desires of swing voters more than those of core supporters (i.e. voters who either vote for the candidate or for his opponent, no matter what policies the candidates choose), and, to the extent that there are different swing voter groups, their relative sizes influence which group candidates cater to most. Socially conservative swing voters are necessarily economic liberals (otherwise, they would be Republican hard core partisans), and socially liberal swing voters are economic conservatives. An increase in the number of a particular swing voter group means that the candidates’ incentive to cater to this group (through their policy choice) is increased.

Our result appears consistent with behavior observed in the last decade in which Republicans were in control of the executive and the legislative branch for most of the time. The aftermath of the terrorist attacks of September 11 conceivably increased the proportion of voters with a non-economic preference for the Republican party (and their intensity of preference), and the Republicans in spite of their small-government rhetoric, increased government spending as a fraction of GDP from 18.2 percent in 2000 to 20.7 percent in 2008.\footnote{Since 2008, Republicans have become more concerned with the deficit, but this is probably due to intraparty effects that are not present in our model: In our model, candidates take positions that maximize their vote share in the general election, while taking as given the support of their core constituencies. While this was likely a good description until 2008, it is quite conceivable that the rise of the “Tea Party” has shifted the focus of Republican politicians from choosing the policy that would be most successful in the general election to choosing a policy that minimizes the probability of being attacked from the right in a primary.}

Next, we analyze how the equilibrium is affected by changes in $r$ and $d$, which can be interpreted as changes in the social policy partisanship of conservatives and liberals, respectively, another measure of polarization. In general, the intuition for the effect of an intensification of cultural preferences is very transparent in our model framework: More intense non-economic preferences among social liberals, for example, imply that the liberal cutoff type must decrease (i.e., dislikes government spending more than before) in order to remain indifferent between Republican and Democrat. As candidates maximize some weighted average of the economic preferences of socially liberal cutoff voter and of the (unchanged) socially conservative cutoff voter, they now have
an incentive to propose lower government spending.

Figure 6 shows the effect of increasing \( d \) graphically. The solid curves represent the original \( E \) curves (for \( t_D' < 1 \) in the left panel and for \( t_D' > 1 \) in the right panel). The dashed curves show \( E \) after \( d \) increases. Note that \( t_D(t_R) \) is independent of \( d \). Thus, (3) implies that \( \theta_L^* \) decreases and \( \theta_C \) remains unchanged. Therefore points on the solid and dashed \( E \) curves that correspond to the same tax rate \( t_R \) are aligned horizontally. The curved arrow along \( E \) indicates the direction of movement as \( t_R \) increases.

Equations (30) and (31) in the Appendix imply that for given \( t_R \), the slope of \( E(t_R) \) increases (i.e., a negative slope becomes less steep) as \( d \) increases. The comparative static result for \( d \) now follows immediately from simple geometric observations. In the left panel, the new equilibrium point is below the horizontal line. Given the direction of rotation of \( E \) indicated by the arrow, this corresponds to a lower \( t_R \) and hence lower \( t_D \). The cutoff \( \theta_C \) decreases. \( \theta_L \) may decrease (as in the graph) or increase, depending on the curvature of \( E \). Finally, as one would expect, increasing \( d \) increases candidate \( D \)'s vote share.

In the right panel, the new equilibrium is above the horizontal line. Again, the rotation direction of \( E \) implies that at this new equilibrium, taxes are lower. Note that \( \theta_C \) increases, while \( \theta_L \) decreases — in the case of \( t_D' > 1 \) there is no ambiguity about the change of cutoffs.

Figure 7 shows that, by analogous arguments, increasing \( r \) leads to the reverse effects of increasing \( d \): Increasing \( r \) moves \( E \) up and results in a steeper slope along vertical lines; note that vertical rather than horizontal lines connect points on the two \( E \) curves with the same tax rate; and
in equilibrium, taxes are increased rather than decreased.

We now summarize our results.

**Theorem 3.** Suppose that Assumption 1 holds, and that the sign of $w'(x)w'''(x) - 2(w''(x))^2$ is the same for all $x > 0$. Then

1. Both candidates increases spending (taxes) if one of the following occurs:
   - The conservative ideology intensity $r$ increases, the liberal ideology intensity $d$ decreases, or
   - the fraction of social liberals to social conservatives, $\pi_L/\pi_C$ decreases.

2. $\theta_C$ increases and $\theta_L$ decreases if one of the following occurs:
   - (a) $w'(x)w'''(x) > 2(w''(x))^2$ for all $x > 0$, and the liberal ideology intensity $d$ or $\pi_L/\pi_C$ increases.
   - (b) $w'(x)w'''(x) < 2(w''(x))^2$ for all $x > 0$, and the conservative ideology intensity $r$ increases or $\pi_L/\pi_C$ decreases

Note that the ideological changes among voters that increase spending in part 1 of Theorem 3 are all changes that are electorally beneficial for the Republican in the sense that an increase in conservative preference for Republicans, or an increase in the number of conservatives increases the equilibrium vote share of the Republican. This implies that, in terms of implemented policy, two effects are possible: First,
We now turn to the question how the results of Theorem 3 influence the structure of voter polarization in society: Do voters separate more along ideological issues (i.e., does the separating line in Figure 1 become steeper?), or more along economic lines? Note that the case described in number 2 of Theorem 3 – $\theta_C$ increases and $\theta_L$ decreases – is a case of stronger ideological polarization because more conservatives and fewer liberals vote for the Republican.

Consider first what happens if the number of social conservatives increases. By itself, this change does not affect the behavior of any given voter type. However, there is an indirect effect because the candidates’ equilibrium policies change. If $t'_D(t_R) < 1$ (by Lemma 4, this is equivalent to $w'(x)w'''(x) < 2(w''(x))^2$), then the Democrat increases taxes by less than the Republican, and so parties become more similar economically. As a consequence, society appears more divided along cultural-ideological issues (i.e., a steeper separating line). In contrast, if $t'_D(t_R) > 1$ (equivalently, if $w'(x)w'''(x) > 2(w''(x))^2$), then the Democrat increases taxes by more than the Republican, so that parties become more economically distinct, and voters become more separated along economic issue preferences (a flatter separating line).

Consider now what happens when the intensity of ideological preferences among social conservatives increases. Again, both candidates’ equilibrium taxes will increase, and whether economic differences between parties increase or decrease depends on whether $t'_D(t_R) > 1$ or $t'_D(t_R) < 1$. In addition, social conservatives feel more intensely about the ideological differences between parties. Thus, if $t'_D(t_R) < 1$, then both the direct and the indirect effect go in the same direction, and society will be more polarized along ideological lines. If $t'_D(t_R) > 1$, then the two effects go in opposite directions, and the total effect is ambiguous in sign.

Finally, it is interesting to connect Theorem 3 with Krasa and Polborn (2012) who analyze the direction and intensity of voter separation in U.S. presidential elections between 1972 and 2008. They show that Democratic and Republican candidates diverged substantially on “cultural” issues since the 1970s; specifically, they estimate that the distance between the candidates’ ideological positions is about 4 times as large in the election in the 2000s than it was in 1976. In terms of our model in this paper, this would translate into an increase of the preferences of liberals and conservatives, $d$ and $r$. Since the U.S. has substantially more conservatives than liberals,\(^{16}\) the overall net effect is the same as if only conservative intensity increased (but by a smaller

\(^{16}\)For example, in the 2004 and 2008 Presidential election exit polls, 21 and 22 percent of voters identify themselves as “liberals”, respectively, in both years. 34 percent call themselves “conservatives” (see http://www.msnbc.msn.com/id/5297138/ and http://www.cnn.com/ELECTION/2008/results/polls/\#USP00p1).
amount). Krasa and Polborn (2012) also show that economic differences between candidates are somewhat larger today than in the 1970s. Along the arguments given discussion above, the increase in ideological differences between parties is consistent with both parties proposing higher spending levels, and the case of $t'(t_R) > 1$, so that the difference between the Democratic and the Republican tax rate increases.

Theorem 4 below analyzes what happens to equilibrium policies and voter cutoffs when the ideological positions of the two candidates diverge. This will change the ideological payoff that liberals and conservatives receive from the election of their ideologically-preferred candidate. Theorem 4 shows that the decisive parameter for the direction of the effect is the net average ideology, $\pi_L\delta_1 + \pi_C\delta_2$. For example, if this is positive, voters on average have a preference for the Republican candidate (remember that we normalized candidate positions such that candidates are located symmetrically around zero). Moreover, candidate polarization affects conservatives more than liberals. As a consequence, the effect of candidate polarization in this case is the same as if only conservatives’ ideological utility had increased: Spending increases, and how the liberal and conservative cutoffs change depends on whether the Republican and the Democratic platform converge or diverge, just like in Theorem 3.

Theorem 4. Let $\hat{\pi}_L = \frac{\delta_2}{\delta_2 - \delta_1}$, so that the average ideological net preference intensity $\pi_L\delta_1 + \pi_C\delta_2$ is positive if and only if $\pi_L < \hat{\pi}_L$. Suppose that the ideological distance between the candidates, $\delta_R - \delta_D$, increases. Then:

1. If $\pi_L = \hat{\pi}_L$, i.e., if the average ideology is zero, then spending remains the same. Cutoff $\theta_C$ increases, while $\theta_L$ decreases.

2. If $\pi_L > \hat{\pi}_L$ (i.e., the average type is liberal), then spending decreases. If $\pi_L < \hat{\pi}_L$ (i.e., the average type is conservative), then spending increases.

3. Let $\hat{\theta}_C, \hat{\theta}_L$ be the cutoff that would obtain in case 1 where $\pi_L = \hat{\pi}_L$. Then

   (a) $\theta_C < \hat{\theta}_C$ and $\theta_L > \hat{\theta}_L$ if $\pi_L > \hat{\pi}_L$ and $w'w''' < 2(w'')^2$, or $\pi_L < \hat{\pi}_L$ and $w'w''' > 2(w'')^2$.

   (b) $\theta_C > \hat{\theta}_C$ and $\theta_L < \hat{\theta}_L$ if $\pi_L < \hat{\pi}_L$ and $w'w''' < 2(w'')^2$, or $\pi_L > \hat{\pi}_L$ and $w'w''' > 2(w'')^2$.

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17 To see this, consider the case that both groups have the same size, $\pi_L = \pi_C$, so that $\delta_2 > |\delta_1|$. Thus, conservatives are farther away from the Democratic candidate than liberals are from the Republican candidate, and since the ideological utility function is convex, the net payoff of conservative voters from their ideologically preferred candidate grows by more than the net payoff of liberal voters.
Figure 8 illustrates the effects of ideological polarization of party platforms on the voting behavior of individuals, using the line that separates Democratic from Republican voters, which we encountered already in Figure 1. Note that, while there are only two ideology types in the analysis so far, Section 6 below demonstrates that the structure of the equilibrium remains unchanged when there are many ideology types, in which case the linearity of the separating line is more meaningful than in the case of just two types.

The left panel of Figure 8 refers to case 3 (a) of Theorem 4, while the right panel refers to case 3 (b). In both cases, the first arrow, from the original line to the dotted line, indicates the change in the separating line that would arise if only the parties’ ideological positions changed, but their economic platforms remained unaffected. The movement from the dotted line to the final line indicates the additional change that arises because of the change in equilibrium economic platforms.

Under the conditions of case 3 (a), exogenous ideological differentiation leads to economic convergence of party platforms. Because of decreased economic differences between parties, voters’ ideology becomes even more decisive for their voting behavior than if there was no change in economic platforms. In contrast, under the conditions of case 3 (b), exogenous ideological differentiation leads to economic divergence of party platforms, which counteracts the original motion of the separating line. In principle, it would even be possible that the indirect effect of economic platforms outweighs the direct ideology effect, so that the final separating line could be flatter than the original line; this will happen if the increased economic divergence between parties is more
important for voters than the increased ideological divergence.

Finally, if the average ideological preference of voters is zero (case 1 of Theorem 4), then exogenous ideological differentiation between parties has no effect on equilibrium economic policies, so that there is only the effect indicated by the first arrow. Consequently, the separating line in Figure 8 becomes steeper (i.e., more liberals vote for the Democrat, and more conservatives vote for the Republican).

6 More than Two Ideology Types

We now generalize our results to more than two ideological voter types. The main insight of this section is that we can define an “average” liberal and conservative ideology type such that the equilibrium of the model with many types is the same as that of the model with the two average types only.

Let $d_i, i \in N_L$, and $r_i, i \in N_C$, be a collection of ideology preferences of liberals, and conservative types, respectively. Some of these ideological voter types may be partisans that always vote for the same candidate, irrespective of their economic preferences. Let $\Gamma$ be the set of all types that are contested, i.e., that have an interior cutoff voter type. As long as $\Gamma$ is non-empty, Lemma 2 implies that, in equilibrium, $t_D$ and $t_R$ are set such that $a_Dw'(g_D) = a_Rw'(g_R)$. Hence, we get the same function $t_D(t_R)$ as in the case of two types. Thus, $E$ is also defined as in the two-type case, and the derivatives of $E$ are given by (5) and Lemma 3.

Let $\pi_\Gamma = \sum_{i \in \Gamma} \pi_i$ be the fraction of all contested types. If all types are contested, then $\Pi_\Gamma = 1$. The necessary condition for an equilibrium is that $E$ is tangent to the constant-vote share hyperplane, i.e., $\nabla_{t_R}K(t_R) \cdot \pi_C = 0$. Suppose that there is at least one contested liberal type and one contested conservative type. Substituting the derivatives of $E$ into this equation yields the first order condition

$$\frac{\sum_{i \in N_C \cap \Gamma} \pi_i \cdot x_i}{\sum_{i \in \Gamma} \pi_i} - \frac{\sum_{i \in N_L \cap \Gamma} \pi_i \cdot d_i}{\sum_{i \in \Gamma} \pi_i} = \frac{w(g_D) - w(g_R)}{a_Rw'(g_R)} - (t_D - t_R).$$

(17)

Define

$$d = \frac{\sum_{i \in N_C \cap \Gamma} \pi_i \cdot d_i}{\sum_{i \in N_C \cap \Gamma} \pi_i}, \quad r = \frac{\sum_{i \in N_L \cap \Gamma} \pi_i \cdot x_i}{\sum_{i \in \Gamma} \pi_i}, \quad \pi_L = \frac{\sum_{i \in N_L \cap \Gamma} \pi_i}{\sum_{i \in \Gamma} \pi_i}, \text{ and } \pi_C = \frac{\sum_{i \in N_C \cap \Gamma} \pi_i}{\sum_{i \in \Gamma} \pi_i}$$

(18)

Then (17) and (18) imply

$$\pi_C r - \pi_L d = \frac{w(g_D) - w(g_R)}{a_Rw'(g_R)} - (t_D - t_R).$$

(19)
Note that (18) is equivalent to the first order condition of a model with a conservative type \( r \) and one liberal type \( d \), where the fraction of conservatives and liberals are given by \( \pi_C \) and \( \pi_L \), respectively. As a consequence, we can use (18) to convert the model with many types into a two-type model, and then use the comparative static results of Section 5.

Now suppose that we increase \( r_i \) for the most conservative contested type such that this type becomes a partisan. As a consequence, (18) implies that \( r \) decreases, \( \pi_L/\pi_C \) increases, while \( d \) remains constant. Theorem 3 implies that the overall effect is a decrease in spending. Analogously, if the most liberal type becomes uncontested, we get an increase in spending.

We now summarize these results:

**Theorem 5.**

1. Equilibrium policies in a model with multiple types only depend on \( d \) and \( r \), the ideological preferences of the average contested liberal and conservative types, respectively, and the fractions \( \pi_L, \pi_C \) of these types in the population, where \( d, r, \pi_L, \pi_C \) are given by (18).

2. If \( d_i \) is sufficiently increased of the most liberal contested type, then spending is increased.

3. If \( r_i \) is sufficiently increased of the most conservative contested type, then spending is decreased.

As a consequence of Theorems 3 and 5, changes in ideological intensity have a non-monotonic effect on the candidates’ equilibrium spending level. For example, a small increase of conservative ideological intensity (that leaves the voter group competitive) leads to an increase in spending. However, if we increase ideological preferences even more, then at some point, this conservative voter group becomes completely partisan, which leaves the remaining competitive cutoff voters more socially liberal (and thus, on average, more economically conservative); this leads to a discrete decrease in equilibrium spending levels. Of course, an analogous non-monotonicity arises for the preference intensity of liberal groups.

7 Conclusion

In this paper, we have developed a model in which voters care about both social ideology (which, in our model, is exogenously given for candidates) and the economic positions that candidates take.
The interaction between these two dimensions is of first-order importance for our understanding of what determines economic policy: In reality, there are considerable differences in candidates’ economic policy platforms, but voter preferences for parties and candidates appear to be influenced by both economic and, probably to an even greater extent, by cultural-ideological positions. A model that explicitly incorporates these non-economic factors provides us with a better understanding of this important interaction, and thus with a better understanding of the determinants of economic policy than a model that abstracts from cultural ideology in order to focus entirely on economic policy issues.

We derive three main results in this framework. First, we identify the swing voters who are decisive for the candidates’ equilibrium choice of economic policies: Among ideological social conservatives, these are voters who are quite economically liberal (i.e., prefer substantial government spending), while among social liberals, swing voters are economically conservative. Candidates focus their equilibrium economic policies to appeal to a weighted average of these swing voters. One key implication of the mechanics of political competition between candidates is that any exogenous change will always have a uniform effect on the two candidates’ equilibrium platforms: Either, both candidates propose higher spending, or both propose lower spending than before.

Second, an intensification of ideological preferences among swing voters – whether brought about by changes in voter preferences, or changes in candidates’ ideological positions – has a somewhat surprising effect in that more intense ideological preferences for the small-government party lead both parties to increase their spending proposals, and vice versa for an intensification of ideological preferences for the big-government party. However, the intensity of ideological preferences in a particular voter group has a non-monotonous effect on the average ideological preferences of swing voters: For example, if the ideological intensity of a group of cultural conservatives becomes too large, they cease to be swing voters, and the average ideological balance among swing voters moves in the culturally liberal (and economically conservative) direction, so that the normal effects of preference intensification are all reversed.

Third, ideological preference polarization affects economic platforms, and can do so in a way that either increases or decreases economic policy differences between the two parties, depending on the voters’ utility functions in a way that we characterize precisely. This has important implications for the effect of ideological polarization on the direction of political conflict. If ideological polarization leads candidates to economic convergence, then voters will split more along ideological lines (and less so along economic preferences), because ideology has become more important.
and economic preferences less so. However, it is also possible that ideological polarization leads to economic divergence, and if the latter effect is strong enough, the overall effect brought about by an initial ideological polarization may be that voters ultimately divide more along economic preference differences and less along their ideological preferences. In Krasa and Polborn (2012), we empirically analyze polarization in U.S. Presidential elections between 1972 and 2008, and show that substantial increase in cultural policy divergence between the parties (presumably being brought about by an increasing involvement of religious and social conservatives in the Republican party) was accompanied by a smaller degree of economic policy divergence. As a consequence, the electorate splits today more along cultural-ideological lines (and less along economic lines) than in the 1970s, but this effect is somewhat muted by the economic divergence.
8 Appendix

Proof of Lemma 1. For fixed $t_D$, consider the curve given by $t_R \mapsto (\theta_L(t_R), \theta_C(t_R))$. Let $S(t_R) = \frac{\partial \theta_L(t_R)}{\partial t_R} / \frac{\partial \theta_C(t_R)}{\partial t_R}$. Thus, (5) and (5) imply that

$$S(t_R) = \frac{(w(g_D) - w(g_R)) - ((t_D - t_R) + \alpha a_R' w'(g_R))}{(w(g_D) - w(g_R)) - ((t_D - t_R) - \beta) a_R w'(g_R)}.$$  \hfill (20)

Let

$$A(t_R) = (w(g_D) - w(g_R)) - (t_D - t_R) a_R w'(g_R)$$  \hfill (21)

Then

$$\frac{\partial S(t_R)}{\partial t_R} = (\tau + \beta) \frac{w'(g_R) a_R' \frac{\partial A(t_R)}{\partial t_R} - A(t) w''(g_R) f_R^2(t_R)}{\left((w(g_D) - w(g_R)) - ((t_D - t_R) - \beta) a_R w'(g_R)\right)^2},$$  \hfill (22)

where

$$\frac{\partial A(t_R)}{\partial t_R} = -(t_D - t_R) f_R^2(t_R) w''(g_R).$$  \hfill (23)

Next, note that

$$f_R^2(t_R) \frac{w'(g_R)}{w''(g_R)} - f_R^2(t_R) \frac{A(t_R)}{\frac{\partial A(t_R)}{\partial t_R}} = a_R' \frac{w'(g_R)}{w''(g_R)} + \frac{w(g_D) - w(g_R)}{(t_D - t_R) w''(g_R)} - a_R' \frac{w'(g_R)}{w''(g_R)}$$  \hfill (24)

Thus,

$$\frac{\partial S(t_R)}{\partial t_R} = -(\tau + \beta) \frac{w''(g_R) f_R^2(t_R)(w(g_D) - w(g_R))}{\left((w(g_D) - w(g_R)) - ((t_D - t_R) - \beta) a_R w'(g_R)\right)^2},$$  \hfill (25)

The signed curvature of candidate $R$’s response function is given by

$$\kappa_R = \frac{\partial w'_R}{\partial t_R} \frac{\partial^2 w_R}{\partial t_R^2} - \frac{\partial w'_R}{\partial t_R} \frac{\partial^2 w_R}{\partial t_R^2} = \left(\frac{\partial w'_R}{\partial t_R} \frac{2}{\partial t_R} (\frac{\partial w'_R}{\partial t_R} \frac{2}{\partial t_R})^{3/2}\right),$$  \hfill (26)

Thus,

$$\kappa_R = \frac{\partial S(t_R)}{\partial t_R} \left(\frac{\partial w'_R}{\partial t_R} \frac{2}{\partial t_R} (\frac{\partial w'_R}{\partial t_R} \frac{2}{\partial t_R})^{3/2}\right),$$  \hfill (27)

As a consequence (22), (23), (24) and (27) imply (8).

Similarly, it follows that the curvature of $t_D \mapsto (\theta_L(t_D), \theta_C(t_D))$ is given by (9).
Proof of Corollary 1. (10) and concavity of $w$, i.e., the fact that $w'$ decreases, implies $g_D' > g_R'$. Since $2\delta(\delta_R - \delta_D) = -d < 0$ and by $\theta^*_L > 0$, equation 3 implies that $t_D' - t_R' < 0$. □

Proof of Lemma 2. Equation (4) implies that the slope of $(\theta^*_L(t_D), \theta^*_C(t_D))$ is

$$
\frac{(w(g_D) - w(g_R)) - ((t_D - t_R) - d)a_Dw'(g_D)}{(w(g_D) - w(g_R)) - ((t_D - t_R) + r)a_Dw'(g_D)}.
$$

(28)

Similarly, (5) implies that the slope of $(\theta^*_L(t_R), \theta^*_C(t_R))$ is

$$
\frac{(w(g_D) - w(g_R)) - ((t_D - t_R) - d)a_Rw'(g_R)}{(w(g_D) - w(g_R)) - ((t_D - t_R) + r)a_Rw'(g_R)}.
$$

(29)

For an interior solution these slopes must be the same, and equal to the slope of the the lines with constant vote share given by $-\pi_L/\pi_C$.

Thus, for an interior solution (28) and (29) must be the same. Clearly, this is the case if $a_Dw'(g_D') = a_Rw'(g_R')$. Conversely, it is easy to see that unless $d = r = 0$, the condition $a_Dw'(g_D') = a_Rw'(g_R')$ is also necessary for the slopes to coincide.

Now suppose that $\theta^*_L$ is not contested, i.e., is on the boundary. Then candidate $R$ chooses $t_R$ to maximize $\theta^*_R$, while candidate $D$ chooses $t_D$ to minimize $\theta^*_R$. This leads to the first order conditions (12) for candidate $D$ and $w(g_D') - w(g_R') = ((t'_D - t'_R) + r)a_Rw'(g_R')$ for candidate $R$. These two conditions immediately imply $a_Dw'(g_D') = a_Rw'(g_R')$.

The case where $\theta^*_C$ is on the boundary is identical. □

Proof of Lemma 3. Substituting $t_D(t_R)$ for $t_D$ in (3), and taking the derivative with respect to $t_R$ yields

$$
\frac{\partial \tilde{\theta}_L}{\partial t_R} = (t'_D - 1)\frac{(w(g_D) - w(g_R)) - ((t_D - t_R) - d)a_Rw'(g_R)}{(w(g_D) - w(g_R))^2} = (1 - t'_D)\frac{\partial \theta^*_L}{\partial t_R}.
$$

(30)

$$
\frac{\partial \tilde{\theta}_C}{\partial t_R} = (t'_D - 1)\frac{(w(g_D) - w(g_R)) - ((t_D - t_R) + r)a_Rw'(g_R)}{(w(g_D) - w(g_R))^2} = (1 - t'_D)\frac{\partial \theta^*_C}{\partial t_R}.
$$

(31)

This and equations (5), and (5) proves the third statement of the Lemma.

Next, note that (30), (31) and (10) imply that the slope $H(t_R) = S(t_R)$, where $S(t_R)$ is given by (20). Thus, the candidates’ reaction functions have the same slope as $(\theta^*_L(t_R), \theta^*_C(t_D(t_R)))$.

Let

$$
B(t_R) = (w(f_D(t_D(t_R)))) - w(f_R(t_R))) - (t_D - t_R)a_Rw'(f_R(t_R))
$$

(32)
Then (20) implies that
\[ \frac{\partial B(t_R)}{\partial t_R} = -(t_D - t_R) f_R^2(t_R) w''(f_R(t_R)). \]
Thus,
\[ \frac{\partial H(t_R)}{\partial t_R} = \frac{\partial S(t_R)}{\partial t_R} = (r + b) \frac{w''(g_R) f_R^2(t_R)(w(g_D) - w(g_R))}{((w(g_D) - w(g_R)) - ((t_D - t_R) - b) a_R w'(g_R))^2}, \]
Let \( \hat{k} \) be the signed curvature of the curve. Then as in equation (27) it follows that
\[ \hat{k} = \frac{\partial H(t_R)}{\partial t_R} \left( \frac{\partial \theta(t_R)}{\partial t_R} \right)^2 \] (35)
Thus, (27), (30), (31), and (35) imply
\[ \frac{\partial S(t_R)}{\partial t_R} \left( \frac{\partial \theta(t_R)}{\partial t_R} \right)^2 \left( \frac{\partial \theta(t_R)}{\partial g_R} \right)^2 = \frac{\kappa_R}{|t_D' - 1|}. \]

Proof of Lemma 4. Suppose that \( w'(x) w'''(x) > 2(w''(x))^2 \) for all \( x > 0 \). Then this implies that \( w''(x)/w'(x)^2 \) is strictly increasing. This, in turn, implies that
\[ \left( \frac{w'(x)}{w'(y)} \right)^2 (-w''(y)) < -w''(x), \text{ for all } y > x, \] (36)
Choose \( g_D \) and \( g_R \) such that \( a_D w'(g_D) = a_R w'(g_R) \). Then \( g_D > g_R \). Substituting \( g_D \) for \( y \) and \( g_R \) for \( x \) in (36) implies \( a_D^2 (-w''(g_D)) < a_R^2 (-w''(g_R)) \). Thus, \( \frac{d}{dt_D} w(a_D t_D - b_D) < \frac{d}{dt_R} w(a_R t_R - b_R) \). Since at \( \frac{d}{dt_D} w(a_D t_D - b_D) = \frac{d}{dt_R} w(a_R t_R - b_R) \) at \( t_D \) and \( t_R \) we get \( t_D = t_D(t_R) \). Moreover, the slope at \( t_D \) changes faster than at \( t_R \). Thus, an increase of \( t_R \) results in a larger increase of \( t_D \) and hence \( t_D(t_R) - t_R \) is increasing, which proves that \( t_D'(t_R) > 1 \).

The proof of the second statement is identical, we just reverse all inequalities.

Proof of Theorem 2. We first prove that there exists \( 0 < \tilde{t}_D, \tilde{t}_R < 1 \) such that
\[ \frac{w(a_D \tilde{t}_D - b_D) - w(a_R \tilde{t}_R - b_R)}{\tilde{t}_D - \tilde{t}_R} = a_D w'(a_D \tilde{t}_D - b_D) = a_R w'(a_R \tilde{t}_R - b_R). \]
(37)
Let \( w_P(t) = w(a_P t - b_P) \) for candidate \( P = D, R \). If \( t_D = 1 \) then candidate \( D \) provides public goods in the amount of \( a_D - a_R \). Thus, \( w'_D(1) = a_D w'(a_D - a_R) \).
Next, note that \( w_D(\bar{t}) = w_R(\bar{t}) \) at \( \bar{t} = \frac{b_D-b_R}{a_D-a_R} \). Assumption 1 implies that \( 0 < \bar{t} < 1 \). At \( \bar{t} \) both candidates provide public goods \( \bar{g} = \frac{a_Db_D-a_Rb_R}{a_D-a_R} \). Thus, the assumption of the Theorem implies that \( w'_D(\bar{t}) = a_Rw'(\bar{g}) > w'_D(1) \). As a consequence, concavity of \( w \) implies that \( t_D(\bar{t}) < 1 \). Since \( w'_D(\bar{t}) = a_Dw'(\bar{g}) > a_Rw'(\bar{g}) = w'_R(\bar{t}) \) it also follows that \( t_D(\bar{t}) > \bar{t} \).

Define \( h(t) = \frac{w_D(t_D(t)) - w_R(t)}{t_D(t) - t} \). Then the argument in the previous paragraph and the concavity of \( w \) implies that \( h(t) > 0 \). We next prove that \( h(t) < 0 \) if \( t \) is close to the tax rate at which no public good is provided, i.e., where \( a_Rt - b_R = 0 \). By assumption \( w'(0) = \infty \). Since \( t_D = t_D(t_R) \) solves \( a_Dw'(t_D) = a_Rw'(t_R) \) it follows that as \( t_R \downarrow \frac{b_R}{a_R} \), we get \( t_D(t_R) \downarrow \frac{b_R}{a_R} \). Since \( w \) is concave we get \( \frac{w_R(t_R) - w_R(t)}{t_R(t) - t} < \frac{w_R(t_R) - w_R(t)}{t_R(t) - t} < w_R'(t_R) \). Hence \( h(t) < 0 \) for small \( t \). By continuity of \( h \) there exists \( t' \) such that \( h(t') = 0 \). Moreover, \( t' < \bar{t} \), and hence \( t_D(t') < t_D(\bar{t}) < 1 \).

Let \( \bar{t}_R = t' \) and \( \bar{t}_D = t_D(t') \). Then (37) is satisfied. The definition of \( t_D(t_R) \) and the assumption that \( w(0)/w'(0) = 0 \) implies

\[
\lim_{t_R \downarrow \frac{b_R}{a_R}} \frac{w(a_Dt_D(t_R) - b_D) - w(a_Rt_R - b_R)}{w'(a_Rt_R - b_R)} = \lim_{t_R \downarrow \frac{b_R}{a_R}} \frac{w(a_Dt_D(t_R) - b_D)}{w'(a_Rt_R - b_R)} - \lim_{t_R \downarrow \frac{b_R}{a_R}} \frac{w(a_Rt_R - b_R)}{w'(a_Rt_R - b_R)} = 0.
\]

Therefore, if \( \delta < \frac{b_R}{a_R} \) it follows that both (5) and (5) are strictly positive for \( t_R \) close to \( b_R/a_R \) and \( t_D = t_D(t_R) \).

Next, let \( t_R = \bar{t} \). Then as shown above \( t_D(t_R) < 1 \) and

\[
w(a_Dt_D(\bar{t}) - b_D) - w(a_R\bar{t} - b_R) > a_Rw'(a_R\bar{t} - b_R)(t_D(\bar{t}) - \bar{t}).
\]

Thus, (5) and (5) are strictly negative at \( t_R = \bar{t}, t_D = t_D(t_R) \) for any \( r \) with

\[
r < \frac{w(a_Dt_D(\bar{t}) - b_D) - w(a_R\bar{t} - b_R)}{a_Rw'(a_R\bar{t} - b_R)} - (t_D(\bar{t}) - \bar{t}).
\]

By assumption \( t_D(\bar{t}) - 1 \) does not change its sign. Thus, Lemma 3 implies that there exists \( 0 < t_1 < t_2 < 1 \) such that \( E \) has negative slope for \( t_R \) with \( t_1 < t < t_2 \), and assumes slope 0 and \( \infty \) at \( t_1 \) or \( t_2 \). By continuity, there exists \( t_R^* \) such that the slope of \( E \) at \( t_R^* \) equals \( -\pi_L/\pi_C \). Let \( t_D = t_D(t_R^*) \).

By construction \( t_D(t_R^*) - t_R^* - r > 0 \). Thus, (3) and (3) imply \( 0 < \theta_C, \theta_C^* \). If \( \bar{\theta} > \max(\theta_C, \theta^*_C) \), then the tangency point is interior. Thus, theorem 1 implies that the solution \( t_R^* \) is a local equilibrium.

Finally, note that since \( E \) has strictly positive curvature, there exists only one point along \( E \) with slope \( t_R^* \) for \( t < \bar{t} \). Finally, for \( t > \bar{t} \) the slope of \( E \) can only change sign again if \( t_D - t_R^* - \bar{g} \) becomes negative, in which case \( \theta_L < 0 \) and we do not have an interior solution. Thus, \( t_D, t_R^* \) is the unique interior equilibrium.
Proof of Theorem 4. If $\delta_R - \delta_D$ increases to $\delta'_R - \delta'_D$ then both $d_r$ and $r$ increase. Thus, (3) implies that $\theta_L$ decreases and $\theta_C$ increases if taxes $t_D$ and $t_R$ remain the same. It remains to prove that the first order condition (11) still holds for the same taxes. This, is the case if $2\pi_L\delta_1(\delta'_R - \delta'_D) + 2\pi_C\delta_2(\delta'_R - \delta'_D) = 0$, which holds since $\pi_L\delta_1 + \pi_C\delta_2 = 0$. □
References


