Agenda-Setter Power Dynamics: Learning in Multi-Issue Bargaining

Renee Bowen†  Ilwoo Hwang‡  Stefan Krasa§

March 23, 2022

Abstract

We study a dynamic bargaining model between a fixed agenda-setter and responder over successive issues. If the responder rejects the setter’s proposal, the setter can attempt to assert her will to implement her ideal and will succeed with a probability that depends on her “personal power”. The players learn about the setter’s power as gridlock persists. Gridlock occurs when the setter’s perceived power is either too high or too low, and the players reach compromise in an intermediate interval of beliefs. The presence of “difficult” issues can induce more compromise as the players have incentives to avoid learning.

JEL Classification Numbers: C78, D72, D74, D83

Keywords: Bargaining, Personal power, Gridlock, Learning.

*We thank Nageeb Ali, Georgy Egorov, Roger Myerson, Juan Ortner for helpful comments and Xiameng Hua for excellent research assistance. We also thank the seminar audiences at the Harris School of Public Policy, University of British-Columbia and the Virtual Seminar in Economic Theory, as well as the participants at the NBER organization economics meeting, POLECONUK conference and the 2nd ETH Workshop on Democracy.

†UC San Diego and NBER; trbowen@ucsd.edu
‡Department of Economics, Seoul National University; ilwoo.hwang@snu.ac.kr
§Department of Economics, University of Illinois; skrasa@illinois.edu
“Macht bedeutet jede Chance, innerhalb einer sozialen Beziehung den eigenen Willen auch gegen Widerstreben durchzusetzen[...]” (Power means every chance within a social relationship to assert one’s will even against opposition[...]) - Weber (1922)

1 Introduction

What does it mean for a decision maker to have power? To answer this question, the theoretical literature has long focused on the ability of agenda-setting at least since the seminal works by Black (1958), Farquharson (1969), and Romer and Rosenthal (1978). In this literature, the power of agenda-setter is derived from his institutional position — for example, in both the US House of Representatives and the UK House of Commons, the agenda is controlled by the party in power, and a bill that is not supported by a majority of that party is virtually never put on the agenda (Cox, 2001). However, there is another source of power which, like agenda-setting power, is derived from the decision maker’s institutional position, but has not been investigated in the literature. It is the ability to cajole, lobby, persuade, or “go above the responder’s head” to get a proposal passed. Unlike agenda-setting power, it is derived mostly from the decision maker’s personal attributes and abilities. The management literature has defined it as personal power to contrast it from the power derived from the position, or position power (Yukl, 1989).

In this paper, we provide a theoretical framework of personal power and investigate its effect on decision-making processes.

Lincoln’s effort to get the 13th amendment passed in the House of Representatives is an example of the interplay between position power and personal power. As Burlingame (2013) reports, in mid-January 1865, Lincoln told a pair of house members that additional votes “needed to be obtained by hook and crook” to obtain the required two-thirds majority. Opponents understood Lincoln’s power of forcing representatives to vote in favor of the amendment: “The wish or order of the president is very potent. He can punish and reward.” In terms of the opening quote of Max Weber, Lincoln was very effective at using power in the sense of “assert[ing] [his] will even against opposition”. While one part of Lincoln’s power was clearly based on his position as president, a president’s personal ability to exercise power also matters. In contrast, his predecessor James Buchanan had the same position power, but was not as skilled in using the power of the office. Using the above terminology, his personal power was low and he is
therefore consistently ranked among the least effective presidents in U.S. history.¹

A crucial characteristic of personal power is that there exists uncertainty surrounding its strengths. This uncertainty implies that the perception of a decision maker’s personal power is inherently dynamic. The managerial literature on leadership points out that solving problems successfully increases the perceived power of the leader, while failure reduces it (Hollander, 1978). Moreover, when choosing whether or not to force a decision, a decision maker must take into account its effect on the dynamics of perceived power, which in turn affects future outcomes. Given these observations, we are interested in addressing the following questions: How does personal power affect the political decision-making process? What is the effect of the players’ incentives to learn about the power on the equilibrium dynamics? How does the existence of tough policy issues, in which conflict necessarily occurs, affect the future outcomes?

We consider a stylized model of dynamic bargaining with a hierarchical difference between players. An agenda-setter (setter hereafter) and a responder bargain over successive issues. A motivating example is the chair of an academic department making proposals to faculty. At each meeting, a curriculum change, hire, promotion, or other issue may arise and must be bargained over. Time is continuous and a new issue arises with some probability in an interval of time. Each issue has at most three possible outcomes: The setter’s ideal, the status quo (the responder’s most preferred outcome), and a compromise outcome (second most preferred outcome for both players). We call a policy issue “easy” if all three outcomes are possible; In contrast, an issue is “difficult” if the compromise outcome does not exist. Once an issue arrives, the setter puts a proposal on the table. The responder can accept at any point in time, or simply wait, which we refer to as rejection. While a proposal is not accepted, the setter can assert her will by pressuring the responder to accept, which may or may not succeed. The period in which the responder rejects and the setter fails to force acceptance is referred to as gridlock. At any point in time, the setter can replace the existing proposal by another one. Time runs out when a new issue arrives, in which case the status quo prevails.

A key assumption in the model is that upon rejection of a proposal by the responder, the setter has an opportunity to assert her will and have her ideal implemented. Whether or not she is successful at asserting her will depends on her unknown personal power. A setter’s personal

power can only be known in the context of a particular institution, and the position power that this institution provides to the setter. For example, prior to Lincoln becoming president, it was unknown whether or not he would be able to use the power of the office effectively. Consistent with this observation, we assume both the setter and responder are symmetrically unaware of the setter’s power at the start of the game. Whenever the setter asserts her will, symmetric learning about the setter’s power occurs. A powerful setter will succeed in asserting her will with some probability, while a powerless setter will never be successful.

Learning about the setter’s power to successfully assert her will is modeled in the spirit of the exponential bandit literature (Keller et al., 2005). There is a common prior belief that the setter is powerful. While the setter is attempting to assert her will, the belief decreases as long as she has not been successful. If the setter succeeds in asserting her will (by implementing her ideal), then the setter and responder know that she is powerful for certain and the belief jumps to one. We refer to the belief that the setter is powerful as the setter’s level of (personal) power. The setter’s power evolves over time as players learn during the period of gridlock.

We first analyze a game in which all issues are easy (i.e., there are three possible bargaining outcomes). We show that there is a unique Markov perfect equilibrium in which players choose to compromise on issues for an intermediate range of beliefs. That is, compromise is possible if the setter is neither too powerful nor too powerless. As is standard in the bandit literature, the upper bound on the belief is driven by the setter’s “exploitation” versus “exploration” tradeoff. In equilibrium, the responder accepts the compromise offer when the level of power is high, in which case the compromise endogenously becomes a “safe” alternative for the setter. The safe alternative may be exploited, or the risky alternative (proposing her ideal) may be explored. The lower bound on the belief is driven by the responder’s incentives. When the setter is sufficiently powerless, the responder knows it is unlikely that the setter can implement her ideal. Therefore, the responder never accepts a compromise proposal, effectively eliminating the “safe” alternative of the setter. Given this, the setter seeks to assert her will on every issue, because she has nothing to lose – there is a small probability that she is successful, and discovers that she is powerful.

When the compromise interval of beliefs is strictly inside [0, 1] this leads to the following dynamics. If the setter is very powerful (i.e. the belief about the setter’s personal power level is close to one) at the beginning of the game, then the setter is unwilling to compromise and
gridlock occurs. If the setter is unsuccessful in using her personal power, the belief about the setter’s power decreases. When that belief hits the upper bound of the compromise region, then the setter switches to the compromise offer, which will be immediately accepted. If the setter is sufficiently powerless at the beginning, then there will be long-run gridlock. The belief about the setter’s power will either continue to decline towards zero, or move to one, if the setter is ever able to overturn a rejection.

We then consider a model in which difficult issues arise with some probability. Recall that an issue is difficult if the compromise outcome does not exist, and thus the players must agree on either the status quo or the setter’s ideal. In this case, we show that two types of equilibria exist. In both types of equilibria, behavior under the easy issues does not change: the players agree on the compromise outcome when the setter’s power is in the intermediate range. Under difficult issues, however, the equilibrium behavior differs qualitatively between the two equilibria, which we call gridlock equilibrium and avoiding-the-issue equilibrium.

In the gridlock equilibrium, the setter always proposes her ideal so there is always gridlock (and learning) when a difficult issue arrives. Therefore, even if beliefs are in the interval where compromise is implemented for easy issues, the players eventually move out of the compromise region, and from this point onward gridlock ensues even for easy issues. This helps to explain why gridlock may arise for issues that may have once been resolvable: As the setter’s power becomes more conclusive during the difficult issue, the players no longer wish to compromise. In this case, the belief tallies the past periods of disagreement, and once the tally is full, there is no longer a chance of compromise. In contrast, in the avoiding-the-issue equilibrium, the setter stops demanding her ideal at the lower bound of the compromise interval. Since the setter knows that further gridlock will push the belief out of the compromise interval—and leading to gridlock even for easy issues—she decides to “avoid the current issue” and propose the responder’s ideal. As a result, long-run compromise ensues once the belief hits the compromise boundary.

While one may expect that the presence of difficult issues always makes compromise harder, this is not always the case. In fact, we show that bundling easy and difficult policy issues can result in less gridlock for both types of issues, as doing so provides additional incentives to avoid learning. Most surprisingly, the presence of difficult issues can induce more compromise even for the easy issues. To see this, observe that compromise on the difficult issues results
in the responder’s most preferred outcome. Thus, even if the belief about the setter’s personal power is low, the responder may be willing to compromise on easy issues, because by not doing so he would lose out on the benefit of compromise when difficult issues arise. Further, bundling helps to generate compromise for the difficult issues. If all issues were difficult, permanent gridlock would arise as the players have no reason to compromise. However, the prospect of future easy issues provides the setter an incentive to compromise on difficult issues in order to remain in the compromise region, and avoid permanent gridlock.

**Literature Review**

There is an extensive literature on dynamic legislative bargaining, now summarized in Eraslan et al. (2020). As in Diermeier and Fong (2011) (and Romer and Rosenthal (1979) in a static setting), we assume that there is a designated agenda setter. This assumption allows us to focus on learning about the type of only one of the players. In contrast, Baron and Ferejohn (1989, 1987) consider the case of a decentralized committee in which each member can be selected to be the agenda setter. These models have been extended to multidimensional policy spaces by Banks and Duggan (2000, 2006), and to dynamic bargaining (c.f., Baron (1996), Kalandrakis (2004), Duggan and Kalandrakis (2012), Bowen and Zahran (2012)). To link policies over time, these models assume that once a policy is enacted it determines the status quo for the next period. In our model, the status quo payoff is exogenous and fixed, but the belief about the setter’s type links issues through time. This allows us to clearly identify the effects of learning in the bargaining process.

The choice between a risky proposal (the setter’s ideal) and a safe alternative (the compromise outcome) is modeled as a bandit problem in the spirit of Keller et al. (2005). In this sense, this project is related to the growing literature on collective experimentation and voting rules, including Strulovici (2010), Anesi and Bowen (forthcoming) and Gieczewski and Kosterina (2019). Like ours, these papers study the interaction between collective choice and experimentation, however, what is uncertain in our paper is the bargaining ability of the setter. In equilibrium, the setter’s ideal action can be considered the risky alternative, while the compromise outcome is the safe alternative. Interestingly, the setter and responder have opposing incentives to experiment. This generates the possibility for an intermediate interval of beliefs such that the safe alternative is implemented. In this interval experimentation is too risky for the setter, while not conveying enough information for the responder to trigger it. Other papers considering policy exper-
imentation and collective choice include Majumdar and Mukand (2004), Volden et al. (2008), Cai and Treisman (2009), Callander (2011), Callander and Hummel (2014), Millner et al. (2014), Hirsch (2016) and Freer et al. (2018). Callander and Hummel (2014) consider a case in which a political party preemptively experiments on policy to affect future decisions of the opposition party. Our paper differs because in our model agents do not learn the type of the policy, but the strength of the setter which endogenously determines the future outcome.

The existing literature on bargaining with incomplete information (Fudenberg et al., 1985; Abreu and Gul, 2000; Deneckere and Liang, 2006; Lee and Liu, 2013) typically focuses on the effect of private information of bargainer(s). In these models, a rejection by the informed bargainer signals that the bargainer has a higher reservation value. In contrast, in our paper players are symmetrically uninformed about the setter’s ability and their conflicts over policy induces social learning. Uncertainty about the bargaining strength of agents has been previously proposed as a rationale for delay in bargaining. This has been explored in the seminal works of Admati and Perry (1987), Cramton (1992) and more recently by Friedenberg (2019). We do not seek to explain delay in this paper, but rather we seek to explain when we expect compromise to arise, or when we expect a challenge to ensue. Our model features delay in the sense that gridlock exogenously implies some delay relative to agreeing to a policy proposal immediately.

Theoretically our paper is also related to a rich literature in continuous time bargaining models, including Ortner (2019); Perry and Reny (1993). Ortner (2019) is closely related as, similar to us, the effect of evolving setter power on bargaining outcomes is considered. Ortner (2019) considers a single issue, whereas we consider multiple. In addition, unlike Ortner (2019), our model features endogenous evolution of proposal power as gridlock is chosen. The setter and responder must therefore consider the trade-off between fighting for their preferred issue (which will imply learning about setter strength) and settling for a less preferred outcome. In earlier work, Powell (2004) also takes up the question of endogenous evolution of setter power, but in a discrete time setting with effectively a single issue being considered.

The remainder of the paper is organized as follows. Section 2 presents the baseline dynamic bargaining model and Section 3 calculates dynamic payoffs for setter and responder for two benchmark outcomes. Section 4 characterizes the equilibrium of the dynamic model and establishes existence of the compromise interval of beliefs. Section 5 extends the baseline model to include the possibility of difficult issues and shows that these can introduce learning that forces
players into permanent gridlock or permanent compromise. Section 6 concludes. All proofs are relegated to the Appendix.

2 Model

We present a stylized model of bargaining over an infinite sequence of issues between an agenda-setter $S$ (House Speaker or department chair) and a responder $R$ (speaker’s party or faculty). We assume that time is continuous $t \in [0, \infty)$ and new issues arise at random times. In particular, in any interval of time $[t, t+dt)$, a new issue arrives with probability $1 - e^{-\xi dt} \approx \xi dt$. Since $\xi$ governs the speed at which new issues arrive, we interpret this as the velocity of the institution. At most one issue is bargained over at any time. Therefore, if a new issue arrives while the present issue is not resolved, then the status quo is implemented for the present issue, which is then abandoned.

The game proceeds as follows. At each instant, the setter makes a proposal $x \in X \subseteq \{x_0, x_c, x_s\}$ for the issue at hand. We denote $x_0$ as the exogenous status quo position, $x_c$ as the compromise outcome, and $x_s$ as the setter’s preferred option. After a proposal is made, the responder chooses to accept or reject the proposal. If the proposal is accepted, it is implemented immediately and the players receive payoffs. If the proposal is rejected, then the setter can choose to assert her will or allow the status quo to be implemented immediately.\(^2\) Then a stochastic outcome is realized—which we describe in detail in the next paragraph—and a new issue stochastically arrives. Figure 1 depicts a heuristic timeline within an interval $[t, t+dt)$.

We model the outcome of the setter’s attempt to assert her will as an exponential bandit in the spirit of Keller et al. (2005). If the setter chooses to assert her will, then her success at implementing her ideal $x_s$ depends on her type $\theta \in \{0, 1\}$. We say that if the setter is powerful or $\theta = 1$, then she is successful at asserting her will with probability $1 - e^{-\lambda dt} \approx \lambda dt$ over time interval $[t, t+dt)$. If the setter is powerless then she is never successful at asserting her will. Note that $\lambda$ governs the setter’s probability of success conditional on being powerful. We thus think of $\lambda$ as related to institutional features (such as access to powerful committees), and thus captures position power.\(^3\) The common prior probability that the setter is powerful is $p_0$. We

\(^2\)The terminology of “asserting one’s will” is from Weber (1922), quoted in the beginning of the introduction of this paper.

\(^3\)In the case of House Speaker, Cooper and Brady (1981) describe how leadership style moved from hierarchical...
Figure 1: Heuristic timeline within an interval \([t, t + dt)\)

refer to the posterior \(p_t\) as the setter’s level of perceived personal power (or simply power) at time \(t\).\(^4\)

Players receive payoffs \(u_S(x)\) and \(u_R(x)\) when position \(x\) is implemented for each issue. We assume that \(R\) strictly prefers \(x_0\) to \(x_s\), and, similarly, \(S\) strictly prefers \(x_s\) to \(x_0\). We assume that \(u_S(x_0) = u_R(x_s) = 0\).\(^5\) Also, let \(u_S(x_s) = \bar{u}, u_S(x_c) = u_c, u_R(x_0) = \bar{v}\) and \(u_R(x_c) = v_c\), where \(0 < u_c < \bar{u}\) and \(0 < v_c < \bar{v}\).\(^6,7\) Note that \(x_c\) is considered a compromise proposal because it is the second-ranked outcome for both players. Note also that in this setting \(v_c + u_c > \max(\bar{u}, \bar{v})\) is sufficient for compromise to be efficient.\(^8\) Finally, utility is discounted at a rate \(r > 0\) and to bargaining, suggesting that \(\lambda\) may have decreased over time.

\(^4\)The way we model “asserting her will” parallels Lee and Liu (2013). They also assume the setter has two types and a powerful setter has higher probability of successfully extracting higher outside payment in the event of no disagreement. Lee and Liu (2013) focus on one-sided learning, whereas we focus on two-sided learning in a simpler setting.

\(^5\)The results of this paper are robust to the case in which \(u_S(x_0)\) and \(u_R(x_s)\) are not too high. If either is sufficiently high, then the players may agree on their worst outcome to reduce the bargaining delay.

\(^6\)This model assumes the status quo as the responder’s best outcome. The 13th amendment example in the introduction fits this assumption, as Lincoln attempted to assert his will to change the status quo. Without this assumption, the equilibrium outcome is straightforward Since the setter’s proposal power dominates and her perceived personal power does not affect the outcome. For example, if the status quo is on the right of \(x_s\), then it is easy to see that the setter always offers \(x_s\), and the responder always accepts it immediately.

\(^7\)Note that commonly used utility functions satisfy these minimal assumptions. For example \(u_i(x) = -(x - x_i)^2\) with \(x \in \mathbb{R}, x_0 < x_s < x_t, x_R = x_0, x_c \in (x_0, x_s)\).

\(^8\)The necessary and sufficient condition for compromise to be always efficient is \(u_c + v_c > \max((\lambda \bar{u} + \xi \bar{v})/(r + 8)\)
players maximize discounted sums of payoffs from all issues bargained over.

**Learning** If the responder accepts the setter’s offer, or if the setter decides not to assert her will, then there is no learning about the setter’s type and beliefs are unchanged. If the responder rejects the offer and the setter attempts to assert her will, the belief changes depending on the outcome of the attempt. If the setter succeeds in asserting her will, then her perceived power jumps to one and there is no learning thereafter. In other words, “good” news is conclusive. As the setter attempts to assert her will without success, then players become more pessimistic about the setter’s power. Formally, while the setter is asserting her will (and there is gridlock), the belief that the setter is powerful changes on the time interval \([t, t+dt]\) via Bayes’ rule:

\[
p_{t+dt} = \frac{p_t(1 - \lambda dt)}{p_t(1 - \lambda dt) + (1 - p_t)}.
\]

Simplifying (1) implies that the power dynamics during gridlock follows \(dp = -\lambda p(1 - p)dt\).

**Markov Strategies** We restrict attention to pure Markovian strategies where the state of the game at period \(t\) is given by the setter’s power \(p_t\). Denote an offer strategy for the setter as \(\chi : [0, 1] \rightarrow X\) which maps a power level \(p_t\) into a proposal in \(X\). An acceptance strategy for the responder is a correspondence \(A : [0, 1] \Rightarrow X\) that gives the set of proposals which the responder will accept given the state \(p_t\). Finally the setter’s strategies of whether or not to “assert his will” is given by \(\beta : [0, 1] \times X \rightarrow \{0, 1\}\), where 1 indicates that the decision to to assert his willing against opposition.

To ensure that strategies are well-defined in the continuous-time setting, we assume admissibility in the sense of Klein and Rady (2011). We further restrict attention to strategies such that the responder accepts proposals when indifferent. This ensures that the equilibrium

\[\xi + \lambda \xi / (r + \xi)\]. This is because of the loss due to delay when there is gridlock.

\[\text{9The strategy profile } \{\chi, A, \beta\} \text{ is admissible if there exists at least one well-defined solution to the corresponding law of motion for posterior beliefs. This is the case if and only if for each initial belief } p_0, \text{ there is a function } t \mapsto p_t \text{ on } [0, \infty) \text{ that satisfies}
\]

\[
p_t = \frac{p_0 e^{-\lambda t} \int_{t_0}^t I_{(p_t, \chi(p_t), A(p_t, \chi(p_t)))} d\tau}{p_0 e^{-\lambda t} \int_{t_0}^t I_{(p_t, \chi(p_t), A(p_t, \chi(p_t)))} d\tau + 1 - p_0}.
\]

As we shall see, admissability condition in our model requires that all belief intervals in which the responder accepts the setter’s proposal must be right-closed.

\[\text{10The is a common assumption in the dynamic bargaining literature, for example, Baron and Ferejohn (1989) or Bowen et al. (2014).}]}
of the continuous-time game is the limit of an equilibrium of a corresponding discrete-time game. These restrictions together imply that equilibrium acceptance sets $A(p)$ will be closed. We consider Markov perfect equilibria which are subgame perfect equilibria in which players use Markov strategies. We refer to a Markov perfect equilibrium with the above restrictions as simply an equilibrium.

3 Benchmark outcomes

Before proceeding to the equilibrium analysis we consider two benchmark outcomes and corresponding dynamic payoffs. It is yet to be determined that these outcomes occur in equilibrium. For now we construct the payoffs and later in Section 4 show that these are equilibrium payoffs for some values of $p$.

3.1 Long-run gridlock

First consider the case in which there is long-run gridlock. In this outcome, the setter proposes her ideal $x_s$, at each instant, this is rejected by the responder, and the setter then chooses to assert her will. Gridlock induces learning in this outcome. If the setter is successful, then both players know the setter is powerful and learning ceases, and the setter continues to assert her will. We denote by $V_{i,g}(p)$ the value function under the long-run gridlock for player $i \in \{R, S\}$.

Suppose first that the setter is known to be powerful, i.e., $p = 1$. Fix a time interval $[t, t + dt)$ with a small $dt > 0$. If gridlock occurs in the time interval $[t, t + dt)$, the setter succeeds in asserting her will with probability $\lambda dt$, in which case the setter and the responder obtain payoffs $\bar{u}$ and 0, respectively, and the players wait for the next issue. When players are simply waiting for a new issue we say that the previous issue is resolved. If the setter is not successful, then with probability $\xi dt$ the issue is replaced with a new one, in which case $S$ and $R$ obtain payoffs of 0 and $\bar{v}$, respectively. Therefore,

$$V_{S,g}(1) = \lambda dt (\bar{u} + \hat{V}_{S,g}(1)) + (1 - \lambda dt)e^{-rdt}V_{S,g}(1),$$
$$V_{R,g}(1) = \lambda dt \hat{V}_{R,g}(1) + (1 - \lambda dt)(\xi dt \bar{v} + e^{-rdt}V_{R,g}(1)),$$

where $\hat{V}_{i,g}(1)$ is player $i$’s value function when the previous issue has been resolved and the players wait for a new issue. To derive the value of $\hat{V}_{i,g}(p)$ for any $p \in [0, 1]$, note that a new
issue arrives with probability $\xi dt$ in the time interval $[t, t + dt)$. Therefore, for $i = S, R,$

$$\hat{V}_{i,g}(p) = e^{-r dt}(\xi dt V_{i,g}(p) + (1 - \xi dt)\hat{V}_{i,g}(p)).$$

Using $1 - r dt \approx e^{-r dt},$ dropping the higher orders of $dt$ and simplifying yields $\hat{V}_{i,g}(p) = \xi V_{i,g}(p)/(r + \xi).$ Using this, we derive the value of $V_{i,g}(1),$ which are given by

$$V_{S,g}(1) = \frac{\lambda(r + \xi)}{r(r + \lambda + \xi)} \bar{u}, \quad V_{R,g}(1) = \frac{\xi(r + \xi)}{r(r + \lambda + \xi)} \bar{v}.$$

When the setter is powerless (i.e., $p = 0),$ she never succeeds in asserting her will (that is, $\lambda = 0$ in the above equations). Therefore, the value functions are given by

$$V_{S,g}(0) = 0, \quad V_{R,g}(0) = \frac{\xi}{r} \bar{v}.$$

Now consider any intermediate level of power $p \in (0, 1).$ Observe that player $i$’s value function $V_{i,g}(p)$ is a convex combination of $V_{i,g}(1)$ and $V_{i,g}(0).$ To understand this, note that the players’ never change their actions in the future, regardless of the outcome – $S$ always proposes $x_s$ which $R$ rejects. Therefore, $\hat{V}_{i,c}$ is player $i$’s value function when the players wait for a new issue. The same argument used to derive $\hat{V}_{i,g}$ shows that $\hat{V}_{i,c} = \xi V_{i,c}/(r + \xi).$ Therefore, the above equations simplify to

$$V_{S,c} = \frac{r + \xi}{r} u_c, \quad V_{R,c} = \frac{r + \xi}{r} v_c. \quad (5)$$

### 3.2 Long-run compromise

The second important benchmark is where players compromise for all future issues. In this case, the setter offers the compromise $x_c,$ and the responder immediately accepts the offer. No learning occurs, and hence the payoffs do not depend on power $p.$ Let $V_{i,c}$ be player $i$’s continuation utility under the long-run compromise. Then the value functions can be written as

$$V_{S,c} = u_c + e^{-r dt}(\xi dt V_{S,c} + (1 - \xi dt)\hat{V}_{S,c}),$$

$$V_{R,c} = v_c + e^{-r dt}(\xi dt V_{R,c} + (1 - \xi dt)\hat{V}_{R,c}),$$

where $\hat{V}_{i,c}$ is player $i$’s value function when the players wait for a new issue. The same argument used to derive $\hat{V}_{i,g}$ shows that $\hat{V}_{i,c} = \xi V_{i,c}/(r + \xi).$ Therefore, the above equations simplify to

$$V_{S,c} = \frac{r + \xi}{r} u_c, \quad V_{R,c} = \frac{r + \xi}{r} v_c.$$
4 Bargaining with Easy Issues

In this section, we analyze equilibria for the model in which all policy issues have three possible outcomes, i.e., $X = \{x_0, x_c, x_s\}$. We refer to these as easy issues as players can compromise on the outcome $x_c$. Later, we introduce difficult issues in which only the two extreme outcomes are feasible.

The next result, Proposition 1, describes a Markov perfect equilibrium of the game. We first explain the players’ behavior and equilibrium dynamics under the profile in Proposition 1. Then we establish the uniqueness of equilibrium under a mild criterion.

Define the two threshold beliefs

$$p = \frac{r + \lambda + \xi}{\lambda} \left(1 - \frac{r + \xi v_c}{\xi}\right),$$

$$\bar{p} = \frac{r + \lambda + \xi}{\lambda} \frac{ru_c}{(r + \lambda)\bar{u} - (r + \lambda + \xi)u_c}.$$

Proposition 1

1. If

$$\frac{v_c}{\bar{v}} \geq \max \left\{ \frac{\xi}{r + \xi} \left(1 - \frac{u_c}{\bar{u}}\right), \frac{\xi}{r + \lambda + \xi} \right\},$$

then there exists an equilibrium in which compromise occurs in an interval of beliefs:

$$\chi(p) = \begin{cases} x_c & \text{if } p \in [p, \bar{p}] \\ x_s & \text{otherwise,} \end{cases} \quad A(p) = \begin{cases} \{x_0, x_c\} & \text{if } p \geq \bar{p} \\ \{x_0\} & \text{if } p < p. \end{cases}$$

2. Otherwise, there exists an equilibrium in which no compromise occurs:

$$\chi(p) = x_s \text{ for any } p \in [0, 1], \quad A(p) = \begin{cases} \{x_0, x_c\} & \text{if } p \geq \bar{p} \\ \{x_0\} & \text{if } p < p. \end{cases}$$

Moreover, the setter always asserts her will after a rejection, i.e., $\beta(p, x) = 1$ for all $p \in [0, 1]$ and $x \in X$.

Proposition 1 states that if compromise occurs in equilibrium (i.e., if (8) holds), then it occurs when the perceived power is within the interval $[p, \bar{p}]$. In other words, compromise occurs when the players have a moderate perception of the agenda-setter’s power. If the setter
is perceived to be too powerful, then the setter prefers to take the risk and induce gridlock, believing that he would succeed in asserting her will. On the contrary, when the setter is too weak, the responder refuses to compromise, as he finds gridlock more beneficial.

Gridlock induces learning on the setter’s power, leading to various equilibrium dynamics depending on the setter’s initial power. If the setter is initially very powerful \( p_0 > \bar{p} \), then gridlock occurs in the beginning of the game. If the setter succeeds in asserting her will, then the power jumps to one and the setter continues to induce gridlock. If the setter cannot assert her will, however, then the power gradually drifts down, and when \( p \) reaches \( \bar{p} \) the setter offers \( x_c \) to make a compromise. If the setter’s power is moderate \( p_0 \in [\bar{p}, \bar{P}] \), then the players compromise for all future issues and no learning occurs. For an initially weak setter \( p_0 < \bar{p} \), the power either jumps to one or gradually drifts down, resulting in permanent gridlock.

The complete proof of Proposition 1 is provided in the Appendix. Here we provide a heuristic argument to derive the lower and upper bound of the compromise set. First, the value of \( \bar{p} \) is derived from the responder’s incentives. Note that if \( p \in (0, 1) \), then the responder is indifferent between long-run compromise and long-run gridlock at \( p = \bar{p} \). A single deviation to reject \( x_c \) would lead to permanent gridlock. Therefore, it must be that \( V_{R,c} = V_{R,g}(\bar{p}) \). From (4) and (5), it follows that the responder is indifferent when (6) holds.

Note that \( p \) is less than 1 if and only if \( v_c/\bar{v} \geq \xi/(r+\lambda+\xi) \), which is necessary for compromise to occur in equilibrium. In order for compromise to occur at \( p = \bar{p} \), the setter must also prefer to compromise (by offering \( x_c \) ) than inducing long-run gridlock (by offering \( x_s \)) at \( p = \bar{p} \). This condition is given by

\[
V_{S,c} \geq V_{S,g}(\bar{p}) \iff \frac{v_c}{\bar{v}} \geq \frac{\xi}{r+\xi} \left( 1 - \frac{u_c}{\bar{u}} \right).
\]

The above arguments imply that compromise occurs at \( p = \bar{p} \in [0, 1] \) if and only if (8) holds.

The setter’s incentive determines the upper bound \( p \) of the compromise set. Specifically, if the compromise region is a nondegenerate interval (i.e., \( \bar{p} > \bar{P} \) and \( \bar{p} < 1 \), then the setter is indifferent between gridlock and compromise at \( p = \bar{p} \). Therefore, the value matching condition (Dixit (2002)) gives

\[
\frac{r+\xi}{r}u_c = \lambda\bar{p}dt(\bar{u} + \hat{V}_{S,g}(1)) + (1 - \lambda\bar{p}dt)e^{-rdt}\frac{r+\xi}{r}u_c,
\]

which can be simplified to (7). Note that \( \bar{p} \) is greater than one if and only if \( u_c/\bar{u} \geq \lambda/(r+\lambda+\xi) \). In this case, the setter prefers to compromise for all \( p > \bar{p} \).
Figure 2 illustrates the parameters such that the equilibrium in Proposition 1 admits compromise. The green region represents the set of parameters such that (8) holds, and thus the equilibrium in this region features compromise at some \( p \in [0, 1] \).

![Diagram](image)

**Figure 2**: An illustration of the equilibrium described in Proposition 1. For parameters in colored regions, compromise occurs for some power level. Labels in each colored region indicate the locations of the compromise set.

It is straightforward to see that the compromise outcome occurs for a broader range of power as the players’ payoff from compromise increases. For the responder, we check from (6) that \( \underline{p} \) is decreasing in \( v_c/\bar{v} \). If \( v_c/\bar{v} > \xi/(r + \xi) \), then \( \underline{p} = 0 \) and the compromise outcome occurs for all low \( p \). Similarly, (7) implies that \( \overline{p} \) is increasing in \( u_c/\bar{u} \), and \( \overline{p} = 1 \) if \( u_c/\bar{u} > \lambda/(r + \lambda + \xi) \), leading to compromise for all high \( p \).

**Uniqueness**  We next show that for any fixed parameter values, the equilibrium described in Proposition 1 is the unique Markov perfect equilibrium under two mild conditions. Let \( \mathcal{A} = \{ p \in [0, 1] | \chi(p) \in A(p) \} \) be the set of beliefs in which agreement is reached in equilibrium. We call \( \mathcal{A} \) a agreement set of a Markovian profile. Observe that for the equilibrium in Proposition 1, the agreement set is either empty or \( [\underline{p}, \overline{p}] \) depending on the parameter values.
Condition 1 $\mathcal{A}$ is closed.

We argue that the closedness of the agreement set is a reasonable equilibrium refinement criterion. First, observe that the admissibility condition on the Markovian profile (footnote 9) requires that all intervals in $\mathcal{A}$ be right-closed. Second, the left-closedness of $\mathcal{A}$ imposes an important incentive constraint. To understand this, consider a profile with $\mathcal{A} = [p_1, p_2]$ for some $p_1$ and $p_2$. Observe that if an instant of (off-the-equilibrium-path) gridlock occurs at $p \in \mathcal{A}$, the resulting dynamics differs depending on whether $p = p_1$ or $p \in (p_1, p_2]$. If $p \in (p_1, p_2]$, the belief never falls below $p_1$ after an instant of gridlock; if $p = p_1$, however, an instant of gridlock makes the belief drift out of the agreement set. Especially, the latter dynamics create different incentives for bargainers at $p = p_1$, since the bargainers now consider “the fear of falling out” of the agreement set. By imposing left-closedness for all intervals in $\mathcal{A}$, we can ensure that the players have such an incentive condition in a game.\footnote{Although we do not explicitly model a discrete-time version of our bargaining game, we argue that any profile with left-open intervals in $\mathcal{A}$ cannot be the limit of equilibrium in the corresponding discrete-time game. Consider a profile with $\mathcal{A} = (p_1, p_2]$. Then for any small discrete-time interval, if the belief is sufficiently close to $p_1$, gridlock for one ‘period’ leads the belief to jump below $p_1$. Therefore, the bargainers’ incentives differ from those in the continuous-time counterpart.} 

Condition 2 For any $p \notin \mathcal{A}$, the setter proposes her most preferred position in $X \setminus \mathcal{A}(p)$.

To see why Condition 2 is mild, recall that the outcome after the responder’s rejection is independent of the setter’s original proposal. Therefore, Condition 2 affects only on the strategies that does not influence the equilibrium outcome.

Proposition 2 All equilibria that satisfies Condition 1 are payoff equivalent. Furthermore, there exists a unique equilibrium that satisfies Conditions 1 and 2.

Comparative Statics We next explore how the compromise interval changes with parameter values. Given the simple expressions for the bounds on power for compromise to occur, we can do straightforward comparative statics.

Proposition 3 Suppose that $\underbrace{\overbrace{\text{Proposition 1}}}_{\text{Proposition 1}}$, 

1. \( \bar{p} \) increases in \( r \); \( p \) decreases in \( r \);

2. both \( \bar{p} \) and \( p \) decrease in \( \lambda \); \( \bar{p} - p \) decreases in \( \lambda \);

3. both \( \bar{p} \) and \( p \) increase in \( \xi \).

Recall that \( \bar{p} \) and \( p \) are the beliefs at which the setter and responder, respectively, are indifferent between compromise and conflict. Increasing the discount rate \( r \) increases the costs of delay. This means that both players are more willing to compromise, which in turn implies that \( \bar{p} \) increases and \( p \) decreases. Of course, this also implies that raising \( r \) increases the size of the set of beliefs, \( \bar{p} - p \), at which compromise can occur.

Increasing \( \lambda \) means that the setter’s position power is increased. Hence, the setter is less willing to compromise, i.e., \( \bar{p} \) decreases. At the same time, the responder is more willing to compromise, which means that \( p \) also decreases. Overall, however, compromise can occur for a smaller set of beliefs, as measured by \( \bar{p} - p \) when \( \lambda \) is increased.

Finally, recall that \( \xi \) is that the rate at which a new issue arises, or the velocity of the institution. If the current issue has not been resolved, then the status quo is retained. Thus, raising \( \xi \) benefits the responder versus the setter. As a consequence, the responder is less willing to compromise, thereby raising \( p \), while the setter is more willing to compromise, which raises \( \bar{p} \). The size of the agreement set \( \bar{p} - p \) itself is in general not monotone.

**Value of information**  To provide further intuition for the result, we use the value functions to determine the value of information for any level of power \( p \). The value of information is simply the expected informational benefit from gridlock.\(^{12}\) For the setter, the benefit is in learning that she is powerful, and for the responder, the benefit is learning that the setter is powerless. The value of information for player \( i = S, R \) in the interval \( [t, t + dt) \) is given by

\[
VI_i(p) = \lambda p (V_i(1) - V_i(p)) - V_i'(p) \lambda p (1 - p),
\]

where \( V_i(p) \) is player \( i \)'s value function. The first term in the expression is the expected benefit from learning that the setter is powerful for certain. The second term is the loss to the setter if no success occurs, or the benefit to the responder if no success occurs. Figure 3 graphs the value of information and the value functions for a parametric example. We describe these below for each interval of \( p \).

\(^{12}\)This is similar to the value of playing the risky alternative in Keller et al. (2005).
For $p < \underline{p}$ we have $V_i(p) = V_{i0}(p)$ from equations (3) and (4). Simplifying (9) shows that the value of information is zero as illustrated in Figure 3 right panel. The reason is that information will not change players’ strategies, as gridlock will ensue even if there is new information. If $p \in [\underline{p}, \overline{p}]$, then the long-run compromise arises. Thus, $V_i(p) = V_{iC}$ from (5), which implies that $V_i'(p) = 0$. For the responder $V_R(1) < V_R(p)$ if and only if $v_c/\bar{v} > \xi/(r + \lambda + \xi)$. For the setter, $V_S(1) > V_S(p)$ if and only if $u_c/\bar{u} < \lambda/(r + \lambda + \xi)$, which is the condition under which $\overline{p} < 1$. In this case the value of information is strictly positive and strictly increasing for the setter, and strictly negative and strictly decreasing for the responder in the compromise region. This reflects the fact that as $p$ gets smaller, the setter’s value of experimenting is decreasing, as there is less the setter is able to do. From the responder’s perspective, her benefit from information is increasing as beliefs drift down, because she approaches the region of beliefs such that she rejects the compromise.

In the Appendix, Section 7.1 we show that if $p > \overline{p}$ then the setter’s and responder’s value functions are convex as illustrated in Figure 3, left panel. If we start with some belief $p > \overline{p}$ at time $t$ and learning occurs, then the belief at $t + dt$ remains above $\overline{p}$. By (9), the strict convexity of $V_i(p)$ implies that the value of information is strictly positive for both players. That is, both players have the ability to modify actions based on information arrival. Note that the setter’s value of information is continuous at $\overline{p}$, but the responder’s is discontinuous at that point. The reason is that it is the setter who chooses to switch from proposing gridlock, to compromise.
When the setter does this the responder’s value of information becomes negative—information can only hurt the responder by causing the setter to revert to gridlock.

5 Difficult issues

The previous analysis showed how power dynamics can result in long-run compromise outcomes, but it is often the case in practice that a pre-existing compromise is disrupted. In this section, we explore one possibility why this might occur. In the organizational or political bargaining process, an issue without a compromise option often arises. For example, the Affordable Care Act was a difficult issue for which compromise was not possible. For the speaker Nancy Pelosi, passage was a signature moment demonstrating her personal power. Regarding bargaining with these “all-or-nothing” policy issues, we ask the following questions: Would any bargaining party ever concede to the opponent’s extreme position? Do the outcomes from the all-or-nothing issues affect the outcomes in the normal issues? We answer both questions in the affirmative.

To model such an environment, we modify our main model and assume that the players may have a difficult policy issue with no compromise alternative. Assume that a difficult issue arises with probability $\alpha > 0$. With the complementary probability, the easy policy issue (with three possible outcomes) arises. Formally, let $\tau \in \{e, d\}$ be the type of a policy issue. Let $X^e = \{x_0, x_c, x_s\}$ and $X^d = \{x_0, x_s\}$ be the set of possible outcomes under easy issues and difficult issues, respectively. Let $\chi^\tau$ be the offer strategy for the setter if the current issue is of type $\tau$. Similarly, let $A^\tau$ and $\beta^\tau$ be the acceptance strategy for the responder and the setter’s strategy to assert her will, respectively.

In this section, we focus on the parameter range where

\[
\frac{\xi}{r + \lambda + \xi} < \frac{v_c}{\bar{v}} < \frac{\xi}{r + \xi}, \quad \text{and} \quad \frac{u_c}{\bar{u}} < \frac{\lambda}{r + \lambda + \xi}.
\]

The condition on $v_c/\bar{v}$ guarantees that the value of $p$ defined in Proposition 1 is in the interior of $[0, 1]$, and the condition on $u_c/\bar{u}$ ensures that gridlock occurs at $p = 1$ regardless of the issue type. In the equilibria constructed in this section, the players’ behavior under the easy policy issue remains qualitatively the same.

We construct two types of equilibria exhibiting qualitatively different behavior under the difficult issues. First, there exists an equilibrium in which players never concede under difficult
issues. We call this equilibrium a gridlock equilibrium. Second, there exists an equilibrium in which the setter concedes by offering \(x_0\) to avoid a long-run gridlock. We call this an avoiding-the-issue equilibrium.

The next proposition formally states the gridlock equilibrium and the parametric conditions for its existence. The proofs for the next two propositions are in the Online Appendix.

**Proposition 4 (gridlock equilibrium)** Suppose that \((10)\) holds, and

\[
\frac{\xi}{r + \xi} \left(1 - \frac{u_c}{\bar{u}}\right) \leq \frac{v_c}{\bar{v}} \leq \frac{\xi}{r + \xi} \left(1 - \frac{\xi(1 - \alpha) u_c}{r + \xi \bar{u}}\right). \tag{11}
\]

Then for some \(\bar{p}_a \in [p, 1]\), there exists an equilibrium in which

\[
\chi^e(p) = \begin{cases} 
  x_c & \text{if } p \in [p, \bar{p}_a), \\
  x_s & \text{otherwise,}
\end{cases} \quad A^e(p) = \begin{cases} 
  \{x_0, x_c\} & \text{if } p \geq \bar{p}_a, \\
  \{x_0\} & \text{if } p < \bar{p}_a,
\end{cases}
\]

\[
\chi^d(p) = x_s \text{ for all } p, \quad A^d(p) = \{x_0\} \text{ for all } p.
\]

Moreover, the setter always asserts her will after a rejection, i.e., \(\beta^\tau(p, x) = 1\) for all \(\tau = e, d\), \(p \in [0, 1]\) and \(x \in X\).

In the gridlock equilibrium, the equilibrium behavior eventually features permanent gridlock. Even when the prior belief sits in the intermediate range so that compromise is initially maintained on easy issues, difficult issues arise and gridlock over those issues leads to learning. Learning makes the belief either jump to one or drift below \(p\), and thus the belief moves out of the compromise region. There is permanent gridlock thereafter in equilibrium.

This helps to explain why gridlock can arise on seemingly easy, non-contentious issues, for which there was no gridlock before. Learning over gridlock for the difficult issues leads to a more extreme level of perceived personal power, which in turn provides either the setter or the responder greater incentives to reject compromise proposals.

The next proposition describes another type of equilibrium in which the setter concedes to avoid future gridlock.

**Proposition 5 (avoiding-the-issue equilibrium)** Suppose that \((10)\) holds, and

\[
\frac{v_c}{\bar{v}} \geq \frac{\xi(1 - \alpha)}{r + \xi(1 - \alpha)} \left(1 - \frac{\xi}{r + \xi \bar{u}} \frac{u_c}{\bar{u}}\right). \tag{12}
\]
Then there exists $p_0$ and $\hat{p}$ with $p_0 \leq \hat{p} \leq p$, such that for any $p_{\alpha} \in [p_0, \hat{p}]$, there is an equilibrium in which

$$
\chi^e(p) = \begin{cases} 
  x_c & \text{if } p \in [p_{\alpha}, p_{\alpha}], \\
  x_s & \text{otherwise},
\end{cases} \\
A^e(p) = \begin{cases} 
  \{x_0, x_c\} & \text{if } p \geq p_{\alpha}, \\
  \{x_0\} & \text{if } p < p_{\alpha},
\end{cases}
$$

$$
\chi^d(p) = \begin{cases} 
  x_0 & \text{if } p = p_{\alpha}, \\
  x_s & \text{otherwise},
\end{cases} \\
A^d(p) = \{x_0\} \text{ for all } p.
$$

Moreover, the setter always asserts her will after a rejection, i.e., $\beta^{\tau}(p, x) = 1$ for all $\tau = e, d$, $p \in [0, 1]$ and $x \in X$.$^{13}$

In the avoiding-the-issue equilibrium, gridlock ensues on difficult issues until the belief reaches the lower bound of the compromise region for the easy issues ($p = p_{\alpha}$). At the lower bound, the setter concedes to the responder by proposing the status quo. After that, the players immediately reach agreements for every issue, with outcomes of $x_c$ and $x_0$ for the easy and difficult type, respectively, and the belief stays at $p_{\alpha}$. The setter has an incentive to concede because doing so would avoid further learning, which leads to a gridlock even for easy issues.

Figure 4 describes the parameter regions under which each type of equilibrium exists. In general, the gridlock equilibrium (shaded region) exists for the lower values of $v_c/\bar{v}$ compared to the avoiding-the-issue equilibrium (yellow region). Intuitively, higher values of the compromise payoff gives the setter a stronger incentive to concede for the current difficult issue to avoid a long-run gridlock.

Note that for $v_c/\bar{v} \geq \xi/(r + \xi)$ there exists a trivial equilibrium in which for easy issues there is gridlock for $p \geq \bar{p}$ and compromise otherwise. In this equilibrium, there is always gridlock for difficult issues. Combining with Propositions 4 and 5, the condition to sustain an equilibrium with the possibility of compromise when there are difficult issues is

$$
v_c/\bar{v} \geq \max \left\{ \min \left\{ \frac{\xi}{r + \xi}, \frac{\xi(1 - \alpha)}{r + \xi(1 - \alpha)} \right\}, \frac{\xi}{r + \lambda + \xi} \right\}. \tag{13}
$$

Comparing condition (13) to condition (8) in Proposition 1, it is straightforward to see that the set of parameters that admits a possibility of compromise with difficult issues, includes those

$^{13}$There exists at least one other equilibrium that yields the same equilibrium outcome described in Proposition 5. In this equilibrium, the setter offers her most preferred outcome under a difficult issue when the belief hits the lower bound of the agreement set ($\chi^d(p_{\alpha}) = x_s$) but chooses not to assert her will after a rejection ($\beta^d(p_{\alpha}, x_s) = 0$).
Figure 4: Parameter range for the equilibrium in Propositions 4 and 5 when $\alpha = 0.5$. The shaded region (resp. yellow region) represents the parameter space in which a gridlock equilibrium (resp. an avoiding-the-issue equilibrium) exists. The red dotted triangle depicts the parameter region where compromise never occurs when there is no difficult issue.

parameters that admit compromise when there are no difficult issues. Indeed, the additional parametric region is precisely those indicated with red dots in Figure 4.

We thus conclude that by bundling difficult and easy issues, compromise on easy issues becomes possible in more environments. Moreover, whereas difficult issues alone would result in perpetual gridlock, agreement on the status quo is possible when difficult issues are combined with easy issues. This results from the unwillingness of the setter to yield power that can be valuable when easy issues arise.

6 Concluding Remarks

In this paper, we provide a model that predicts the dynamics of bargaining when the agenda-setter power evolves endogenously. We show that agents will compromise when the setter is neither too powerful nor too powerless. A powerful setter is never willing to offer a compromise, while the responder is never willing to compromise when the setter is too powerless. The
incentive constraints of both the setter and responder determine an intermediate range of beliefs such that compromise occurs. In this interval, the setter believes she is too powerless to assert her will successfully, and the responder prefers to compromise rather than learn about the setter’s type.

We seek to understand how the evolution of power may explain puzzling bargaining outcomes. That is, observing gridlock on issues that may have previously been a relatively easy issue to settle. We show that when difficult issues arise that force the agents into gridlock, these issues also forces learning about the setter’s strength. If the setter learns either that she is powerful with certainty, or becomes too powerless, then gridlock ensues on every issue. On the other hand, we also show the existence of an equilibrium where the setter accommodates the responder on difficult issues when the belief about her type is too low. This avoids learning and allows compromise to be sustained for easy issues. We think this helps explain instances where the setter “avoids the issue”.

We believe that our model is a first step in analyzing the effect of personal power in political decision-making processes, and that there are several interesting future research directions. First, one can consider a model with endogenous sequence and timing of easy and difficult issues. If either the setter or the responder is allowed to choose the issue sequence, their choice would be certainly affected by its effect on learning.\(^{14}\) Second, it is possible that the policy issues have various degrees of importance, and it may affect the players’ incentive to create gridlock and learn about personal power. Thus, it would be fruitful to analyze a case in which issues have heterogeneous payoffs. Third, while a perfect good news model is a natural representation of the evolution of personal power, one can consider a model with generalized information structures, in which gridlock may generate either a good or bad news. Finally, the model lends itself naturally to empirical or experimental tests. One can test if compromise occurs more often with higher position power than lower position power, or if compromise occurs more often when difficult issues are bundled with easy issues as opposed to in isolation.

\textbf{References}


\(^{14}\)Some of these questions are explored in a companion paper (Hwang and Krasa, 2020).


7 Appendix

7.1 Proof of Proposition 1

In this proof, we show that there exists an equilibrium described in Proposition 1. Define  and  as

\[
p = \begin{cases} 
\frac{r + \lambda + \xi}{\lambda} \left(1 - \frac{r + \xi}{\xi} \frac{v_c}{\bar{v}}\right) & \text{if } \frac{\bar{u}}{\bar{v}} < \frac{\xi}{r+\xi} \text{ and } \frac{u}{\bar{u}} < \frac{1}{r + \lambda + \xi}, \\
\frac{r + \lambda - v_c}{\bar{v} - v_c} & \text{if } \frac{\bar{u}}{\bar{v}} < \frac{\xi}{r+\xi} \text{ and } \frac{u}{\bar{u}} \geq \frac{1}{r + \lambda + \xi}, \\
0 & \text{if } \frac{\bar{u}}{\bar{v}} \geq \frac{\xi}{r+\xi},
\end{cases}
\]

(14)

and \( \bar{p} = \max(\hat{p}, p) \), where

\[
\hat{p} = \begin{cases} 
\frac{ru_c}{\lambda} - (r + \lambda)\bar{u} & \text{if } \frac{u}{\bar{u}} < \frac{\lambda}{r + \lambda + \xi}, \\
1 & \text{if } \frac{u}{\bar{u}} \geq \frac{\lambda}{r + \lambda + \xi}.
\end{cases}
\]

(15)

Note that these are precisely the values calculated in (6) and (7) when \( \frac{\bar{u}}{\bar{v}} \in (\frac{\xi}{r+\lambda+\xi}, \frac{\xi}{r+\xi}) \) and \( u_c/\bar{u} < \lambda/(r + \lambda + \xi) \). Recall that \( \mathcal{A} = \{ p \in [0, 1] : \chi(p) \in A(p) \} \) is the agreement set of a Markovian profile. Then under the strategy profile described in Proposition 1 we have \( \mathcal{A} \) for the following cases:

- **Case A:** condition (8) holds and \( \frac{u}{\bar{u}} < \frac{\lambda}{r + \lambda + \xi} \). In this case, \( \mathcal{A} = [p, \bar{p}] \) where \( \bar{p} < 1 \).
- **Case B:** condition (8) holds and \( \frac{u}{\bar{u}} \geq \frac{\lambda}{r + \lambda + \xi} \). In this case, \( \mathcal{A} = [p, 1] \).
- **Case C:** condition (8) does not hold. In this case, \( \mathcal{A} = \emptyset \).

In what follows, we first derive the players’ value functions under the conjectured strategy profile in each case. Then, we complete the proof by verifying each player’s incentive conditions.

**Value Functions**  **Case A:** First consider the profile in Case A, with \( \mathcal{A} = [p, \bar{p}] \) where \( \bar{p} < 1 \). Observe that the belief dynamics imply that whenever \( p < \bar{p} \), the players will never reach a compromise in the future and long-run gridlock occurs. Thus, the players’ value functions are \( V_i(p) = V_{i,\emptyset}(p) \) for any \( p < \bar{p} \). Moreover, for any \( p \in [\bar{p}, \bar{p}] \), the players always compromise at the moment a new issue arrives, and thus \( V_i(p) = V_{i,c}(p) \), where \( V_{i,c}(p) \) is the player \( i \)'s payoff under no gridlock. Therefore, it remains to derive the value functions for \( p > \bar{p} \).

First, consider the setter’s value function \( V_S(p) \) for \( p > \bar{p} \). In the interval \([t, t + dt]\), the probability that the setter successfully asserts is given by \( \lambda dt \). In this case, the issue is resolved and the setter receives utility \( \bar{u} \). In addition, the setter’s type is now known to be high, i.e., the belief jumps up to 1. With the complementary probability, the issue is not resolved in \([t, t + dt]\),
and the belief drifts down to $p + dp$ following Bayes’ rule given by (1). Therefore, $V_S(p)$ is determined recursively by the equation

$$V_S(p) = p\lambda dt \left(\bar{u} + e^{-rdt}(\xi dt V_S(1) + (1 - \xi dt)\hat{V}_S(1))\right) + (1 - p\lambda dt)e^{-rdt}V_S(p + dp).$$

As in Section 3, $\hat{V}_i(p)$ is the dynamic payoff for player $i$ after an issue has been resolved and players await a new issue. By the same arguments, it follows that

$$\hat{V}_i(p) = \frac{\xi}{r + \xi} V_i(p),$$

for any $p \in [0, 1]$.

Note that $V_S(p + dp) = V_S(p) + V_S'(p)dp$, $e^{-rdt} \approx 1 - rdt$, and the belief dynamics in (1) simplifies to $dp = -\lambda p(1 - p)dt$. Therefore,

$$(1 - (1 - p\lambda dt)(1 - rdt))V_S(p) = p\lambda dt \left(\bar{u} + (1 - rdt)(\xi dt V_S(1) + (1 - \xi dt)\hat{V}_S(1))\right) - (1 - p\lambda dt)(1 - rdt)V_S'(p)\lambda p(1 - p)dt.$$ 

Note that we use $1 - rdt \approx e^{-rdt}$. Dropping terms with order of $dt^2$ or higher, and substituting the value of $\hat{V}_S(1)$ yields the differential equation

$$(r + p\lambda)V_S(p) = p\lambda(1 + \frac{\xi\lambda}{r(r + \lambda + \xi)}) - \lambda p(1 - p)V_S'(p).$$

Solving the differential equation we get

$$V_S(p) = \frac{(1 - p)^{1+\frac{r}{\lambda}}}{p^\frac{r}{\lambda}} K_S + V_{S,\gamma}(p),$$

(16)

where $K_S$ is the constant of integration. The boundary condition is given by $V_S(p) = V_{S,c}$, which is the value matching condition at $p = \overline{p}$.

We now determine the responder’s value function $V_R(p)$ for $p > \overline{p}$. Using a similar argument as for the setter, the responder’s value function is defined recursively as follows:

$$V_R(p) = p\lambda dt e^{-rdt}(\xi dt V_R(1) + (1 - \xi dt)\hat{V}_R(1)) + (1 - p\lambda dt)(\xi dt \bar{v} + e^{-rdt}V_R(p + dp)).$$

Simplifying the above equation yields the following differential equation

$$(r + p\lambda)V_R(p) = \frac{p\lambda \xi^2 \bar{v}}{r(r + \lambda + \xi)} + \xi \bar{v} - \lambda p(1 - p)V_R'(p).$$

Solving the differential equation yields

$$V_R(p) = \frac{(1 - p)^{1+\frac{r}{\lambda}}}{p^\frac{r}{\lambda}} K_R + V_{R,\gamma}(p),$$

(17)
where $K_R$ is the constant of integration. Similar to the setter’s case, the boundary condition that determines $K_R$ is given by the value-matching condition at $p = \overline{p}$, which is $V_R(\overline{p}) = V_{R,c}$.

**Case B:** Next, consider the strategy profile in Case B, in which $\mathcal{A} = [p, 1]$. For $p \geq \overline{p}$, it is straightforward that $V_i(p) = V_{i,c}$ for $i = S, R$. For $p < \overline{p}$, the setter’s value function is recursively written as

$$V_S(p) = p\lambda dt\left(\overline{u} + e^{-rdt}(\xi dtV_{S,c} + (1 - \xi dt)\hat{V}_{S,c})\right) + (1 - p\lambda dt)e^{-rdt}V_S(p + dp).$$

Note that if the setter successfully asserts her will, then the belief jumps to one and the players compromise for all future issues. Simplifying the above equation yields

$$(r + p\lambda)V_S(p) = p\lambda(\overline{u} + \hat{V}_{S,c}) - \lambda p(1 - p)V_S'(p).$$

Solving the differential equation yields

$$V_S(p) = \frac{(1 - p)^{1+r}}{p^r}K_S + \frac{\lambda}{r + \lambda}(\overline{u} + \hat{V}_{S,c})p,$$

where $K_S$ is an integration constant. Since the boundary condition is $V_S(0) = V_{S,g} < \infty$, it must be that $K_S = 0$, and thus the

$$V_S(p) = \frac{\lambda}{r + \lambda}(\overline{u} + \hat{V}_{S,c})p. \quad (18)$$

Similar to the setter’s case, the responder’s value function is recursively written as

$$V_R(p) = p\lambda dt e^{-rdt}(\xi dtV_R(1) + (1 - \xi dt)\hat{V}_R(1)) + (1 - p\lambda dt)(\xi dt\overline{v} + e^{-rdt}V_R(p + dp)), $$

which simplifies to

$$(r + p\lambda)V_R(p) = p\lambda\hat{V}_{R,c} + \xi \overline{v} - \lambda p(1 - p)V_R'(p).$$

Solving the differential equation and applying the boundary condition $V_R(0) = V_{R,g} < \infty$ yield

$$V_R(p) = \frac{\xi}{r} - \frac{\lambda}{r + \lambda} \left(\frac{\xi}{r} - \hat{V}_{R,c}\right)p. \quad (19)$$

**Case C:** The players’ value functions in Case C—part 2 of Proposition 1—are straightforward. Since the players never reach a compromise in the future,

$$V_i(p) = V_{i,g}(p) \quad \text{for any} \ p \in [0, 1],$$

where $V_{i,g}(p)$ is player $i$’s expected payoff under the long-run gridlock given in (3) and (4).
Equilibrium verification  We are ready to verify the optimality of the candidate equilibrium profile in each case.

Case A: Consider the candidate equilibrium profile in Case A. We proceed our analysis in each of four belief regions: (i) \( p < \bar{p} \); (ii) \( p = \bar{p} \); (iii) \( p \in (\bar{p}, \bar{p}] \); and (iv) \( p > \bar{p} \).

Case A1: \( p < \bar{p} \). In this case, the responder rejects any offer from the setter, and the setter offers \( x_s \). Given the responder’s behavior, the setter’s incentive condition is trivially satisfied. Therefore, it suffices to check if the responder rejects a compromise offer \( x_c \) if the setter deviates and makes such proposal. This requires that

\[
V_{R,g}(p) \geq v_c + \hat{V}_{R,g}(p).
\]

This inequality simplifies to

\[
V_{R,g}(p) \geq V_{R,c} \quad \text{for any } p < \bar{p}. \tag{20}
\]

Since \( V_{R,g}(p) \) is decreasing in \( p \), the above inequality is satisfied if and only if

\[
V_{R,g}(p) \geq V_{R,c}. \tag{21}
\]

From (6) if \( \frac{\bar{p}}{\gamma} \in (\frac{\xi}{\gamma + \lambda + \xi}, \frac{\xi}{\gamma}) \), then \( p \) is such that \( V_{R,g}(p) = V_{R,c} \), so this is satisfied. If \( \frac{\bar{p}}{\gamma} \leq \frac{\xi}{\gamma + \lambda + \xi} \), then \( p \geq 1 \). At \( p = 1 \) using (4) and (5) we have (21) simplified to \( \frac{\xi(r+\xi)}{r(r+\lambda+\xi)} \hat{V} \geq \frac{r+\xi}{\gamma} v_c \) or \( \frac{\bar{p}}{\gamma} \leq \frac{\xi}{\gamma + \lambda + \xi} \) so this is satisfied. If \( \frac{\bar{p}}{\gamma} \geq \frac{\xi}{r+\xi} \), then \( p = 0 \) and the case of \( p < \bar{p} \) does not exist.

Case A2: \( p = \bar{p} \). We check two incentive conditions for the responder: (a) incentive to accept \( x_c \); and (b) incentive to reject \( x_s \).

If the responder accepts the setter’s offer \( x_c \), then the belief stays the same and there will be an agreement at \( x_c \) for all future periods. If the responder rejects the offer, then learning occurs. With probability \( \lambda dt \) the setter is successful and the issue is resolved with position \( x_s \). With the complementary probability, however, the setter is not successful, in which case the current issue continues and the belief declines. Therefore, the responder is better off accepting the offer \( x_c \) if

\[
V_R(p) \geq p \lambda dt e^{-\rho dt} (\xi dt V_R(1) + (1 - \xi dt) \hat{V}_R(1)) + (1 - p \lambda dt) \left( \xi dt \bar{v} + e^{-\rho dt} V_R(p + dp) \right). \tag{22}
\]

Since \( p = \bar{p} \), if the responder rejects \( x_s \) and the setter fails to overturn, then the belief goes out of the compromise region, after which the players engage in the permanent gridlock. Therefore, the incentive condition (22) becomes

\[
V_{R,c} \geq p \lambda dt e^{-\rho dt} (\xi dt V_R(1) + (1 - \xi dt) \hat{V}_R(1)) + (1 - p \lambda dt) \left( \xi dt \bar{v} + e^{-\rho dt} V_{R,g}(p + dp) \right).
\]

Dropping the terms with orders of \( dt \) and higher, we have

\[
V_{R,c} \geq V_{R,g}(p). \tag{23}
\]
As before if \( \frac{\nu}{v} \in (\frac{\xi}{r+\lambda+\xi}, \frac{\xi}{r+\xi}) \), then \( p \) is such that \( V_{R,0}(p) = V_{R,c} \), so this is satisfied. If \( \frac{\nu}{v} \geq \frac{\xi}{r+\lambda+\xi} \), then \( p \geq 1 \), so this does not apply for Case A. If \( \frac{\nu}{v} \geq \frac{\xi}{r+\xi} \), then \( p = 0 \) and (23) simplifies to \( r\xi v e \geq \xi \bar{v}/r \) or \( \frac{\nu}{v} \geq \frac{\xi}{r+\xi} \).

We also need to check that the responder prefers to reject \( x_i \) when it is offered by the setter, i.e., \( x_i \notin A(p) \). The responder’s payoff from accepting \( x_i \) is given by \( \hat{V}_R(p) \), and her payoff from rejecting \( x_i \) is identical to the right-hand side of (22). Therefore, \( V_R(p) \) must satisfy

\[
\hat{V}_R(p) \leq p\lambda dt e^{-\lambda dt} (\xi dt V_R(1) + (1 - \xi dt) \hat{V}_R(1)) + (1 - p\lambda dt) \left( \xi dt \bar{v} + e^{-\lambda dt} V_R(p + dp) \right).
\]

Dropping the terms with order of \( dt \) and higher from the above inequality, we have

\[
\hat{V}_R(p) \leq V_R(p),
\]

which is trivially satisfied.

Next, consider the setter’s incentive at \( p = \overline{p} \). Given that the responder accepts \( x_c \), the setter prefers to offer \( x_c \) rather than to offer \( x_i \) if

\[
V_{S,c} \geq p\lambda dt (\bar{u} + e^{-\lambda dt} \hat{V}_{S,0}(1)) + (1 - p\lambda dt) e^{-\lambda dt} V_{S,0}(p + dp).
\]

Dropping the terms with orders of \( dt \) and higher yields \( V_{S,c} \geq V_{S,0}(p) \), or

\[
\frac{v_c}{v} \geq \frac{\xi}{r + \xi} \left( 1 - \frac{u_c}{\bar{u}} \right).
\]

This is satisfied since condition (8) holds.

Also, note that the argument in Case 2 shows the necessity of condition (8): If (8) does not hold, then either (23) or (24) would be violated, and thus the candidate profile in part 1 of Proposition 1 cannot be an equilibrium.

**Case A3**: \( p \in (p, \overline{p}) \). Next, we analyze the players’ incentive condition in the “compromise interval” \( (p, \overline{p}) \). Note that this interval exists only if \( p < \overline{p} \).

First observe that the responder’s incentive condition to accept \( x_c \) for \( p \in (p, \overline{p}) \) is identical to (22). However, in contrast to Case 2, the belief after the setter’s failure for a small period of time is still in the compromise region. Therefore, (22) becomes

\[
V_{R,c} \geq p\lambda dt e^{-\lambda dt} (\xi dt V_{R,0}(1) + (1 - \xi dt) \hat{V}_{R,0}(1)) + (1 - p\lambda dt) \left( \xi dt \bar{v} + e^{-\lambda dt} V_{R,c} \right).
\]

Dropping terms with orders of \( (dt)^2 \) and higher, we get

\[
p \lambda \left( V_{R,c} - \hat{V}_{R,0}(1) \right) + rV_{R,c} \geq \xi \bar{v}.
\]

A simple calculation shows that (25) holds for all \( p > \overline{p} \) as long as \( p < 1 \) which is the case when \( p \in (p, \overline{p}) \). Furthermore, the responder’s incentive condition to reject \( x_i \) is identical to the one in Case 2, and thus is trivially satisfied.
Next, consider the setter’s incentive constraints for \( p \in (\underline{p}, \overline{p}] \), in which the setter must prefer offering \( x_c \) (which is immediately accepted) to offering \( x_s \) (which is rejected). Since the belief after the setter’s failure remains in the agreement set, then we require
\[
V_{S,c} \geq p\lambda dt \left( \bar{u} + e^{-\lambda dt}(\xi dt V_{S,g}(1) + (1 - \xi dt)\bar{V}_{S,g}(1)) + (1 - p\lambda dt)e^{-\lambda dt}V_{S,c} \right).
\]
Eliminating terms with orders of \( dt^2 \) and higher and reorganizing yield
\[
p\lambda \left( (r + \lambda) - (r + \lambda + \xi)\frac{\lambda_c}{\bar{u}} \right) \leq r(r + \lambda + \xi)\frac{\lambda_c}{\bar{u}}.
\]
(26)
It is easy to check that (26) holds if and only if \( p \leq \hat{p} \), where \( \hat{p} \) is defined in (15). Since Case 3 assumes the case where \( \hat{p} = \bar{p} \), it follows that the setter’s incentive is satisfied for any \( p \in (\underline{p}, \bar{p}] \).

Equations (14) and (15) imply that given that (8) holds (so that \( \underline{p} < 1 \)), \( \underline{p} < \hat{p} \) if and only if
\[
\frac{v_c}{\bar{u}} > \frac{\xi}{r} \frac{(r + \lambda) - (2r + \lambda + \xi)\frac{\lambda_c}{\bar{u}}}{r + \xi} \frac{(r + \lambda) - (r + \lambda + \xi)\frac{\lambda_c}{\bar{u}}}{r + \lambda + \xi}\frac{\lambda_c}{\bar{u}}.
\]
(27)
Therefore, the compromise region is a nondegenerate interval (i.e., \( \underline{p} < \bar{p} \)) if and only if (27) holds.

Case A4: \( p > \bar{p} \). First, observe that the integration constraints \( K_S \) and \( K_R \) in equations (16) and (17) must be nonnegative. To see this, note that it is straightforward to show \( K_R \geq 0 \) from (17): At \( p = \bar{p} \), the boundary condition is given by \( V_R(\bar{p}) = V_{R,c} \); but \( V_{R,c} \) is no less than than \( V_{R,g}(\bar{p}) \) since \( \underline{p} \leq \bar{p} \). Similarly, one can easily check that \( V_{S,c} \geq V_{S,g}(\bar{p}) \), which implies that \( K_S \) is nonnegative.

For \( p > \bar{p} \), the responder accepts \( x_c \) when it is offered. Therefore, her incentive condition is \( v_c + \hat{V}_R(p) \geq V_R(p) \), or
\[
V_{R,c} \geq V_R(p) \quad \text{for all } p > \bar{p}.
\]
(28)
This condition is immediately satisfied since \( K_R \) is nonnegative, which implies (from (17)) that \( V_R(p) \) is decreasing and strictly convex for \( p > \bar{p} \). Given the responder’s strategy, the setter prefers to offer \( x_s \) rather than to offer \( x_c \). This incentive condition is \( u_c + \hat{V}_S(p) \leq V_S(p) \), which is equivalent to
\[
V_{S,c} \leq V_S(p) \quad \text{for all } p > \bar{p}.
\]
(29)
Again, the fact that \( K_S \) is nonnegative implies that the incentive condition is satisfied.

Case B: Next, we consider the case in which (8) holds and \( u_c/\bar{u} \geq \lambda/(r + \lambda + \xi) \), and check the optimality of the candidate profile that \( \mathcal{A} = [\underline{p}, 1] \). Much of our analysis here will be based on the results in Case A. First, consider the responder’s incentive. The analysis in Cases 1A and 1B imply that the responder’s incentives at \( p < \underline{p} \) and \( p = \underline{p} \) are satisfied if and only if \( V_R(p) = V_{R,c} \). From (19), we have
\[
\underline{p} = \frac{r + \lambda \bar{u} - \frac{r + \xi}{\bar{u}}v_c}{\lambda \bar{u} - v_c}.
\]
which is identical to (14). For the setter’s incentive, it suffices to check if the setter prefers to offer \( x_c \) than to offer \( x_s \) when \( p = 1 \). This condition is given by

\[
V_{S,c} \geq \lambda dt \left( \bar{u} + e^{-\lambda dt}(\xi dt V_{S,c} + (1 - \xi dt)\hat{V}_{S,c}) + (1 - \lambda dt)e^{-\lambda dt}V_{S,c} \right).
\]

Cancelling out terms with orders of \( dt \) or higher and simplifying yields \( u_c/\bar{u} \geq \lambda/(r + \lambda + \xi) \), and thus the setter’s incentive is satisfied.

**Case C:** It remains to verify that if (8) is violated, the candidate profile in Case C is optimal for each player. Note that the argument in Case A1 implies that the responder accepts \( x_c \) if and only if \( p \geq \bar{p} \). For the setter, her incentive condition for \( p < \bar{p} \) is trivial (and identical to that of Case A). Therefore, it remains to verify the setter’s incentive condition for \( p \geq \bar{p} \).

For \( p \geq \bar{p} \), the setter prefers to offer \( x_s \) than to offer \( x_c \) if

\[
u_c + \hat{V}_{S,s}(p) \leq V_{S,s}(p) \iff V_{S,s}(p) \geq V_{S,c}.
\]

Since \( V_{S,s}(p) \) increases in \( p \), it suffices to check the condition at \( p = \bar{p} \). Plugging in \( p = \bar{p} \) yields the condition (24) with the reversed inequality. Therefore, the setter’s incentive condition holds if (8) is violated.

### 7.2 Proof of Proposition 2

Recall that an agreement set of a Markovian strategy profile is defined as \( \mathcal{A} \equiv \{ p \in [0, 1] \mid \chi(p) \in A(p) \} \). We first prove the following lemma.

**Lemma 1** In any Markov perfect equilibrium, the following is satisfied:

1. For any \( p \in [0, 1] \), \( x_0 \in A(p) \) and \( x_s \notin A(p) \);
2. For any \( p \in \mathcal{A} \), \( \chi(p) = x_c \) and \( A(p) = \{x_0, x_c\} \);
3. For any \( p \notin \mathcal{A} \), \( \chi(p) = x_s \); \( A(p) = \{x_0, x_c\} \) if \( V_R(p) \leq V_{R,c} \) and \( A(p) = \{x_0\} \) otherwise.

**Proof of Lemma 1.** **Item 1:** First we show that \( x_0 \in A(p) \) for any \( p \in [0, 1] \). For any \( p \in \mathcal{A} \), accepting \( x_0 \) is obviously the responder’s optimal choice. For \( p \notin \mathcal{A} \), the responder prefers to accept \( x_0 \) if \( \bar{v} + \hat{V}_R(p) \geq V_R(p) \), or \( V_R(p) \leq (r + \xi)\bar{v}/r \). This inequality is always satisfied, since the responder’s best possible payoff is \( (r + \xi)\bar{v}/r \) (when \( x_0 \) is implemented without any delay).

To show that \( x_s \notin A(p) \) for any \( p \), suppose to the contrary that there exists a Markov perfect equilibrium in which \( x_s \in A(p) \) for some \( p \). Then it must be optimal for the setter to offer \( x_s \), since doing so would yield the best possible payoff for him. Therefore \( V_R(p) = 0 \), and thus rejecting the offer is a profitable deviation for the responder.

**Item 2:** Given the behavior of the responder in Item 1, the setter would never offer \( x_0 \) for any \( p \), since doing so would yield a payoff of zero to him. Then it follows trivially that whenever \( p \in \mathcal{A} \) (i.e., \( \chi(p) \in A(p) \)), it must be that \( (\chi(p), A(p)) = (x_c, \{x_0, x_c\}) \).
**Item 3:** Consider any \( p \notin \mathcal{A} \). It is straightforward that \( \chi(p) = x_i \) if \( A(p) = \{x_0,x_i\} \). If \( A(p) = \{x_0\} \), then the setter is indifferent between proposing \( x_c \) and \( x_s \), then our equilibrium requirement implies that he proposes \( x_c \).

For the responder, he prefers to accept \( x_c \) for any \( p \notin \mathcal{A} \) if and only if

\[
v_c + \hat{V}_R(p) \geq V_R(p),
\]

which is equivalent to \( V_R(p) \leq V_{R,c} \). □

Given the result in Lemma 1, we focus on finding the equilibrium agreement set \( \mathcal{A} \), which enables us to characterize the equilibrium profile. The next lemma is the first step in characterizing the equilibrium \( \mathcal{A} \).

**Lemma 2** If a Markov perfect equilibrium has non-empty agreement set \( \mathcal{A} \), it must be that \( \inf \mathcal{A} = p \), where \( p \) is given in (14).

**Proof of Lemma 2.** Let \( p^* = \inf \mathcal{A} \). Since \( \mathcal{A} \) is closed, it follows that \( p^* \in \mathcal{A} \). Then by Lemma 1, \( \chi(p^*) = x_c \) and \( \chi(p) = x_i \) for all \( p < p^* \). Then the responder’s incentive conditions at \( p < p^* \) and \( p = p^* \) are given by (21) and (23), respectively, with \( p \) replaced with \( p^* \). Therefore, it must be that \( V_{R,c} = V_{R,\xi}(p^*) \), which implies that \( p^* = p \). □

Lemma 2 implies that the agreement set of any Markov perfect equilibrium must be one of the following three types: (1) \( \mathcal{A} = \emptyset \); (2) \( \mathcal{A} = \{p\} \); and (3) \( \mathcal{A} \supseteq \{p\} \) and \( \inf \mathcal{A} = p \). Below, we show that there exists a unique candidate equilibrium profile for each type, and derive the parametric conditions for existence of each type of equilibrium.

**Case 1:** \( \mathcal{A} = \emptyset \). In this case, Lemma 1 implies that \( x(p) = x_i \) for any \( p \). Since the equilibrium features a permanent gridlock, the players’ value functions are \( V_i(p) = V_{i,\xi}(p) \). Then by the definition of \( p \) (equation 14), \( V_R(p) \geq V_{R,c} \) if and only if \( p \geq \hat{p} \). Therefore, by Lemma 1, \( A(p) = \{x_0,x_i\} \) for \( p \geq \hat{p} \) and \( A(p) = \{x_0\} \) for \( p < \hat{p} \), providing the unique candidate equilibrium profile. Note that this profile is the one analyzed in the proof of Proposition 1. The proof shows that this profile exists if and only if (8) does not hold.

**Case 2:** \( \mathcal{A} = \{p\} \). Next, consider a strategy profile with \( \mathcal{A} = \{p\} \). By Lemma 1, \( x(p) = x_c \) if \( p = p \) and \( x(p) = x_s \) otherwise. Then the responder’s value function under this profile is given by \( V_R(p) = V_{R,\xi}(p) \). Then by the same logic as Case 1, \( A(p) = \{x_0,x_c\} \) for \( p \geq \hat{p} \) and \( A(p) = \{x_0\} \) for \( p < \hat{p} \), which provides the unique candidate equilibrium profile. Again, the existence condition for this equilibrium profile is analyzed in the proof of Proposition 1: The unique candidate profile in Case 2 exists if and only if (8) holds and (27) is violated.

**Case 3:** \( \mathcal{A} \supseteq \{p\} \) and \( \inf \mathcal{A} = p \). We claim that in Case 3, there exists a unique equilibrium agreement set, which is an interval \([p,\hat{p}]\) where \( \hat{p} \) is given in (15).

Let \( p^{**} = \sup \mathcal{A} > \hat{p} \), then \( p^{**} \in \mathcal{A} \) since \( \mathcal{A} \) is closed. Since the players never reach an agreement for any \( p > p^{**} \), the responder’s value function for \( p > p^{**} \) is given by (17), with the
boundary condition $V_R(p^{**}) = V_{R,c}$. Since $p^{**} > p$, it must be that $V_{R,d}(p^{**}) < V_{R,c}$. Therefore, it follows from (17) that $K_R > 0$ and that $V_R(p) < V_{R,c}$ for all $p > p^{**}$. Then Lemma 1 implies that $A(p) = \{x_0, x_c\}$ for $p \geq p^{**}$. Since the responder’s behavior is constant for $p \in [p^{**}, 1]$, the setter’s value function $V_S(p)$ must satisfy both value-matching and smooth-pasting conditions at $p = p^{**}$. Since the setter’s value function for $p > p^{**}$ is given by (16). The value-matching and smooth-pasting conditions are given by

\[
V_{S,c} = \frac{(1 - p^{**})^{1+\frac{\bar{\gamma}}{2}}}{(p^{**})^{\frac{\bar{\gamma}}{2}}} K_S + V_{S,d}(p^{**}),
\]

\[
0 = -\frac{(1 - p^{**})^{\frac{\bar{\gamma}}{2}}}{(p^{**})^{1+\frac{\bar{\gamma}}{2}}} \left( p^{**} + \frac{r}{\lambda} \right) K_S + V'_{S,d}(p^{**}),
\]

respectively. Solving above system shows that

\[
p^{**} = \frac{r + \lambda + \xi}{\lambda} \frac{ru_c}{(r + \lambda)\bar{u} - (r + \lambda + \xi)u_c},
\]

which coincides with $\hat{p}$ given in (15). If the above formula is above one, then the boundary conditions do not hold at any $p \in [0, 1]$ and it must be that $p^{**} = 1$.

Next, we finish our claim by showing that there does not exist $p^* \in (p, \bar{p})$ such that $p^* \not\in \mathcal{A}$. Suppose to the contrary that there exists such $p^*$. Then the closedness of $\mathcal{A}$ implies that $(p^* - \epsilon, p^* + \epsilon) \subset [0, 1] \setminus \mathcal{A}$ for sufficiently small $\epsilon > 0$. Let $p'$ be an infimum of a connected interval in $[0, 1] \setminus \mathcal{A}$ which contains $p^*$. Then since $p' \geq p$, the same logic used to show $p^{**} = \bar{p}$ implies that the responder should accept $x_c$ for $p \in (p^* - \epsilon, p^*)$, which in turn implies that the setter’s value function must satisfy the both value-matching and smooth-pasting conditions at $p = p'$. But then it implies that $p' = \hat{p}$, leading to a contradiction.

This argument provides the unique candidate equilibrium profile in Case 3:

\[
\chi(p) = \begin{cases} 
  x_c & \text{if } p \in [p, \hat{p}] \\
  x_s & \text{otherwise,}
\end{cases} \quad A(p) = \begin{cases} 
  \{x_0, x_c\} & \text{if } p \geq p \\
  \{x_0\} & \text{if } p < p.
\end{cases}
\]

Again, this profile is analyzed in proof of Proposition 1; this profile exists if and only if both (8) and (27) hold.

### 7.3 Proof of Proposition 3

First, it is straightforward from (14) and (15) that $\underline{p}$ and $\bar{p}$ is continuous in all parameters. Define

\[
\underline{p}_a = \frac{r + \lambda + \xi}{\lambda} \left( 1 - \frac{r + \xi v_c}{\xi \bar{u}} \right), \quad \bar{p}_a = \frac{r + \lambda}{\lambda} \frac{\bar{u} - \frac{r + \xi}{\xi}v_c}{\bar{v} - v_c},
\]

and

\[
\hat{p}_a = \frac{r + \lambda + \xi}{\lambda} \frac{ru_c}{(r + \lambda)\bar{u} - (r + \lambda + \xi)u_c},
\]

34
Then we show the desired results by analyzing the comparative statics of $p_a$, $p_b$, and $\hat{p}_a$.

**Item 1:** Consider the comparative statics with respect to $r$. First, $\hat{p}_a$ is strictly increasing in $r$, because

$$
\frac{\partial \hat{p}_a}{\partial r} = \frac{u_c((r + \lambda)^2 + \xi \lambda)\bar{u} - (r + \lambda + \xi)^2u_c)}{((r + \lambda)\bar{u} - (r + \xi + \lambda)u_c)\lambda} > 0. \tag{30}
$$

In particular, (30) is strictly positive because

$$
\frac{u_c}{\bar{u}} < \frac{\lambda}{r + \lambda + \xi} \leq \frac{(r + \lambda)^2 + \xi \lambda}{(r + \lambda + \xi)^2}.
$$

Taking derivatives of $p_a$ and $p_b$ with respect to $r$ yields

$$
\frac{\partial p_a}{\partial r} = \frac{\bar{v} - \frac{2r + 2\xi + \lambda}{\xi}v_c}{\lambda\bar{v}}, \quad \frac{\partial p_b}{\partial r} = \frac{\bar{v} - \frac{2r + \xi + \lambda}{\xi}v_c}{\lambda(\bar{v} - v_c)}.
$$

Both derivatives are strictly negative since (8) implies that $v_c/\bar{v} > \xi/(r + \xi + \lambda)$.

**Item 2:** It is straightforward that $p_a$ and $p_b$ are strictly decreasing in $\lambda$. For $\hat{p}_a$,

$$
\frac{\partial \hat{p}_a}{\partial \lambda} = \frac{ru_c((r + \lambda + \xi)^2u_c - ((r + \lambda)^2 + (2\lambda + r)\xi)\bar{u})}{((r + \lambda)\bar{u} - (r + \xi + \lambda)u_c)\lambda^2} < 0, \tag{31}
$$

because

$$
\frac{u_c}{\bar{u}} < \frac{(r + \lambda)^2 + \xi \lambda}{(r + \lambda + \xi)^2} \leq \frac{(r + \lambda)^2 + (2\lambda + r)\xi}{(r + \lambda + \xi)^2}.
$$

**Item 3:** Taking the derivatives with respect to $\xi$ yields

$$
\frac{\partial p_a}{\partial \xi} = \frac{r(r + \lambda)v_c + \xi^2(\bar{v} - v_c)}{\bar{v}\xi^2}\lambda > 0,
$$

$$
\frac{\partial p_b}{\partial \xi} = \frac{r(r + \lambda)}{\lambda\xi^2(\bar{v} - v_c)} > 0,
$$

$$
\frac{\partial \hat{p}_a}{\partial \xi} = \frac{u_c\bar{u}r(r + \lambda)}{((r + \lambda)\bar{u} - (r + \xi + \lambda)u_c)\lambda} > 0,
$$

showing the desired results.
Online Appendix: Proof of Propositions 4 and 5

We first derive the players’ value functions in each type of equilibrium. Observe that the two types of equilibrium differ only in the behavior at \( p = \frac{p_0}{p_1} \). Therefore, the differential equations underlying both value functions are identical, and they differ only in the boundary conditions at \( p = \frac{p_0}{p_1} \). After obtaining the value functions, we verify each type of equilibrium by investigating the players’ incentive conditions.

**Value functions** Let \( V_i(p), i = S, R \) be the value function under the easy issues, and let \( W_i(p) \) be the value function under the difficult issues. Also, for notational simplicity, define \( Z_i(p) \) to be the value function when the new issue arises, and define \( \hat{Z}_i(p) \) as the value function when the current issue is resolved (but the new issue has not yet appeared). Then it is straightforward that

\[
Z_i(p) = (1 - \alpha) V_i(p) + \alpha W_i(p),
\]

\[
\hat{Z}_i(p) = \frac{\xi}{r + \xi} ((1 - \alpha) V_i(p) + \alpha W_i(p)).
\]

Observe that for \( p < \frac{p_0}{p_1} \), regardless of the issue type, permanent gridlock occurs in equilibrium. Therefore, \( V_i(p) = W_i(p) = V_{i,g}(p) \).

Next, we derive the value functions for \( p \in (\frac{p_0}{p_1}, \frac{p_0}{p_1}] \). Consider first the setter’s value function. Under the easy issue, the setter offers \( x_c \) and the responder accepts the offer. Therefore, \( V_S(p) \) satisfies

\[
V_S(p) = u_c + e^{-rdt}(\xi dt Z_S(p) + (1 - \xi dt) \hat{Z}_S(p)).
\]

Cancelling the terms with order \( dt \) or higher, applying (32) and (33) and simplifying yield

\[
V_S(p) = \frac{rV_{S,c} + \xi \alpha W_S(p)}{r + \xi \alpha}.
\]

Under the difficult issue, the setter offers \( x_s \) and the responder rejects the offer. Therefore, \( W_S(p) \) satisfies

\[
W_S(p) = p \lambda dt \left( \hat{V}_S(p) + e^{-rdt} (\xi dt V_{S,g}(1) + (1 - \xi dt) \hat{V}_{S,g}(1)) \right) + (1 - p \lambda dt) e^{-rdt} \left( (1 - \xi dt) W_S(p) + (1 - \xi dt) W_S(p + dp) \right).
\]

Cancelling the terms with order \( dt^2 \) or higher, applying (32) and (33), and reorganizing yields

\[
\lambda p (1 - p) W_S'(p) = p \lambda (\hat{V}_S(1)) + \xi (1 - \alpha) V_S(p) - (p \lambda + r + \xi (1 - \alpha)) W_S(p).
\]

Plugging in (34) and solving the differential equation yield

\[
W_S(p) = f(p) K_S + \frac{a - b}{\mu + 1} p + b,
\]

36
where
\[
f(p) = \frac{(1 - p)^{p+1}}{p^p}, \quad \mu = \frac{r(r + \xi)}{\lambda(r + \xi\alpha)}, \quad a = \tilde{u} + \hat{V}_{S,g}(1), \quad b = (1 - \alpha)\hat{V}_{S,c} \tag{37}
\]
and $K_S$ is an integration constant.

Now consider the responder. Similar to (34), the responder’s value function under the easy issues are given by
\[
V_R(p) = \frac{rV_{R,c} + \xi\alpha W_R(p)}{r + \xi\alpha}.
\tag{38}
\]

Under the difficult issues, the setter offers $x_e$ and the responder rejects the offer. Therefore, $W_R(p)$ satisfies
\[
W_R(p) = p\lambda dt e^{-rdt}(\xi dt V_{R,g}(1) + (1 - \xi dt)\hat{V}_{R,g}(1))
+ (1 - p\lambda dt)\Big(\xi dt\tilde{u} + e^{-rdt}(\xi dt Z_R(p + dp) + (1 - \xi dt)W_R(p + dp))\Big).
\]

Cancelling the terms with order $dt^2$ or higher, applying (32) and (33), and reorganizing yields
\[
\lambda p(1 - p)W'_R(p) = p\lambda \hat{V}_{R,g}(1) + \xi\tilde{u} + \xi(1 - \alpha)V_R(p) - (p\lambda + r + \xi(1 - \alpha))W_R(p).
\]

Solving this equation with (38) gives
\[
W_R(p) = f(p)K_R + \frac{1}{\mu + 1} \left[ (\mu + 1 - p) \left( \frac{r + \xi\alpha}{r + \xi} V_{R,g}(0) + (1 - \alpha)\hat{V}_{R,c} \right) + \hat{V}_{R,g}(1)p \right], \tag{39}
\]
where $K_R$ is an integration constant.

For $p > \overline{p}_\alpha$, gridlock arises in both the easy and the difficult policy issues. For the setter, $V_S(p)$ satisfies
\[
V_S(p) = p\lambda dt \Big( \tilde{u} + e^{-rdt}(\xi dt V_{S,g}(1) + (1 - \xi dt)\hat{V}_{S,g}(1))
+ (1 - p\lambda dt)e^{-rdt}(\xi dt Z_S(p + dp) + (1 - \xi dt)V_S(p + dp)) \Big),
\]
and $W_S(p)$ satisfies (35). Simplifying, we have
\[
\lambda p(1 - p)V'_S(p) = \lambda ap + \xi\alpha W_S(p) - (p\lambda + r + \xi\alpha)V_S(p),
\lambda p(1 - p)W'_S(p) = \lambda ap + \xi(1 - \alpha)V_S(p) - (p\lambda + r + \xi(1 - \alpha))W_S(p), \tag{40}
\]
where $a$ is defined in (37). Similarly, $V_R(p)$ and $W_R(p)$ jointly solve
\[
\lambda p(1 - p)V'_R(p) = \lambda \hat{V}_{R,g}(1)p + \xi\tilde{u} + \xi\alpha W_R(p) - (p\lambda + r + \xi\alpha)V_R(p),
\lambda p(1 - p)W'_R(p) = \lambda \hat{V}_{R,g}(1)p + \xi\tilde{u} + \xi(1 - \alpha)V_R(p) - (p\lambda + r + \xi(1 - \alpha))W_R(p). \tag{41}
\]
Solving the systems (40) and (41) yield the value functions

\[ V_S(p) = K_{S1} h_1(p) + K_{S2} h_2(p) + \frac{\lambda}{r + \lambda} a p \]  
(42)

\[ W_S(p) = K_{S1} \frac{\alpha - 1}{\alpha} h_1(p) + K_{S2} h_2(p) + \frac{\lambda}{r + \lambda} a p \]  
(43)

\[ V_R(p) = K_{R1} h_1(p) + K_{R2} h_2(p) + \frac{\lambda}{r + \lambda} \left( \hat{V}_{R,\alpha}(1) - \frac{\xi}{r} \right) p + \frac{\xi}{r} \]  
(44)

\[ W_R(p) = K_{R1} \frac{\alpha - 1}{\alpha} h_1(p) + K_{R2} h_2(p) + \frac{\lambda}{r + \lambda} \left( \hat{V}_{R,\alpha}(1) - \frac{\xi}{r} \right) p + \frac{\xi}{r}, \]  
(45)

where \( K_{S1}, K_{S2}, K_{R1}, K_{R2} \) are integration constants and

\[ h_1(p) = \frac{(1 - p)^{\tau + 1}}{p^\tau}, h_2(p) = \frac{(1 - p)^{\nu + 1}}{p^\nu}, \tau = \frac{r + \xi}{\lambda}, \nu = \frac{r}{\lambda}. \]

**Equilibrium Verification: Gridlock Equilibrium** In the first type of equilibrium—gridlock equilibrium—the setter induces gridlock for any \( p \) when the current policy issue is a difficult one. Therefore, if players currently face a difficult policy issue and \( p = p^\alpha \), there will be permanent gridlock. Thus, the boundary conditions for \( W_i \) are given by \( W_i(p^\alpha) = V_{i,\alpha}(p^\alpha) \). Then from (34) and (38), we have

\[ V_S(p^\alpha) = \frac{r V_{S,c} + \xi \alpha V_{S,\alpha}(p^\alpha)}{r + \xi \alpha}, \]  
(46)

\[ V_R(p^\alpha) = \frac{r V_{R,c} + \xi \alpha V_{R,\alpha}(p^\alpha)}{r + \xi \alpha}. \]  
(47)

Given this boundary condition, let us verify the incentive constraints of each player.

**Case 1:** \( p < p^\alpha \). In this case, we only need to verify the responder’s incentive condition to reject \( x_c \) under easy issues. Same as the benchmark model, this condition is given by

\[ V_{R,c} \leq V_{R,\alpha}(p) \quad \text{for any } p < p^\alpha. \]

Since \( V_{R,\alpha}(p) \) is decreasing in \( p \), the above inequality holds if and only if

\[ p^\alpha \leq p. \]  
(48)

where \( p \) is given in equation (14).

**Case 2a:** \( p = p^\alpha \) under a easy policy issue. First, consider the responder’s incentives. Under easy issues, the responder must accept \( x_c \) at \( p = p^\alpha \). This condition is given by

\[ V_{R}(p^\alpha) \geq p \lambda d t e^{-r d t} (\xi d t V_{R,\alpha}(1) + (1 - \xi d t) \hat{V}_{R,\alpha}(1)) + (1 - p \lambda d t) (\xi d t \bar{v} + e^{-r d t} V_{R,\alpha}(p^\alpha + d p)). \]
Deleting terms with order $dt$ or higher and reorganizing yield

$$v_c + \frac{\xi}{r + \xi}((1 - \alpha)V_R(p) + \alpha W_R(p)) \geq V_{R,\alpha}(p).$$

Plugging in $W_R(p) = V_{R,\alpha}(p)$ and (47), and simplifying yield

$$V_{R,c} \geq V_{R,\alpha}(p) \iff p_{\alpha} \geq p. \quad (49)$$

Combining (48) and (49) implies that

$$p_{\alpha} = \frac{p + \lambda + \xi}{\lambda} \left(1 - \frac{r + \xi v_c}{\xi \bar{v}}\right). \quad (50)$$

Since the setter prefers to offer $x_c$ than to offer $x_s$ at $p = p$ under easy issues, the setter’s incentive condition is given by $V_S(p) \geq V_{S,\alpha}(p)$. By (46), this condition simplifies to $V_{S,c} \geq V_{S,\alpha}(p)$, or

$$\frac{v_c}{\bar{v}} \geq \frac{\xi}{r + \xi} \left(1 - \frac{u_c}{u}\right). \quad (51)$$

Note that (51) is identical to (24), the corresponding condition in the benchmark case.

Given the result of (50), we denote $p$ instead of $p_{\alpha}$ in the remaining of the proof.

**Case 2b:** $p = p$ **under a hard policy issue.** The responder must prefer to reject $x_s$ than to accept the offer, and thus her incentive condition is given by

$$V_{R,\alpha}(p) \geq \frac{r}{r + \xi}((1 - \alpha)V_R(p) + \alpha W_R(p)).$$

Since (50) implies that $V_{R,\alpha}(p) = W_{R,\alpha}(p) = V_{R,\alpha}(p)$, the above condition simplifies to $V_{R,c} \geq \hat{V}_{R,c}$, which is trivially satisfied.

Consider the setter’s incentive to offer $x_s$ at $p = p$. Because she must prefer offering $x_s$ to offering $x_0$, her incentive condition is given by

$$V_{S,\alpha}(p) \geq e^{-\xi dt \xi Z_S(p)} + (1 - \xi dt)\hat{Z}_S(p)).$$

Plugging in (46) and (50), and simplifying yield

$$\frac{v_c}{\bar{v}} \leq \frac{\xi}{r + \xi} \left(1 - \frac{\xi(1 - \alpha) u_c}{r + \xi \bar{u}}\right), \quad (52)$$

which is given by condition (11) in Proposition 4.

**Case 3a:** $p \in (p, p_{\alpha}]$ **under a easy policy issue.** In this case, the responder prefers to accept $x_c$ than to reject. Therefore, her incentive condition is

$$V_R(p) \geq p\lambda dt e^{-\xi dt \hat{V}_{R,\alpha}(1)} + (1 - p\lambda dt)(\xi dt \bar{v} + e^{-\xi dt Z_R(p + dp)}) + (1 - \xi dt) e^{-\xi dt V_R(p + dp)}.$$

39
We show that the left side of (55) is concave. First, observe that $f(p)$ is convex. Second, note that $K_S$ must be positive. If $K_S$ is negative, then $V_S(p)$ must be concave by (34), but this leads to a negative value of information under a easy policy issue. This is a contradiction because there cannot be any loss of learning from the belief drifting down but there exists the upside gain when the belief jumps to one. Therefore, if (55) is satisfied at $p = p_0$, then there exists $\overline{p}_0 \in (p, 1]$ such that (55) is satisfied for $p \in (p, \overline{p}_0)$. If (55) is not satisfied at $p = p_0$, then (since (51) holds) the players compromise only at $p = p_0$.

**Case 3b: $p \in (p, \overline{p}_0)$ under a hard policy issue.**

In this case, the responder’s incentive condition to reject $x_s$ is identical to that in Case 2b. given by

$$W_R(p) \geq e^{-\xi dt}(\xi dt Z_R(p) + (1 - \xi dt)\hat{Z}_R(p)).$$

Cancelling out terms with orders of $dt$ or higher, plugging in (34) and reorganizing yield

$$W_R(p) \geq (1 - \alpha)\hat{V}_{R,e},$$

Simplifying yields

$$(r + p\lambda + \xi \alpha)V_R(p) \geq p\lambda\hat{V}_{R,e}(1) + \xi\hat{v} + \xi\alpha W_R(p) - \lambda p(1 - p)V_R'(p)$$

Using (34), we reorganize the condition as

$$rV_{R,e} - p\lambda\hat{v} = \xi\alpha\left(V_{R,e}(0) - V_{R,e} + \frac{p\lambda}{r}V_{R,e}(1) - W_R(p) - (1 - p)W_R'(p)\right).$$

Note that if $\alpha = 0$, (53) becomes identical to (25), which is the corresponding incentive condition in the benchmark model. Then the corresponding argument in the proof of Proposition 1 (page 31) shows that (25) is satisfied for all $p > p_0$. Moreover, a calculation shows that the right side of (53) is negative for all $p > p_0$, implying that (53) is satisfied for all $p > p_0$. Intuitively, the responder has a stronger incentive to accept $x_e$ when $\alpha > 0$ compared to the case with $\alpha = 0$, because rejecting might lead to the replacement of a difficult policy issue.

Next, consider the setter’s incentive condition. Since the setter prefers to offer $x_e$ then to offer $x_s$, her incentive condition is given by

$$V_S(p) \geq p\lambda dt(\bar{u} + e^{-\xi dt}\hat{V}_{S,e}(1)) + (1 - p\lambda dt)e^{-\xi dt}(\xi dt Z_S(p + dp) + (1 - \xi dt)V_S(p + dp))$$

Simplifying and plugging in (34), we have

$$rV_{S,e} - p\lambda(a - V_{S,e}) \geq \xi\alpha\left(-V_{S,e} + \frac{p\lambda}{r}(a - W_S(p) - (1 - p)W_S'(p))\right).$$

where $a$ is defined in (37). From (36), $a - W_S(p) - (1 - p)W_S'(p) = f(p)\frac{d}{r}K_S + \frac{\mu}{\mu + 1}(a - b)$. Therefore, (54) becomes

$$rV_{S,e} - p\lambda(a - V_{S,e}) + \xi\alpha\left(V_{S,e} - r + \xi\alpha\left(f(p)K_S + \frac{\mu}{\mu + 1}b\right)\right) \geq 0.$$
which is trivially satisfied. The setter’s incentive condition to propose \( x_s \) is

\[
W_S(p) \geq e^{-rdt} (\xi dt Z_S(p) + (1 - \xi dt)\hat{Z}_S(p)).
\]

Plugging in value functions and simplifying yields

\[
W_S(p) \geq (1 - \alpha) \hat{V}_{S,c}.
\]

The analysis in Case 2b shows that the above condition holds at \( p = \bar{p} \) if and only if (52) is satisfied. Then since \( W_S(p) \) is strictly decreasing in \( p \), it follows that the above condition is satisfied for all \( p \in (p, \bar{p}_a] \) if (52) holds.

**Case 4: \( p > \bar{p}_a \).** Let us consider the setter’s incentive condition under the easy issue. He must prefer to offer \( x_s \) than \( x_c \).

\[
V_S(p) \geq u_c + e^{-rdt} (\xi dt Z_S(p) + (1 - \xi dt)\hat{Z}_S(p)).
\]

Similar to above, cancelling out terms with order of \( dt \) or above and reorganizing yield

\[
(r + \xi \alpha)V_S(p) - \xi \alpha W_S(p) \geq rV_{S,c}. \tag{56}
\]

Plugging in (42) and (43) and reorganizing yield

\[
K_{S1}h_1(p) + \frac{r}{r + \xi} \left(K_{S2}h_2(p) + \frac{\lambda}{\lambda + r}ap \right) \geq u_c. \tag{57}
\]

We claim that (57) holds for all \( p \geq \bar{p}_a \) given that \( \bar{p}_a < 1 \). Note that by (34) and the continuity of \( V_S(p) \) and \( W_S(p) \) at \( p = \bar{p}_a \), (56) holds with equality at \( p = \bar{p}_a \). Therefore, it suffices to show that the left-hand side of (57) is increasing in \( p \).

Note that since \( V_S(\bar{p}_a) > W_S(\bar{p}_a) \), it follows from (42) and (43) that \( K_{S1} > 0 \). Now suppose to the contrary that the left-hand side of (57) is strictly decreasing at \( p = p' > \bar{p}_a \). Since \( K_{S1} > 0 \), it must be that \( K_{S2} < 0 \). Then comparing the left-hand side of (57) and \( V_S(p) \) in (42) implies that \( V_S(p) \) must be strictly decreasing at \( p = p' \). However, \( V_S(p) \) must be strictly increasing for all \( p > \bar{p}_a \) since having a higher \( p \) does not incur any cost to the setter, leading to a contradiction.

**Equilibrium Verification: Avoiding-the-issue Equilibrium** In the avoiding-the-issue equilibrium, the setter offers \( x_0 \) to induce compromise at \( p = \bar{p}_a \) under difficult policy issues. Then the value of \( V_S(p_a) \) and \( W_S(p_a) \) satisfy the following system of equations:

\[
V_S(p_a) = u_c + \frac{\xi}{r + \xi} ((1 - \alpha)V_S(p_a) + \alpha W_S(p_a))
\]

\[
W_S(p_a) = \frac{\xi}{r + \xi} ((1 - \alpha)V_S(p_a) + \alpha W_S(p_a))
\]
Similarly, $V_R(p_{α})$ and $W_R(p_{α})$ jointly solve

$$V_R(p_{α}) = v_c + \frac{ξ}{r + ξ}(1 - α) V_R(p_{α}) + α W_R(p_{α})$$

$$W_R(p_{α}) = \bar{v} + \frac{ξ}{r + ξ}(1 - α) V_R(p_{α}) + α W_R(p_{α})$$

Solving the above systems yields the boundary conditions at $p = p_{α}$:

$$V_S(p_{α}) = u_c + \frac{ξ(1 - α)}{r} u_c, \quad V_R(p_{α}) = v_c + \frac{ξ}{r}((1 - α)v_c + α\bar{v}),$$

$$W_S(p_{α}) = \frac{ξ(1 - α)}{r} u_c, \quad W_R(p_{α}) = \bar{v} + \frac{ξ}{r}((1 - α)v_c + α\bar{v}).$$

**Case 1:** $p < p_{α}$. For $p < p_{α}$, the players’ incentive conditions are identical to those in the gridlock equilibrium. Therefore, the profile must satisfy (48), or equivalently,

$$p_{α} \leq p$$

where $p$ is defined in Proposition 1.

**Case 2b:** $p = p_{α}$ **under a easy policy issue.** First, the responder must prefer to accept $x_c$ at $p = p_{α}$. Same as the gridlock equilibrium, this condition is given by

$$V_R(p_{α}) \geq pλdte^{-r dt}ξ dt V_{R,α}(1) + (1 - ξ dt)\hat{V}_{R,α}(1) + (1 - pλ dt)(ξ dt\bar{v} + e^{-r dt} V_{R,α}(p_{α} + dp)).$$

Deleting terms with order $dt$ or higher, plugging in the value of $W_R(p_{α})$ and $V_R(p_{α})$ and simplifying yields

$$V_{R,c} + \frac{ξα}{r}(\bar{v} - v_c) \geq V_{R,α}(p_{α}).$$

Note that the above incentive condition is strictly weaker than the corresponding condition in the gridlock equilibrium (equation 49). Solving for $p_{α}$, we have

$$p_{α} \geq p_{0} \equiv \frac{r + λ + ξ}{λ} \left(1 - α \right) - \frac{r + ξ(1 - α)v_c}{ξ\bar{v}}.$$ (60)

It is straightforward to check that $p_{0} < p$ for any $α > 0$.

For the setter, she must prefer to offer $x_c$ than to offer $x_s$ at $p = p_{α}$, and thus her incentive condition is

$$V_S(p_{α}) \geq V_{S,α}(p_{α}).$$

As we shall see, this condition is strictly weaker than the corresponding condition under the hard policy issue (equation 61).
Case 2b: $p = p_{\alpha}$ under a hard policy issue. In this case, the responder prefers to accept $x_0$, and thus her incentive condition is given by $W_R(p_{\alpha}) \geq V_{R,g}(p_{\alpha})$. It is straightforward to show that this condition is strictly weaker than (59).

Next, consider the setter’s incentive to offer $x_0$ at $p = p_{\alpha}$. Since she must prefer offering $x_0$ than offering $x_s$, her incentive condition is given by

$$rW_S(p_{\alpha}) \geq V_{S,g}(p_{\alpha}).$$

(61)

Reorganizing with respect to $p_{\alpha}$ yields

$$p_{\alpha} \leq \bar{p} \equiv \frac{r + \lambda + \xi}{\lambda} \cdot \frac{\xi(1 - \alpha) u_c}{r + \xi u}$$

(62)

Combining (58), (60), and (62) implies that $p_{\alpha} \in [p_0, \min(p, \bar{p})]$. Such $p_{\alpha}$ exists if and only if $p_0 \leq \bar{p}$, or

$$\frac{v_c}{\bar{v}} \geq \frac{\xi(1 - \alpha)}{r + \xi(1 - \alpha)} \left(1 - \frac{\xi u_c}{r + \xi u}\right),$$

which is the binding condition in Proposition 5.

Case 3 & 4: $p > p_{\alpha}$. For each $p_{\alpha} \in [p_0, \min(p, \bar{p})]$, an argument identical to that for the gridlock equilibrium shows that the incentive conditions for $p > p_{\alpha}$ are satisfied.