The Binary Policy Model

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June 5, 2008

Abstract

We introduce a general framework in which politicians choose a sequence of binary policies. The two competing candidates are exogenously committed to particular positions on a subset of these issues, while they can choose any policy for the remaining issues to maximize their winning probability. We show that the binary policy model provides a tractable multidimensional model of candidate competition that can generate policy divergence and adoption of minority positions by candidates. We also use our framework to analyze plurality rule and runoff rule in elections with three candidates.

JEL Classification Numbers: D72, D60.

Keywords: Multidimensional policy, voting, citizen-candidate, normative analysis of political competition

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We would like to thank Ernesto Dal Bo, Matthias Messner, Dan Bernhardt, Cagdas Agirdas and seminar participants at UC Berkeley, University of Toronto, Simon Fraser University, Michigan State University, University of Bonn, University of Munich, University of Bern and the European Summer Symposium for Economic Theory 2007 for helpful comments. The most current version of this paper is available at https://netfiles.uiuc.edu/polborn/www/
1 Introduction

The one-dimensional policy model with office motivated candidates based on the seminal contributions of Hotelling (1929) and Downs (1957) is the most widely used and successful model framework for a formal analysis of political equilibria. We will call this model the *standard model* in the following. Yet, there are some tensions within the model, and between the model and some real-world observations.

First, in the one-dimensional standard model, there is a strong tendency for candidates to converge to the same, moderate position that appeals to the “median voter” (mitigated only if the candidates care about policy and to the extent that the position of the median is uncertain); furthermore, all voters, including those with extreme preferences, are, in equilibrium, indifferent between the two candidates, as they propose the same policy. Yet, in reality, candidates often run on considerably divergent policy platforms, and voters often intensely favor one candidate over the other.

Second, while the standard model is one-dimensional and continuous, policy in reality is often multidimensional (there are many policy issues) and binary (e.g., candidates are either for withdrawing troops from Iraq or against it). In fact, a widespread casual interpretation of policy in the one-dimensional model is based on multidimensional policies. For example, the statement that “Hillary Clinton used her support for the Iraq war to move towards the political center,” implies that her initial position (on other issues, say on health care) is left-of-center, but by adopting a conservative position on another issue, she can move to the right on the policy line.

This informal multi-issue interpretation of the one-dimensional model is somewhat problematic regarding the treatment of moderates. For example, suppose we accept the notion that support for state-provided health care is a liberal position and support for the Iraq war is a conservative position, and suppose that these are the only two issues and that they are equally important. Then both “Hillary” (with positions as described above) and a voter who opposes both state-provided health care and the Iraq war would be considered “moderate” with a position in the center in a one-dimensional model, suggesting that the voter is likely to support her. Yet, the voter could plausibly prefer another candidate with a “more extreme” position, say, someone who supports state-provided health care but opposes the Iraq war.

We develop a model that directly treats policy as multidimensional and binary. Each dimension corresponds to a position on a particular policy issue, and each voter has a preferred position on each issue. A voter’s utility from Candidate \( j \) is calculated by identifying those issues in which candidate and voter agree, weighing them with a factor to measure the importance of each issue, and adding up.

We assume that each candidate is exogenously fixed on some issues. These fixed positions can be interpreted as characteristics of the candidate (such as party affiliation, incumbency, gender, race or experience in previous elected office), or political issues on which a candidate has taken a stand in the past and where commitment to a different position is not credible and/or not helpful. On the remaining issues, candidates are free to choose any position. Our model is the first to provide a framework in which candidates have both fixed and selectable positions. We thus provide a middle ground between Downsian models, in which candidates are free to choose any position, and the citizen candidate model,
in which no commitment is possible.\footnote{In the citizen candidate literature pioneered by Osborne and Slivinski (1996) and Besley and Coate (1997), candidates are policy motivated and cannot commit to any other position than their ideal one. While the citizen candidate model can, in principle, handle multiple policy dimensions, most papers in this literature only look at a standard one-dimensional framework.} Moreover, we show that this combination enhances our understanding of political economy models significantly: Some core efficiency results of existing models depend on the (seemingly innocuous and certainly unrealistic) assumption that all candidates share the same characteristics or positions on fixed issues.

The most important result for political competition in the standard model is that office-motivated candidates propose policies that appeal to the median voter. In our multi-dimensional model, there is no geometric notion of a median, but our concept of majority-efficiency captures the same fundamental idea of moderation. A policy is \textit{majority-efficient} if a majority of voters prefers the proposed policy to any other policy that a candidate could choose. In other words, a majority-efficient policy is a Condorcet winner among a candidate’s policies, subject to the constraints imposed by his fixed positions.

We show that majority-efficient policies exist in the binary policy model for many distributions of voter preferences, and we characterize sufficient (and generic) conditions for existence. This result differs significantly from Plott (1967), who has shown that Condorcet winners exist in a multidimensional Euclidean setting only for a highly non-generic configuration of preferences (in a setting without fixed positions, the concept of a Condorcet winner is equivalent to majority-efficiency). This problem, and the difficulty to characterize equilibria, has severely limited the applicability of the multidimensional version of the standard model as a tool to study multidimensional policy. For example, many papers that apply political economy arguments to particular fields essentially analyze policy as if there were only one political issue in which the median voter’s opinion is decisive. In contrast, in the binary policy model, if a majority-efficient position exists for a particular voter distribution, it generically also exists for a slightly perturbed distribution of voters. In many cases, the binary policy model therefore provides a tractable framework for the analysis of multidimensional policies.

The key to this result is the discreteness of our policy space. We choose a binary setup (rather than a more general one where candidates have some finite number $P_i$ of feasible positions on issue $i$) mainly for notational convenience, and because this is the simplest multidimensional framework for the analysis of the effects of fixed positions. Moreover, while many economists are used to continuous choice variables, we would argue that a setup with very few feasible positions on each issue is actually a quite realistic description of political campaigns. Generally, candidates can only communicate a limited amount of information in the campaigns and must therefore communicate (as well as commit to) clearly defined, and often binary, positions. In fact, candidates are typically not very successful communicating nuanced positions to the electorate: Kerry’s attempt, in 2004, to explain his preferences over different types of funding the Iraq war to voters (“I voted for the 84 Billion Dollars before I voted against them”) demonstrates that point. Voters often like to know clearly where a candidate stands on the issues.

In the standard model, the equilibrium policy is obviously majority-efficient. The same is true in the binary policy model if there is no difference between the candidates’ fixed positions. In contrast, however, the multidimensional nature of the binary policy model fundamentally changes this, if candidates have different fixed positions. In equilibrium, a candidate may propose majority-inefficient policies,
because adopting minority positions may *increase* his winning probability.

If candidates differ in their fixed positions, some voters will strictly prefer one of the candidates, no matter which policies the candidates choose on flexible issues. The candidates effectively only compete for the votes of the remaining *swing voters* (i.e., those individuals whose vote depends on the policies proposed by the two candidates). We identify two fundamental reasons for inefficiency results. First, the preference distribution among swing voters may differ from that of the population at-large. Since candidates care about pleasing swing voters, they may do so even if such a position goes against the wishes of the majority of the overall electorate.

Second, and more surprisingly, majority-inefficient choices can even arise if the preference distribution among swing voters is the same as in the population at large. The reason is that candidates compete for different groups of swing voters. If Candidate 0 has fewer swing voters who prefer him than Candidate 1, if both choose the same policy on flexible issues, then Candidate 0 may benefit if he chooses to deviate to a minority position. While he will lose a majority of his previous swing voter support, and win only a minority of his opponent’s previous supporters, the asymmetry between different swing voter groups implies that the effect on Candidate 0’s total vote may still be positive. We also show that asymmetric swing voter distributions arise very naturally in a setting where all voters place the same weight factors on the different issues, and ideal positions are independently distributed.

Our model also contributes to the interpretation of policy divergence. In the standard model, neglecting the median voter in order to choose policies that please minorities reduces a candidate’s winning probability, so that platform divergence cannot be rationalized as an electoral success strategy in that model. The observation that candidates, in the real world, propose divergent policies has been interpreted in the sense that the fundamental assumptions in the basic standard model, in particular policy motivation of candidates, have to be modified in order to be consistent with observed candidate behavior. For example, policy motivation of candidates and uncertainty about the position of the median voter can generate policy divergence. In contrast, in the binary policy model, policy divergence can arise in a complete information setting with purely office-motivated candidates.

The binary policy model also yields novel results when comparing the performance of plurality rule and runoff rule electoral systems in elections with more than two candidates. In standard models, candidates can either commit on all issues, or on none; in this case, we show that runoff rule weakly dominates plurality rule: There are voter preference distributions such that a majority of voters prefers the election outcome under runoff rule to the election outcome under plurality rule, but never the reverse. In contrast, if candidates differ in their fixed positions and can choose positions on some other issues, then the set of swing voters is smaller than the total electorate. We show that there are instances in which candidates choose more moderate positions under plurality rule, and where a majority of voters strictly prefers the election outcome under plurality rule to the one under runoff rule.

More generally, if the set of candidates is exogenous, or if candidates can choose all of their positions, then an electoral system that always selects the Condorcet winner from the set of candidates is very desirable.\(^2\) In contrast, our surprising result shows that, when candidates can choose some (but

not all) positions, then it may be the case that another electoral system that does not always choose the Condorcet winner (such as plurality rule) leads to a majority-preferred electoral outcome.

We present the model in the next section, and derive an axiomatic foundation of weighted-issue preferences in Section 3. In Section 4, we introduce the concept of majority-efficiency and derive the main results for competition between two candidates. Section 5 analyzes plurality rule and runoff rule in the binary policy model with three candidates. Section 6 concludes. All proofs are in the Appendix.

2 The model

2.1 Setup

Two candidates, \( j = 0, 1 \), compete in an election. Candidates are office-motivated and receive utility 1, if elected, and utility 0, otherwise, independent of the implemented policy. There are \( I \) issues, the set of issues is denoted by \( \mathcal{I} = \{1, \ldots, I\} \). Candidate \( j \), if elected, implements a policy described by \( a^j = (a^j_i)_{i \in \mathcal{I}} \), where each \( a^j_i \in \{0, 1\} \) denotes Candidate \( j \)'s position on issue \( i \) (0 can be interpreted as opposition to a particular proposal, and 1 as support of that proposal).

Candidate \( j \) can freely choose a policy on a subset of issues \( S^j \subset \mathcal{I} \), while on the remaining issues no commitment is possible. Thus, Candidate \( j \)'s type is given by \( a = (a_i)_{i \in \mathcal{I}} \), while his platform is given by \( a = (a^j_i)_{i \in S^j} \). Candidate \( j \)'s policy consists of the combination of his type and platform, so that his set of feasible policies is given by \( A^j = \{(a^j_i)_{i \in S^j} | a_i = a^j_i \text{ for all } i \notin S^j \text{ and } a_i \in \{0, 1\} \text{ for } i \in S^j \} \).

Let \( T \) be the set of voter preference types, with typical element \( \tau \). We allow \( T \) to be finite or infinite. Each voter type \( \tau \) has preferences \( \succeq_\tau \) on \( A \). Let \( \mu \) be the distribution of voter types, a probability on \( T \).

Note that this is just a frequency distribution which is known to the candidates (if \( T \) is finite then \( \mu(\{\tau\}) \) is the percent of voters in the population that are of type \( \tau \)). The timing of the game is as follows:

Stage 1 Candidates \( j = 1, 2 \) simultaneously announce policies \( a^j \in A^j \). A mixed strategy by agent \( j \) consists of a probability distribution \( \sigma^j \) over \( A^j \).

Stage 2 Each individual votes for his preferred candidate, or abstains when he is indifferent between both candidates.\(^3\) Candidate \( j \) wins if \( \mu(\{\tau | a^j \succ_\tau a^j-1\}) > \mu(\{\tau | a^j \prec_\tau a^j-1\}) \). In case of a tie between the candidates, each wins with probability 0.5.

Clearly, mixed strategy equilibria always exist since each \( S^j \) is finite.

2.2 Interpretation: Binary issues

We assume that, in each issue, candidates have (or take) one of two positions. For issues where the candidates’ positions are fixed, the binary setup is without loss of generality even if the space of generally feasible positions or characteristics is larger. Consider, for example, voters’ ethnic or racial preferences.

\(^3\)If a voter has a strict preference, then it is a weakly dominant strategy to vote for the preferred candidate. If an agent is indifferent, he could in principle vote for any candidate or abstain, but the assumption of abstention is quite natural, and none of the results in this paper depends critically on it.
Each candidate belongs to one of several different races, and individual voters have a (possibly strict) preference ranking over all realizations of this characteristic. However, if (say) one of the candidates is white, while the other one is African-American, then it is irrelevant how voters would feel about, say, an Asian candidate. Without loss of generality, for the given pair of candidates, we can model the racial characteristics of candidates as binary. This argument applies more generally for fixed positions.

A binary framework is implicitly behind several internet-based political comparison programs. For example, smartvote.ch (a cooperation project of several Swiss universities) collects the political positions of candidates in national elections by asking candidates a number of yes/no questions on different political issues. Voters can answer the same questions on a website (and also choose a weight for each issue) and are given a list of those candidates who agree with them most.\footnote{Similar programs exist for Germany (http://www.wahl-o-mat.de), Austria (http://www.wahlkabine.at/) and the Netherlands (http://www.stemwijzer.nl/)}

For some issues, it appears plausible that there are only two possible positions that candidates can take. Consider, for example, the issue of embryonic stem-cell research. Some people think that this research involves killing unborn babies, while other people do not consider blastocysts to be morally equivalent to babies and stress the potential of the research to lead to cures for awful diseases. There are only two logically consistent positions on admitting stem-cell research. In other issues, there may be more than two, but only few, possible positions. The key assumption of our model is that the possible positions are clearly distinct from each other, and we adopt the binary representation merely for technical convenience.\footnote{For the weighted-issue representation derived below, the preferences of a voter can be captured by his preferred position, and a parameter measuring the importance of each issue for him. In contrast, for issues that admit more than two positions, we would also need to know how much a voter likes his intermediately-preferred positions relative to his most and least preferred positions.} It appears intuitive that the main results from the binary model would continue to hold in a framework where each issue admits a finite number of distinct positions.

We would also argue that the binary (or, at least, discrete and small) set of choices is a realistic assumption even in issues where, in principle, very many policies can be implemented. Many economists’ favorite example of political campaign promises when explaining the Downsian model to students concerns tax rates (with a continuum of possible positions). Yet, in practice, candidates rarely commit to a particular tax rate. For example, in the 2008 Democratic presidential primary campaign, the only major tax issue was whether to suspend the federal gas tax for the summer or not — a binary issue. Similarly, in the Republican presidential primary campaign, none of the candidates “promised” to set a particular tax to some explicitly specified (intermediate) rate. Those candidates with the most concrete tax plans rather promised \textit{structural} reform, say, replacing the income tax with a consumption tax (Mike Huckabee) or abolishing the income tax (Ron Paul). A possible reason is that such promises are much more meaningful: If a candidate who makes such a promise is elected, it is easy to determine ex-post whether he followed through. On the other hand, minor changes of tax rates could mostly be undone ex-post by changing other rules in the tax code, and so the commitment quality of, e.g., a promise to lower a particular tax rate from 25\% to 23\%, is very low. This may be the reason why candidates in practice rarely campaign on explicitly promising a particular tax rate.\footnote{Also note that, when candidates do talk about taxes, they focus much more on their own or their opponent’s previous}
2.3 Interpretation and discussion: Fixed issues

A key feature of our model, made possible by the multidimensional structure, is that candidates can commit to a policy on some issues, while they are fixed to an exogenously given position in other dimensions. Thus, our model combines the commitment assumption of the standard Downsian model (with respect to $S_j$) with the assumption in citizen candidate models (Osborne and Slivinski (1996), Besley and Coate (1997)) that no commitment to a policy other than the candidate’s ideal point is possible. This appears to be a reasonable convex combination of these two central models in the literature. In reality, candidates have commitment power on some issues. If a candidate makes a promise a central campaign theme (say, to end a war), then breaking that promise is at least very costly for the candidate, and, counterexamples not withstanding, most candidates keep their central election promises.

However, there are other dimensions in which candidates are exogenously fixed and cannot easily commit to different positions. This is obviously true for characteristics of the candidate that matter for at least some voters, like gender, race, religious affiliation, experience in previous elected office. These characteristics can be interpreted as “fixed positions”, at least in the short run, which is the focus of our analysis. Other fixed positions may correspond to political issues in which a candidate has taken a stand in the past and where a commitment to a different position is not credible and/or not helpful. For example, a candidate who took a strong pro-choice stand in his past legislative voting record may not be able to credibly commit to a pro-life platform, and therefore is essentially fixed to his previous position on the abortion issue.

One important committed issue is party affiliation. When a candidate runs as a Democrat for the U.S. Congress and wins, he is committed to support his fellow Democrats in committee appointments (i.e., even if the candidate chooses to run on a conservative platform, his seat counts for determining whether the Democrats are the majority party in Congress, with the associated privileges for possibly more liberal Democratic party leaders). This may make it difficult for a Democrat to win in very conservative districts, even if he adopted a very conservative platform. For example, in the 2006 elections, many Republican House candidates tried to tie their Democratic opponents to “liberal Nancy Pelosi”, the prospective Speaker of the House in case of a Democratic majority.

Also, note that most senators from states that usually vote for Democrats in the presidential election are Democrats and vice versa. In a naive Downsian model without constraints on the policy platforms,
candidates adopt the position of the median voter in their respective district, and win with equal probability. Hence, while the Downsian model predicts that both Democratic and Republican candidates in conservative districts adopt more conservative positions than in liberal districts, it cannot explain why Republicans win significantly more of the conservative districts than Democrats and vice versa. In contrast, the fixed positions in our framework can generate this result in a natural way.

3 Weighted Issue Preferences

We now narrow down the class of preferences that we consider in our analysis. We have in mind a setting in which there are several, clearly distinct, issues. For example, suppose that the only issues in a campaign are (1) school vouchers for private schools, or (2) restrictions on gun ownership. Further, suppose that both candidates have the same position on gun ownership, but that Candidate 0 wants to issue school vouchers, which candidate 1 opposes. Then it appears reasonable to suppose that a voter’s preference for the two candidates should not depend on which common position the candidates have on gun control. In other words, if a voter prefers school vouchers to be provided if candidates oppose gun control, then this preference should not change if the candidates support gun control. Note that we do not make any assumption on the correlation of ideal position on different issues. For example, it could be the case that voters who support school vouchers are more likely to oppose gun control than those who oppose school vouchers.

We will now show that we can represent such preferences in a very simple additive form: Each voter type \( \tau \) consists of an ideal policy \( \theta \in \mathbb{A} \) that describes the voter’s most preferred policy, and a collection of issue weights \( \lambda_i, i \in \mathcal{I} \) that measure the relative importance of each issue. In the following, we will refer to such preferences as weighted-issue preferences.

To prove this result, we need an independence assumption similar to the one used by Savage to establish the existence of a Bernoulli utility function. Let \( \mathbb{A} = [0, 1]^I \) be a collection of independent lotteries over each policy issue. That is, let \( \alpha = (\alpha_i)_{i \in \mathcal{I}} \in \mathbb{A} \), with \( \alpha_i \) being the probability that position 1 is chosen on issue \( i \). We assume that each voters’ preferences \( \succeq_\tau \) can be extended from \( \mathbb{A} \) to \( \mathbb{A} \). If a voter prefers policy lottery \( \alpha \) over \( \alpha' \), then we assume that the voter would also prefer a lottery that gives \( \alpha \) with probability \( p \) and some other lottery \( \alpha'' \) with probability \( 1 - p \), to a lottery that gives \( \alpha' \) with probability \( p \) and \( \alpha''' \) with probability \( 1 - p \).

**Definition 1** A voter’s preferences \( \succeq_\tau \) on \( \mathbb{A} \) satisfy independence of irrelevant alternatives if and only if for all \( \alpha, \alpha', \alpha'' \in \mathbb{A} \) with \( \alpha \succeq_\tau \alpha' \) it follows that \( p\alpha + (1 - p)\alpha'' \succeq \alpha' + (1 - p)\alpha''' \) for any \( p \in [0, 1] \).

The following Proposition 1 uses in part the classical argument of Savage.

**Proposition 1** Suppose that a voters’s preferences can be extended from \( \mathbb{A} \) to \( \mathbb{A} \), and are complete, transitive, continuous, and satisfy independence of irrelevant alternatives on \( \mathbb{A} \). Then there exists \( \theta \in [0, 1]^I \) and a vector of weights \( \lambda \in \mathbb{R}_+^I \) such that preferences \( \succeq_\tau \) on \( \mathbb{A} \) are described by the utility function

\[
    u_\tau(a) = -\sum_{i=1}^I \lambda_i|\theta_i - a_i|,
\]

(1)
Corollary 1 If the assumption of Proposition 1 is satisfied for all voters, then the type space $T$ can be represented by $\Lambda \times \Theta$, where $\Lambda \in \mathbb{R}_I^+$ is the space of issue weights and $\Theta \in \{0, 1\}^I$ is the set of ideal points. If, additionally, all voters have the same issue weights then $T$ can be represented by $\Theta$.

4 Majority efficiency

4.1 Definition

The central result for political competition in the standard framework is that candidates propose policies that appeal to the median voter. This median voter result corresponds to a notion that political competition forces candidates to propose “popular” policies in order to win elections. “Moving toward the median” (from a non-median initial position) is popular in the standard model because it is preferred by a majority of voters. While there is no geometric notion of a “median voter” in our model, the following concept captures the same fundamental idea. A candidate’s policy is majority-efficient if a majority of voters prefers this policy to any other policy that the candidate could choose. Thus, a majority-efficient policy is the most popular policy a candidate can choose, subject to the constraints imposed by his fixed positions. The central question is whether candidates choose majority-efficient positions in equilibrium.

We first define majority preferences over policies. Because some voters may be indifferent between $a$ and $a'$, we cannot solely require that at least 50% of voters find $a$ at least as good as $a'$: If some voters are indifferent, it can be the case that also more than 50% of voters find $a'$ at least as good as $a$. Thus, our definition compares the number of voters who weakly prefer $a$ to $a'$ with the number of voters who weakly prefer $a'$ to $a$.

Definition 2 Policy $a$ is majority preferred to $a'$, denoted by $a \succeq^* a'$, if and only if $\mu(\{\tau | a \succeq^* \tau a'\}) \geq \mu(\{\tau | a' \succeq^* \tau a\})$. Furthermore, if $a \succeq^* a'$ but not $a' \succeq^* a$, we say that $a$ is strictly majority preferred to $a'$, denoted by $a >^* a'$.

A policy is majority-efficient if and only if it is a Condorcet winner relative to the set of the candidate’s feasible policies.

Definition 3 Candidate $j$’s policy $a^* \in A_j$ is majority-efficient if and only if $a^* \succeq^* a$ for all $a \in A_j$.

In general, the smaller is the set of feasible policies, the more likely it is that a majority-efficient policy in such a set exists. In fact, it is straightforward to find examples in which majority-efficient policies exist, but there is no Condorcet winner in the traditional sense (where all positions can be chosen). Thus, recognizing that candidates are in practice fixed on many positions, and restricting the comparison set to a candidate’s feasible policies, makes majority-efficiency a more applicable concept than that of a Condorcet winner.

While majority-efficiency is a candidate-specific concept, there is a clear relationship to the “majority-efficiency” of the equilibrium outcome relative to the complete set of all feasible policies, $A^0 \cup A^1$. In a pure strategy equilibrium, there is no feasible policy for either candidate that would make a majority of the voters better off if and only if the winner’s platform is majority-efficient.
Proposition 2  Let \((a^0, a^1)\) be a pure strategy equilibrium and assume without loss of generality that Candidate 0 wins the election, so that policy \(a^0\) is implemented. There is no \(\tilde{a} \in A^0 \cup A^1\) such that \(\tilde{a} \succ^* a^0\) if and only if \(a^0\) is majority-efficient. 

Note that this proposition does not claim that the candidates’ equilibrium strategies are necessarily majority-efficient. Indeed, we will show that this is not always the case, even in pure strategy equilibria.

4.2 Existence

We now show that majority-efficient policies exist in all propositions and examples considered in this paper. Thus, our results that point to policy divergence or adoption of majority of inefficient positions are not caused by lack of existence of a majority-efficient policy. We start with a result that is the main tool to show existence.

Proposition 3  Suppose that all voters have the preferences of the form given by (1) and that \(\mu\) and \(\mu'\) are distributions of voter types \(\tau \in T\). Let \(a^j \in A^j\) be a majority-efficient policy given a distribution \(\mu\) over voter types \(\tau \in T\). Define \(C(\lambda, \theta) = \{(\lambda, \theta') | \theta_i = a_i \Rightarrow \theta'_i = a_i, \text{ for all } i \in \mathcal{I}\}\). Suppose that there exists a transition probability \(v\) with \(v(\tau, C(\tau)) = 1\) for \(\mu\) a.e \(\tau \in T\) and \(\mu'(d\tau) = \int v(t, d\tau) d\mu(\tau)\). Then \(a^j\) is also majority efficient given type distribution \(\mu'\).

Proposition 3 is easiest to understand in situation with finitely many voters, where there is only one voter of each type. Suppose that \(a^j = (1, 1, \ldots, 1)\) is majority efficient. Then \(C(\lambda, \theta) = \{(\lambda, \theta') | \theta' \geq \theta\}\). If we move

and that we change each voter’s ideal point \(\theta\) to \(\theta'\) where \(\theta'_i \geq \theta_i\) for all issues \(i \in \mathcal{I}\). Then Proposition 3 implies that \((1, 1, \ldots, 1)\) remains majority efficient.

Proposition 3 suggests that the existence of majority-efficient policies is relatively robust in the binary policy model, in the sense that the distribution of voters can be changed in a generic way without affecting the existence of a majority-efficient policy. This robustness result contrasts sharply with the multidimensional Euclidean model of Plott (1967), in which each voter is indifferent between all policies that have the same distance from his bliss point. In that model, a majority-efficient policy corresponds to a Condorcet winner (as there are no fixed positions). Plott shows that a Condorcet winner exists if and only if the distribution of voter ideal points is radially symmetric around one voter’s ideal point (i.e., that voter is the “median in all directions”). This existence condition is highly non-generic: Starting from a radially symmetric distribution and changing the ideal point of only one voter usually destroys radial symmetry. This is true even if, in the spirit of our Proposition 3, we move that voter’s ideal point closer to the previous median, as long as we don’t move him exactly on the straight line that connects the median with the voter’s previous ideal point.\(^\text{10}\) In contrast, in the binary policy model, we can change an initial distribution that admits a majority-efficient policy in a large number of ways and preserve existence.

\(^{10}\)Equilibrium non-existence also obtains generically in the Euclidean model with policy-motivated candidates, if the number of policy dimensions is larger than two (see Duggan and Fey (2005)).
The key reason for robustness in our model is the discreteness of the feasible policy space, as it ensures that the number of voters who prefer the majority-efficient position is bounded away from 50%. Formally, let \( a^\ast \in A^j \) be majority-efficient. Discreteness of \( A^j \) ensures that \( \min_{a \in A^j \setminus \{a^\ast\}} \mu(\{\tau | a^\ast \succeq_{\tau} a\}) > 0.5 \) for generic distributions \( \mu \). Clearly, perturbing \( \mu \) slightly does not change this inequality, ensuring that \( a^\ast \) remains majority-efficient. The same would apply in a two-dimensional Downsian model with Euclidean preferences if the feasible policy space is finite. In contrast, if \( A^j \) is the entire Euclidean space, then this minimum is 0.5, and thus we can change \( \mu \) slightly to \( \mu' \) such that \( \mu'(\{\tau | a^\ast \succeq_{\tau} a^j\}) < 0.5 \) for some \( a^j \in A^j \), i.e., \( a^j \) is preferred by a majority of voters to \( a^\ast \), and \( a^\ast \) is no longer majority-efficient.

A technical advantage of our model becomes apparent in the proof of Proposition 3 (as well as in the proof of Proposition 6 below). For any two policies, a voter’s net-preference for the first policy is a random variable, and more specifically, a weighted sum of binary random variables, as preferences are of the weighted-issue type. Comparing two policies is then equivalent to determining the median of this weighted sum of binary random variables. Thus, instead of facing a possibly difficult problem of discrete mathematics, we can apply standard tools of probability theory when analyzing our model.

Corollary 2 below shows that, if there are only two pledgeable issues and if all voters have the same issue weights in their respective utility functions, then majority-efficient policies always exist.

**Corollary 2** Suppose that preferences are given by (1). Then majority-efficient policies exist for \( S^j \leq 1 \). If all voters have the same issue weights then majority-efficient policies also exist for \( |S^j| = 2 \).

The significance of Corollary 2 is that a number of interesting examples can already be generated with one or two flexible issues, and in all these applications, existence is guaranteed.\(^{11}\)

The next corollary shows that, if all voters have the same issue weights and the distribution of preferred positions is independent across issues, then majority-efficient policies always exist. It is easy to see that all policies are majority-efficient if all voters have the same issue weights and the distribution of ideal points is uniform. We then increase the proportion of voters who prefer position 1 on issue \( i \) to \( p_i > 0.5 \). Such a transformation satisfies the condition of Propositions 3, so that a policy \( a \) with \( a_i = 1 \) remains majority-efficient. Repeated application of this argument proves the following result.

**Corollary 3** Suppose that preferences are given by (1), that voters have the same issue weights, and that all marginal distributions \( \mu_{\Theta_i} \) are independent. Then policy \( a^j \) is majority-efficient if and only if \( a^j_i = 1 \) when \( \mu_{\Theta_i}(1) > 0.5 \) and \( a^j_i = 0 \) when \( \mu_{\Theta_i}(1) < 0.5 \) for all \( i \in S^j \).

Finally, even if a preference distribution satisfies none of the sufficient conditions stated above, it is straightforward to check whether a majority-efficient position exists: First, find the majority-preferred position on each (pledgeable) issue. The corresponding policy is the unique candidate for a majority-efficient position (for any other platform, one could change a majority-inefficient position in one issue.

\(^{11}\)A result similar to Corollary 2, but based on very different assumptions, is derived by Bade (2006). She shows, in a two-dimensional Euclidean model, that an equilibrium of the game between candidates exists (located at the median in each dimension), if candidates are uncertain about the shape of voters’ indifference curves and are uncertainty-averse (rather than expected-utility maximizers).
and a majority would prefer the new platform). Second, compare this platform with any other feasible platform that differs from the candidate in positions on at least two issues; if no other platform is majority-preferred, then the candidate platform is majority-efficient.

4.3 Adoption of minority positions in the binary policy model

Identical fixed positions. To our knowledge, all existing deterministic voting models in which candidates have a majority-efficient platform have the feature that both candidates choose their majority-efficient policy (if one exists) as their equilibrium platform. This is certainly true for the one-dimensional Downsian model, in which both candidates choose the median voter’s bliss point. Also, while a majority-efficient position rarely exists in a multidimensional Downsian model, if it does, then both candidates choose it as a platform. These results suggest that in general, if candidates can choose majority-efficient positions, they will always do so in equilibrium. Perhaps surprisingly, this conjecture turns out to be false in the binary policy model, if candidates differ in their fixed positions. Only if candidates have exactly the same fixed positions, then it is guaranteed that they choose a majority-efficient platform, provided that one exists.

Proposition 4 Suppose that $A^0 = A^1$.

1. Then $(a^0, a^1)$ is a pure strategy equilibrium if and only if both $a^0$ and $a^1$ are majority-efficient.

2. Suppose a majority-efficient policy exists. If $(\sigma^0, \sigma^1)$ is a mixed strategy equilibrium, then every policy in the support of $\sigma^0$ and $\sigma^1$ is majority-efficient.

Proposition 4 highlights the role of fixed positions for all majority-inefficiency results in this paper, and indeed, for related results when voters do not necessarily have weighted-issue preferences. One interesting issue is, for example, whether there are any models, with general preferences for voters, in which office-motivated candidates have a strict incentive to differentiate from their opponent for electoral gain. Proposition 4, point 1, shows the crucial role of fixed positions for any such model. If there are no differences between the candidates’ fixed positions, then a pure strategy equilibrium with differentiation exists only if there are two (or more) majority-efficient policies. Moreover, even in this case (say, there are two majority-efficient policies, $a$ and $b$), there are also equilibria in which both candidates choose the same policy (that is, $(a, a)$ and $(b, b)$ are equilibria), and in all equilibria, candidates are indifferent between playing $a$ and $b$, so there is never a strict incentive for candidates to differentiate. The same is obviously true for symmetric mixed strategy equilibria.

Swing voters. We now analyze whether candidates select majority-efficient platforms if they differ in their fixed positions. Our main points can already be made in a very simple setup where both candidates are flexible on only one (and the same) issue. However, it will be clear that the basic principles identified here do not rely on this simple structure and apply more generally.

\footnote{We are grateful to Ernesto Dal Bo for raising this question.}
Without loss of generality, assume that both candidates are flexible on issue 1, while they are fixed in all of the first $I - 1$ issues. It is useful to define the notion of a swing voter as a marginal supporter. We say that a voter is a swing voter for Candidate 1 if he prefers Candidate 1 if both candidates choose the same policy on issue 1, but prefers Candidate 2 if Candidate 2 proposes the voter’s preferred position on issue 1 while Candidate 1 proposes the opposite position.

**Definition 4** Voter type $\tau = (\lambda, \theta)$ is a swing voter for Candidate $j$ if

$$- \sum_{k=1}^{I-1} \lambda_k |\theta_k - a_k^j| < - \sum_{k=1}^{I-1} \lambda_k |\theta_k - a_k^j| < - \sum_{k=1}^{I-1} \lambda_k |\theta_k - a_k^j| + \lambda I.$$  

Let $SV_j$ denote the number of swing voters for Candidate $j$, i.e., $SV_j = \mu(\{\tau \mid \tau \text{ satisfies } (2)\})$. Furthermore, let $\xi_j = \mu(\{\tau \mid \tau \text{ satisfies } (2) \text{ and } \theta_I = 0\})$ denote the percentage of voters who prefer position 0 in issue 1 among the swing voters of Candidate $j$.

A voter is either a swing voter, or a core supporter of one of the candidates (i.e., prefers the fixed positions of one candidate so much that he would never vote for his opponent, independent of the candidates’ positions on the flexible issue). Note that, without fixed issues, all voters are swing voters.

Obviously, equilibria with minority positions can occur if one candidate is so much stronger than his opponent that he wins, no matter what his policy positions are. To rule out such trivial cases of majority-inefficient equilibria, Proposition 5 concentrates on vote-maximizing equilibria where the candidates choose positions that maximize the number of swing voters (and hence, of voters in general) who vote for them, given their opponent’s position. The number of swing voters who vote for the two candidates, depending on their platforms, is given by the following matrix. For example, if Candidate 0 plays 0 and Candidate 1 plays 1, then all swing voters who prefer 0 on issue 1 vote for Candidate 0 and vice versa.

<table>
<thead>
<tr>
<th></th>
<th>Cand. 0</th>
<th>Cand. 1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cand. 0</td>
<td>$0$</td>
<td>$0$</td>
</tr>
<tr>
<td>Cand. 1</td>
<td>$SV_0, SV_1$</td>
<td>$SV_0, SV_1$</td>
</tr>
<tr>
<td></td>
<td>$\xi_0 SV_0 + \xi_1 SV_1, (1 - \xi_0) SV_0 + (1 - \xi_1) SV_1$</td>
<td>$(1 - \xi_0) SV_0 + (1 - \xi_1) SV_1, \xi_0 SV_0 + \xi_1 SV_1$</td>
</tr>
</tbody>
</table>

Figure 1: Swing voters voting for Candidate 0 and 1

The following Proposition 5 provides necessary and sufficient conditions for pure strategy vote-maximizing equilibria.

**Proposition 5** Suppose that candidates are flexible only in issue 1 and maximize the number of voters who vote for them. $(0, 0)$ is a vote-maximizing equilibrium if and only if

$$\frac{SV_1}{SV_0} \in \left[ \frac{1 - \xi_0}{\xi_1}, \frac{\xi_0}{1 - \xi_1} \right].$$

$(1, 1)$ is a vote-maximizing equilibrium if and only if

$$\frac{SV_1}{SV_0} \in \left[ \frac{\xi_0}{1 - \xi_1}, \frac{1 - \xi_0}{\xi_1} \right].$$
(0, 1) and (1, 0) are vote-maximizing equilibria if and only if

\[ SV_1 = SV_0 \text{ and } \xi_0 + \xi_1 = 1. \]  

(5)

In the latter case, (0, 0) and (1, 1) are also equilibria.

Clearly, (5) holds only in highly non-generic circumstances. Disregarding this case, at most one of

\[ \text{equations (3) and (4) can hold, since the lower limit of the interval in (3) is the upper limit of the interval in (4), and vice versa (so that, generically, one of the two intervals is an empty set, as its lower limit is a higher number than its upper limit). Without loss of generality, suppose that the interval on the right hand side of (3) is non-empty. The proposition tells us that, for (0, 0) to be an equilibrium, the ratio of swing voters must neither be too low nor too high. We now use Proposition 5 to identify two fundamentally different incentives for candidates to choose majority-inefficient policies in equilibrium.}

Non-representative swing voters. The first reason for equilibrium majority-inefficiency is quite straightforward: The preferences of swing voters (as captured by \( \xi_0 \) and \( \xi_1 \)) may not be representative for the preferences of the population at large. The following example illustrates this point.

Example 1 There are two issues. The first issue is fixed at 0 for Candidate 0 and at 1 for Candidate 1. Both are flexible on issue 2. A voter’s type, consisting of issue weights and ideal positions, is given by \((\lambda_1, \lambda_2, \theta_1, \theta_2)\). Suppose there are only four types \( T = \{\tau_1, \tau_2, \tau_3, \tau_4\} \), where \( \tau_1 = (\bar{\lambda}, \lambda, 0, 0), \tau_2 = (\lambda, \bar{\lambda}, 1, 0), \tau_3 = (\lambda, \bar{\lambda}, 0, 1), \text{ and } \tau_4 = (\lambda, \bar{\lambda}, 1, 1), \) where \( \bar{\lambda} > \lambda \). That is, \( \tau_1 \) and \( \tau_2 \) consider the first, fixed issue, to be the more important one and therefore are core supporters of the candidates. In particular, \( \tau_1 \) always votes for Candidate 0 and \( \tau_2 \) always votes for Candidate 1, no matter what policies the candidates choose on issue 2. In contrast, \( \tau_3 \) and \( \tau_4 \) put a high weight on issue 2 and are thus the swing voters for Candidate 0 and 1, respectively: If candidates choose different positions on issue 2, they would vote for the candidate who picks their preferred position.

Since all swing voters prefer \( a_2 = 1 \) (i.e. \( \xi_0 = \xi_1 = 0 \)), it is clear that both candidates choose \( a_2 = 1 \) in the unique vote-maximizing equilibrium. Candidate 0 receives the votes of types \( \tau_1 \) and \( \tau_3 \), while Candidate 1 receives the votes of \( \tau_2 \) and \( \tau_4 \). This equilibrium is not majority-efficient, if a majority of voters prefers position 0 on the second issue (i.e., if \( \mu(\{\tau_1\} \cup \{\tau_2\}) > \mu(\{\tau_3\} \cup \{\tau_4\}) \)).

There may also be equilibria in which candidates do not maximize votes and in which one or both candidates choose policy \( a_2 = 0 \). For example, if \( \mu(\tau_1) > 1/2 \), then Candidate 0 wins whatever policies the candidates choose, so that all strategy combinations are equilibria. However, if \( \mu(\tau_1) < 1/2 \) and \( \mu(\tau_2) < 1/2 \), then, in any equilibrium, the election winner chooses \( a_2 = 1 \), and thus, by Proposition 2, the utility of a majority of voters could be increased relative to the equilibrium.

The effect of Example 1 is also present in the probabilistic voting model pioneered by Lindbeck and Weibull (1987) (henceforth PVM; see also Lindbeck and Weibull (1993), Coughlin (1992) or Persson and Tabellini (2000) for a review of the various developments of this literature). In the PVM, voters are divided in different groups according to their utility from policies (that are choice variables for candidates). In addition, all voters receive a common utility shock (like valence), and an idiosyncratic
“ideology” shock. In equilibrium, both candidates choose the same policy platform, which maximizes a weighted sum of utility of different groups in society. The weight of a group in the candidates’ objective function is higher than its population share, if voters in the group are more “movable”, i.e. if they are relatively likely to switch to a candidate who offers them a more favorable policy position.

Example 1 is based on a analogous effect, but provides the result in a much simpler setting: In the PVM, there are a number of exogenous random shocks, and members of a group with the same interests about issues must be sufficiently differentiated “ideologically” (i.e., in a dimension that cannot be addressed by the candidates) for an equilibrium to exist. In contrast, Example 1 is deterministic.

Asymmetric swing voter distribution. However, even if preferences of swing voters are representative, majority inefficient policies may be adopted. For example, suppose that 60% of the population prefers issue 0, and that the same is true among swing voters, i.e., \( \xi_0 = \xi_1 = 0.6 \). Then, \((0,0)\) could be an equilibrium, but only if the distribution of swing voters is sufficiently balanced, i.e., if \( 2/3 \leq SV_1/SV_0 \leq 1.5 \). In contrast, if Candidate \( j \) has 1/3 fewer swing voters to defend than his opponent, then Candidate \( j \) will deviate and select position 1 on issue \( I \).

Consider Figure 2, where we focus only on swing voters. Panel (A) shows the distribution of swing voters when both candidates adopt the majority-efficient position 0 on issue \( I \). The black part of each bar indicates the voters who prefer 0 on issue \( I \), whereas the gray portion stands for the minority that prefers position 1. Note that all swing voters to the left of the dashed vertical line vote for Candidate 0, while those to the right vote for Candidate 1. Now suppose that Candidate 0 adopts policy 1 on issue \( I \). Then he loses the black parts of both swing voter groups, and wins both gray parts. Since Candidate 1 starts out with more swing voters in (A), this results in the net gain for Candidate 0 indicated in (B).

To analyze this effect we now consider a situation in which the distribution of preferences over issues is independent (as in Corollary 3). Thus, the percentage of voters in the general electorate who prefer position 0 on issue \( I \) is equal to the corresponding percentage among both swing voter group, i.e. \( \mu(\tau|\theta_i = 0) = \xi_0 = \xi_1 \). Proposition 6 first provides sufficient conditions that guarantee that a candidate
can win by choosing a majority-efficient policy. In general, each candidate will have some “strong” characteristics, i.e., those issues on which a majority prefers his fixed positions. Intuitively, a candidate is the better, the more important these issues are and the higher the majority that supports the candidate’s position. Suppose that issues can be paired such that, for each issue in which Candidate 0 is weak, there is another, more important (in terms of $\lambda$) issue in which he is strong, and the majority preferring Candidate 0 in his strong issue is larger than the majority favoring his opponent in Candidate 0’s weak issue. In this case, Proposition 6 shows that the better Candidate 0 can win by choosing a majority-efficient policy.

In contrast, if such ranking of candidates is not possible, the distribution of swing voters may become sufficiently asymmetric such that majority inefficient positions become attractive. The second part of Proposition 6 shows that there are robust cases in which majority inefficient policies are attractive, even if the preference distribution is independent. More precisely, consider two candidates with given strengths and weaknesses in fixed issues. For any distribution of ideal points there exists an open set of issue weights such that one candidate can (only) win by choosing a majority inefficient policy against an opponent who chooses a majority-efficient policy. In these cases, both candidates choosing a majority-efficient policy is not an equilibrium.

**Proposition 6** Suppose that preferences are given by (1) and that voters have the same issue weights so that type space $T = \Theta$. Assume that all marginal distributions $\mu_\theta$ are independent. Assume that both candidates are fixed on the same set of issues $F$. Suppose that for each $i \in F$ on which candidate 1 is fixed to the majority preferred position (i.e., $\mu(\{\theta | \theta_i = a_1^i\}) \geq 0.5$), there exists an issue $\phi(i) \in F$ on which candidate 0’s position is preferred by a larger majority (i.e. $\mu(\{\theta | \theta_{\phi(i)} = a_0^{\phi(i)}\}) > \mu(\{\theta | \theta_i = a_1^i\})$), where $\phi$ is a one-to-one mapping.

1. If $\lambda_{\phi(i)} > \lambda_i$ for all $i$ with $\mu(\{\theta | \theta_i = a_1^i\}) \geq 0.5$ then Candidate 0 wins by choosing a majority-efficient policy.

2. Let $j \notin F$. There exists an open set $\Lambda \times M_j \subset \mathbb{R}_+^{|F|} \times (0.5, 1]$ such that for all utility weights $(\lambda_1, \ldots, \lambda_I) \in \Lambda$ and all $\mu$ with $\mu(\{\theta | \theta_j = a_j\}) \in M_j$, the following holds:

   If Candidate 1 selects a majority-efficient policy, then Candidate 0 wins if and only if he selects a majority inefficient policy.

To get an intuition for Proposition 6, it is useful to consider special cases with only a few fixed positions. First, consider the case of only one fixed position. (In order to make this situation fit the condition in Proposition 6, we can add a spurious second issue with $\lambda_2 = 0$). It is easy to see that Candidate 0, who has the advantage on the fixed issue, can guarantee himself a victory by choosing the majority-preferred position on the flexible issue: His opponent can either also choose the majority-preferred position on issue 3, in which case Candidate 0 wins with the support of the majority that prefers his fixed position; or Candidate 1 can take the other position on issue 3, in which case, either nothing changes (if $\lambda_1 > \lambda_3$), or, if $\lambda_1 < \lambda_3$, Candidate 0 is supported by the majority of people who prefer the majority-preferred position on issue 3.
Second, consider the case of two fixed positions and one flexible one. Suppose that candidate 0 is fixed to (0, 0), while candidate 1 is fixed to (1, 1). Except for relabeling of candidates, there are three possibilities: (a) for both issues 0 is preferred by majority of voters; (b) a majority prefers 0 on the first and a smaller majority prefers 1 on the second issue, and \( \lambda_1 > \lambda_2 \); (c) again a majority prefers 0 on the first and a smaller majority prefers 1 on the second issue, and \( \lambda_1 < \lambda_2 \). Proposition 6 implies that candidate 0 wins using a majority-efficient position on the third issue in cases (a) and (b). We now focus on case (c) which provides and example for the second statement of Proposition 6 that it is possible that candidate 0’s policy on the third issue is majority inefficient.

Suppose that \( \mu(\tau|\theta_1 = 0) = 0.7 \) and \( \mu(\tau|\theta_2 = 1) = 0.6 \), and that \( \lambda_1 < \lambda_2 \); thus, a majority prefers Candidate 1’s position in the more important fixed issue, but the majority of people who prefer Candidate 0 in the less important issue is even larger. If both candidates adopt the same position on issue 3 (in particular, if both adopt the majority-efficient position), then 60% of voters vote for Candidate 1 (namely, all \( \theta_3 = 1 \) types). If \( \lambda_2 - \lambda_1 < \lambda_3 < \lambda_2 + \lambda_1 \), then Candidate 0’s swing voters have ideal point (1, 0), and the mass of this type is \( 0.3 \cdot 0.4 = 0.12 \). Candidate 1’s swing voters have ideal point (0, 1), with mass \( 0.7 \cdot 0.6 = 0.42 \). If \( \mu(\tau|\theta_3 = 0) = 0.5 \), then it is clearly attractive for Candidate 0 to adopt a different position than Candidate 1, as this wins half of Candidate 1’s swing voters while losing half of Candidate 0’s own swing voters. The net gain of 15% is sufficient for Candidate 0 to win the election.

More generally, one can check that Candidate 0 wins votes by adopting the majority-inefficient position 1 as long as \( \mu(\tau|\theta_3 = 0) \in (1/2, 7/9) \). The additional votes are sufficient to swing the election in Candidate 0’s favor as long as \( \mu(\tau|\theta_3 = 0) \in (1/2, 16/27) \). In this case, candidates play effectively a matching pennies game in which Candidate 1 (0, respectively) wins if the candidates choose the same (different, respectively) positions. Thus, both candidates choose the majority-inefficient position with probability 1/2, and the election winner implements the majority-inefficient position with probability 1/2.

Proposition 6 also demonstrates clear differences between the PVM and the binary policy model. In the PVM, candidates choose the same policy in equilibrium, and the only reason for why candidates may “cater” to particular groups (more than corresponds to these groups’ population weights) is that they may care more about a particular issue and hence are electorally more responsive than the general population. In contrast, all voters in the binary policy model have the same issue weights in their utility functions. Thus, the result that it may be optimal for a candidate to cater to the minority is based on a different reasoning. Also, an equilibrium in which one candidate has an incentive to cater to a minority is in mixed strategies, and therefore candidates’ proposed policies diverge with probability 1/2.

The multidimensional structure of our model is crucial for the potential optimality of majority-inefficient positions for candidates. In the one-dimensional standard model, there is only one group of swing voters (i.e., those at or close to the median of the distribution). Note that this is true even if candidates are constrained to choose their position only from a subset of the policy interval. Candidates have to deliver a policy that is popular with the median voter. Furthermore, a policy that the median voter likes is also preferred by a majority of the population. Therefore, in the equilibrium of the standard model, candidates choose majority-efficient policies in order to maximize their chance of winning. This leads to a presumption that candidates should pick popular positions in order to maximize their probability of winning, but this argument logically only applies in a one-dimensional framework.
There is a large literature that tries to explain, within the Downsian model, the empirical observation that candidates often propose considerably divergent policies. Candidates may diverge even though this decreases their winning probability, because they care about the implemented policy (see, e.g., Wittman (1983), Calvert (1985), Roemer (1994), Martinelli (2001), Gul and Pesendorfer (2006)). Other papers obtain policy divergence with office-motivated candidates, but assume incomplete information among voters about candidates characteristics (Callander (2008), Kartik and McAfee (2006)) or among candidates about the position of the median voter (Aragones and Palfrey (2002), Castanheira (2003), Bernhardt, Duggan, and Squintani (2006)).

In contrast to all previous papers, policy divergence can arise in the binary policy model in a full information environment, and, unlike in models with policy-motivated candidates, divergence increases a candidate’s probability of winning.

5 Plurality versus runoff elections

5.1 Motivation and previous literature

In this section, we depart from the two-candidate setting in order to provide another application in which the binary policy model provides novel results that cannot be obtained in standard models. When three or more candidates compete in an election, the problem arises that the Condorcet winner (i.e., the candidate who is preferred by a majority against any opponent) may not win the election. While “third party candidates” (i.e., in the U.S., candidates neither belonging to the Democratic nor the Republican party) often attract only a small number of votes, they can nevertheless affect the election outcome. For example, in three out of the last four U.S. presidential elections, the election winner did not receive an absolute majority of the votes cast, indicating the importance of votes for third party candidates. This has created concern that “spoiler candidates” can, in general, change the election outcome under plurality rule away from the Condorcet winner.

A number of alternative electoral systems have been proposed to deal with this perceived problem of plurality rule. For example, several local jurisdictions in the U.S. have switched to “instant runoff voting” (IRV) as electoral system for municipal elections (e.g. Minneapolis, San Francisco, Oakland). Another electoral system that has many supporters in the academic community is approval voting (see, e.g., Brams and Fishburn (1978), Cox (1987), Myerson and Weber (1993)).

Our analysis differs in two respects from the existing literature: First, virtually all comparisons of

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13A third branch of literature, which is less directly related to this paper, explains policy divergence as entry deterrence by two dominant parties (e.g., Palfrey (1984), Callander (2005)).

14IRV is a voting system for single winner elections in which voters rank candidates in order of preference. If no candidate receives an overall majority of first preferences in an IRV election, the candidate with the fewest votes is eliminated, and his votes are transferred according to his voters’ second preference. All votes are retallied, and the procedure is repeated until one candidate achieves a majority.

15Under approval voting, each voter is free to vote for as many candidates as he chooses. The candidate with the most votes is elected. The intuition for the advantage of approval voting is as follows: Under plurality rule, voters may ignore a moderate candidate and focus on two extreme candidates, if they fear that the moderate has no chance of winning. In contrast, under approval voting, voters can vote for both their preferred extreme candidate and the moderate candidate, so that they don’t have to fear to waste their vote. This should make it easier for moderate candidates to win.
alternative voting institutions and the effects of third-party candidates are set within a one-dimensional framework. Second, candidates are either assumed to be able to commit to any position, or not at all. In contrast, in the binary policy model, we can analyze both the effect of the electoral system on the election winner, and the effect of a third party candidate on the endogenous part of the platforms of major party candidates. This new endogenous effect can be of crucial importance for the welfare comparison of electoral systems. In fact, we show that this effect can overturn the standard welfare comparison of runoff versus plurality electoral systems.

The effect of third party candidates has been analyzed by the theoretical literature in the Downsian model and the citizen candidate framework. In a Downsian model with three candidates, no pure strategy equilibrium exists when candidates choose simultaneously, assuming that voters vote sincerely. Thus, many models assume some exogenous distinction between candidates to obtain pure strategy equilibria. In Palfrey (1984) and Callander (2005), one candidate (who is interpreted as the “third party candidate”) chooses his platform after the two “main” candidates. In Palfrey, the threat of entry by the third party candidate forces the two main candidates to choose positions that are equidistant from the median. The third party candidate (who is supposed to maximize his vote share if he cannot win) chooses a more extreme position than either candidate, and one that is as close as possible to one of the two main candidates, and loses for sure. He also induces the loss of the candidate next to whom he chooses to position himself. Thus, in this framework, policy-motivated third party candidates would either run on a platform opposite to what they really prefer (if they can commit), or not run at all (if they cannot commit).

Osborne and Slivinski (1996) analyze the issue of third party candidates in a citizen candidate model, and also compare plurality rule and runoff rule voting systems. The set of equilibrium positions in two candidate races is more moderate under runoff rule than under plurality rule, so that, from the point of view of the median voter and the majority of the population, runoff rule is a better electoral system than plurality rule. Under plurality rule, there can be equilibria with a spoiler candidate who enters the election in spite of having no chance of winning. This happens if the spoiler candidate is located between the two main candidates and draws more votes from the candidate who is farther away from the spoiler (this is possible only if the distribution of voters is asymmetric).

5.2 Plurality vs. runoff when all positions or no positions can be chosen

As a benchmark for our analysis we start with the Downsian assumption that candidates can choose all of their positions (or, equivalently, that all candidates have the same fixed positions). In all of the following, we assume that voters vote “sincerely”, i.e., for their most preferred candidate. In elections with three or more candidates, there are usually many Nash equilibria in undominated strategies. How-

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16To our knowledge, the entire literature that compares the effects of electoral systems assumes sincere voting because voting equilibria in plurality elections with more than two candidates abound (and cannot be significantly narrowed down through standard refinements). Therefore no conclusions could be reached with the assumption of strategic voters. Also note that sincere voting is a Nash equilibrium in undominated strategies under plurality rule.
ever, the equilibrium in which voters vote sincerely is a natural focal point. Note that Propositions 7 and 8 in this section hold for any general voter preferences, not just when voters have weighted-issue preferences.

**Proposition 7** Suppose that there are $n$ candidates with the same choice set: $A^0 = A^1 = \cdots = A^n$. Assume that there exists a unique majority-efficient position $a^* \in A^0$.

1. Under run-off rule, there exists an equilibrium in which all candidates choose $a^*$.

2. Let $\tilde{A}$ be the set of all policies that are preferred to $a^*$ by more than $1/n$ of the voters. If $\tilde{A}$ is non-empty, then there is no equilibrium under plurality rule such that all candidates always play $a^*$, and the probability that a candidate with a majority-inefficient position wins the election is strictly positive. If $\tilde{A}$ is empty, then the unique equilibrium under both plurality and run-off rule is that all candidates choose $a^*$.

When all opponents choose $a^*$ under run-off rule, then the best response is to play $a^*$ as well: Clearly, there is no majority-inefficient policy with which a candidate could win an outright majority in the first round, and to have a chance of winning in the second round against an opponent who chooses $a^*$, a candidate has to choose $a^*$ as well.

In contrast, there is usually no equilibrium under plurality rule in which all candidates choose majority-efficient positions. If all $n-1$ opponents choose the majority-efficient position, then a candidate can win for sure by playing some element of $\tilde{A}$. As a consequence, the winning position is majority-inefficient with a strictly positive probability.

As an example, suppose that there are three candidates and only one issue, and that a proportion $p \in (1/2, 2/3)$ prefers position 0, while the remainder of the electorate prefers position 1. Under run-off rule, the unique equilibrium is that all three candidates choose position 0.

Under plurality rule, a candidate is guaranteed to be the winner if both opponents take the opposite position, and each candidate wins with probability $1/3$ if all candidates take the same position. Clearly, there is no pure strategy equilibrium. It is easy to see (from the symmetry of the two strategies with respect to the candidates’ winning chance) that, in the unique mixed strategy equilibrium, each candidate randomizes between both policies with probability $1/2$ each. Consequently, the probability that the majority-inefficient policy 1 wins is $1/2$ in equilibrium.

We now turn to the case that politicians differ in their fixed positions and cannot choose positions on any issue. Again, run-off rule again leads to (weakly) better results than plurality rule in this setup.

**Proposition 8** Let $a_i$ denote the (entirely fixed) position of Candidate $i$, and suppose that there is a Condorcet loser $a_n$ (i.e., Candidate $n$ would lose a two-way race against any other candidate). Under plurality rule, the election winner may be any policy, while $a_n$ is not a possible election outcome under

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17 Degan and Merlo (2006) suggest that, empirically, most observed voting behavior is consistent with the assumption that voters vote sincerely.

18 This assumption is clearly satisfied if there is a transitive majority preference ranking of candidates $d_0 >^* a_1 >^* \cdots >^* a_n$, but our assumption is more general.
runoff rule. Also, the election outcome under runoff rule, \(a_R\), is weakly majority preferred to the election outcome under plurality rule, \(a_P\).

The Condorcet loser \(a_n\) can certainly have the most first preferences in the electorate (and hence win under plurality rule), but cannot receive an outright majority in the first round of a runoff system, and loses the runoff against any opponent (hence, cannot win in a runoff system). The intuition for why \(a_R\) is always (at least weakly) preferred to \(a_P\) is that the plurality rule winner is at least guaranteed to proceed to the second round in a runoff system; thus, for a majority of voters, the outcome under runoff is at least as good as the plurality outcome.

### 5.3 Runoff versus plurality with some fixed and some flexible positions

Together, the results of the previous section show that runoff rule generally dominates plurality rule in terms of electoral outcome, if candidates can either choose all of their positions, or none of them. Since these setups are the only ones that can be analyzed in standard models, it is tempting to conclude that runoff rule is generally (at least weakly) better for society than plurality rule. However, our results so far warn against drawing such a conclusion prematurely.

Indeed, we now also show here that, if candidates differ in their sets of feasible policies, then qualitatively different results can arise in the binary policy model. The intuition is that the set of relevant swing voters differs between plurality rule and runoff rule. In our example, the set of swing voters is smaller under plurality rule than under runoff rule, but it is nevertheless more representative for the population at large.

The following Example 2 is a somewhat more elaborate version of Example 1, in which the two candidates cater to the minority, because minority types are more willing to change their voting behavior than majority types. In other words, among swing voters, the (overall) minority is the majority. We then introduce a third candidate who has no chance of winning, but changes the composition of swing voters who are relevant for the two main candidates in a way that now the overall majority in the population is also the majority of swing voters; thus, under plurality rule, the main candidates now choose the majority-efficient policy. In contrast, under runoff rule, the main candidates essentially ignore the third candidate because they care only about their showdown against each other in the second round, after the third candidate is eliminated, and the equilibrium has the same inefficient features of Example 1.

**Example 2** Candidate 0 is fixed to \((0, 0)\), and Candidate 1 is fixed to \((1, 0)\), on the first two issues. Both these candidates can freely choose their position on the third issue. Candidate 2 is fixed at \((1, 1, 1)\). The following Table 1 gives the proportions and issue weights of all voter types.

Note that 82% of the population strongly dislike Candidate 2’s position on the second issue and issue two is very important compared to the other issues. Thus, Candidate 2 is truly a “spoiler” who has no hope of actually winning the election: Only 18% of the population would ever vote for Candidate 2, so he can neither be the top vote getter in a plurality election nor win the first or second round in a runoff system. On the third issue, 72% of the population prefer policy 0 while 28% prefer policy 1, so that policy 0 is majority-efficient (for both Candidate 0 and 1).
Table 1: Voter distribution

If only the first two candidates stand for election, the first two voter types will always vote for their respective candidate, independent of the positions that candidates take on issue 3; in contrast, the remaining voters are the potential swing voters between Candidates 0 and 1. Note also that, among swing voters, a majority (28% versus 22%) prefers policy 1 on the third issue. Depending on the policies that the candidates choose, the vote shares are given in Table 2. Note that a candidate’s vote share is always 3% higher when he chooses \( a_3 = 1 \). Thus, in vote shares, policy \( a_3 = 1 \) is strictly dominant, and it is weakly dominant in terms of the winning probability. In the equilibrium in weakly dominant strategies, both candidates therefore choose policy \( a_3 = 1 \).  

Table 2: Candidates’ vote shares

Under runoff rule with three candidates, the same logic applies for the policy choice of Candidates 0 and 1: Both are guaranteed to proceed into the second round, because each has a core support (from one of the first two types) that is larger than the 18% that Candidate 2 gets from the last three preference groups. Thus, in their policy choice under runoff rule, Candidates 0 and 1 face exactly the same problem as if Candidate 2 did not exist, and therefore will choose the same positions as in the two-candidate election above: Thus, \( a_3^0 = 1 \) and \( a_3^1 = 1 \) under runoff rule, and Candidate 0 wins with policy (0, 0, 1).

Now consider what happens under plurality rule in a three candidate election: Candidate 2 attracts the votes of the last three voter types and effectively removes them from the set of swing voters who are relevant for Candidates 0 and 1. Choosing policy 1 instead of 0 on the third issue would now only attract 6%, but, at the same time, lose 10%. The unique equilibrium in undominated strategies has \( a_3^0 = 0 \) and

<table>
<thead>
<tr>
<th>Proportion</th>
<th>Preferred policy</th>
<th>( \lambda_1 )</th>
<th>( \lambda_2 )</th>
<th>( \lambda_3 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>26%</td>
<td>(0, 0, 0)</td>
<td>2</td>
<td>5</td>
<td>1</td>
</tr>
<tr>
<td>24%</td>
<td>(1, 0, 0)</td>
<td>2</td>
<td>5</td>
<td>1</td>
</tr>
<tr>
<td>10%</td>
<td>(0, 0, 0)</td>
<td>1</td>
<td>5</td>
<td>2</td>
</tr>
<tr>
<td>10%</td>
<td>(1, 0, 0)</td>
<td>1</td>
<td>5</td>
<td>2</td>
</tr>
<tr>
<td>6%</td>
<td>(0, 0, 1)</td>
<td>1</td>
<td>5</td>
<td>2</td>
</tr>
<tr>
<td>6%</td>
<td>(1, 0, 1)</td>
<td>1</td>
<td>5</td>
<td>2</td>
</tr>
<tr>
<td>9%</td>
<td>(0, 1, 1)</td>
<td>1</td>
<td>5</td>
<td>2</td>
</tr>
<tr>
<td>7%</td>
<td>(1, 1, 1)</td>
<td>1</td>
<td>5</td>
<td>2</td>
</tr>
<tr>
<td>2%</td>
<td>(1, 1, 0)</td>
<td>1</td>
<td>5</td>
<td>2</td>
</tr>
</tbody>
</table>

\(^{19}\)Since candidate 1 always loses, \( a_3^0 = 1 \) and \( a_3^1 = 0 \) is also a Nash equilibrium. Because the implemented policy is the same as in the equilibrium in weakly dominant strategies, the welfare comparison between plurality and runoff rule would be the same as in the text.
\[ a_3^1 = 0 \] under plurality rule, and Candidate 0 wins with policy \((0, 0, 0)\).

Note that the implemented policy under plurality rule with three candidates, \((0, 0, 0)\), is majority-preferred to \((0, 0, 1)\), which is both the equilibrium policy when there are only two candidates and the equilibrium policy under runoff rule. Thus, Example 2 provides both an example where plurality rule leads to better results than runoff rule, and where the existence of a spoiler who cannot win improves welfare for a majority of voters.

Note that, under plurality rule, the spoiler candidate likes the effect of his entry on the policy that is implemented: The identity of the winning candidate is unchanged, but the winner proposes a policy on the third issue that the spoiler prefers relative to the policy that would be proposed, if there are only two candidates. Thus, Example 2 is robust if we endogenize the entry decision of the spoiler candidate, provided that this candidate’s policy motivation is sufficiently large in comparison to the cost of running.

There is certainly no guarantee that plurality rule is better than runoff rule (in the sense of majority-efficiency) in the case that candidates differ in their fixed positions. Indeed, it is simple to adjust the examples given in the last subsection to include some trivial fixed differences between candidates, and runoff rule would still generate better results than plurality rule. However, Example 2 demonstrates that plurality rule may (in a robust example) be strictly better than runoff rule when candidates differ in their fixed positions and can choose a position on some other issues. As the results of the previous subsection show, this result cannot be obtained if candidates can choose all positions, or no position at all. Since these two cases are the only ones that can arise in a one-dimensional framework, this shows the usefulness of a multidimensional framework.

While we have focused our comparison of electoral systems on plurality rule and runoff rule, the fundamental insight we obtain applies more generally. Suppose that there is an electoral system, call it system A, that always selects the Condorcet winner from any given set of candidates. In Example 2, system A corresponds to runoff rule, but this could also be approval voting with strategic voters and some refinement, say the “voting equilibrium” concept of Myerson and Weber (1993), or other electoral systems suggested in the literature. Now suppose that we compare the efficiency of electoral system A with electoral system B (plurality rule in our example) that does not always select the Condorcet winner from a given set of candidates. If the positions of candidates are fixed in all issues, or if candidates are flexible in all issues, then system A is indeed better than system B, in the sense that, if the two systems produce different outcomes, then a majority of voters prefers the outcome under electoral system A. However, as Example 2 shows, this is not anymore true, if some positions are fixed and others are flexible for the candidates. Our result thus shows that a search for “the best” electoral system (in the sense of selecting the Condorcet winner for the largest possible set of voter preference profiles) is not necessarily a useful objective: Even if we were to find such an optimal electoral system for fixed positions, the outcome under this system might be dominated by the outcome under plurality rule (or, some other “non-optimal” system).

We now summarize these results:

**Definition 5** Let \( U \) be a collection of utility profiles for all voters \( \tau \in T \).
An electoral system $\Phi$ is a function that maps each vector $(u_\tau)_{\tau \in T} \in \mathcal{U}$ of voter preferences and a collection of policies $(a^1, \ldots, a^n)$ of $n$ candidates into one of the policies, i.e., $\Phi: \mathcal{U} \times \prod_{i=1}^n A^i \rightarrow \bigcup_{i=1}^n A^i$.

An electoral system $\Phi$ is Condorcet optimal if and only if $\Phi$ selects a Condorcet winner if one exists, i.e., for all preference profiles $(u_\tau)_{\tau \in \Theta} \in \mathcal{U}$ and all $(a^1, \ldots, a^n) \in \prod_{i=1}^n A^i$ it follows that $\Phi((u_\tau)_{\tau \in T}, (a^1, \ldots, a^n))$ is a Condorcet winner with respect to voter preferences $(u_\tau)_{\tau \in \Theta}$ if a Condorcet winner exists.

**Proposition 9** There exists a robust voting game in the binary policy model, where the winner under plurality rule is strictly majority preferred to the winner under a Condorcet optimal election system.

6 Conclusion

The binary policy model provides an intuitive and tractable framework for the analysis of multidimensional policy choice. It is the first model that allows us to study what happens when candidates’ positions are fixed in some dimensions (possibly to different policies), while they can commit on other issues. This combination of these two central political economy models in the literature, the Downsian model and the citizen candidate model, is both realistic and yields truly novel results.

In a two-candidate framework, policy divergence can arise with two office-motivated candidates and no uncertainty about the distribution of voters. While there are many previous models that generate policy divergence in a Downsian framework, they usually require two deviations from the standard Downsian model, namely policy-motivation of candidates and uncertainty about the distribution of voters. In contrast, our model differs from the Downsian model only in the description of the policy space and generates policy divergence in a simpler framework. Therefore, two standard results of the Downsian model — policy convergence of candidates, and movement of candidates “into the middle”, i.e., in a direction that is preferred by a majority of the electorate — are actually generated by the one-dimensional structure of the Downsian model. Our result also casts doubt on the interpretation that observed policy divergence between candidates must imply that candidates are policy-motivated rather than office motivated.

We also show that candidates may choose minority-preferred positions in equilibrium. In contrast to the probabilistic voting model, this result can even arise when both majority and minority put the same weight on the issue in question. Again, this observation leads to important differences in interpretation: While the adoption of minority positions appears “efficient” in the probabilistic voting model (as the minority cares more about the issue than the majority), this does not hold in the binary policy model.

The most interesting of our results arise from the interplay of multidimensionality and fixed issues. The fact that we focus on binary positions in each issue simplifies the model, in particular the description of voter preferences. However, it is intuitive that the main insights from our binary model would continue to hold.

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20For example, if an issue has three possible positions, then, in order to describe a voter’s preference, we need to specify a ranking of these positions, as well as how much the voter likes his second-most preferred position relative to his top and least-preferred choice. The more possible positions there are, the more complicated this becomes.
Alternatively, one could study the effect of fixed issues in a Euclidean framework with one dimension in which candidates are flexible. For a given location of the opponent on the flexible issue, a candidate may not necessarily have an incentive to locate close to his opponent: The reason is that the swing voters are not necessarily located (only) between the positions of the two candidates, as voters have preferences over the candidates’ other, fixed dimensions. In fact, our examples of policy divergence from the binary model can easily be embedded in the continuum model. However, a characterization of equilibria of the continuum would be very challenging, as pure strategy equilibria only arise in special cases. Analyzing mixed strategy equilibria would again require moving to a discrete setting.

In a framework with more than two candidates, runoff rule – or any other rule that selects the Condorcet winner more often than plurality rule – weakly majority dominates plurality rule, if candidates either cannot commit at all (the citizen-candidate case), or are completely flexible on all dimensions (the Downsian case). These results, while new in the binary policy framework, mirror intuitions in the previous literature. However, when candidates have some fixed positions and are flexible in the remaining issues, then the opposite case may arise: Even though runoff rule (in our framework) always selects the Condorcet winner from a given set of candidates, and plurality rule does not, the effect of the electoral system on the policies that candidates choose can be such that a majority strictly prefers the equilibrium outcome under plurality rule than under runoff rule. This result casts doubt on whether there is an “optimal” electoral system (or even just one that is always “better” than plurality rule in the sense of majority-efficiency) for a meaningfully large set of preferences.
7 Appendix

Proof of Proposition 1. The idea of the proof is to show that independence of irrelevant alternatives implies that, in the space of lotteries \( \mathfrak{L} \), indifference curves are parallel hyperplanes. The normal vector to this hyperplane is the vector of utility weights.

**Step 1** There exist \( \alpha^1, \alpha^2 \in \mathfrak{L} \) with \( \alpha^1 \sim \alpha^2 \) and \( \alpha^1 \neq \alpha^2 \).

Suppose by way of contradiction that \( \alpha^1 \not\sim \alpha^2 \) for all \( \alpha^1, \alpha^2 \in \mathfrak{L} \) with \( \alpha^1 \neq \alpha^2 \). Then there exist \( \alpha^1, \alpha^2, \alpha^3 \) in the interior of \( \mathfrak{L} \) such that \( \alpha^1 \prec \alpha^2 \prec \alpha^3 \). Also, we can assume that

\[
\alpha^2 \notin L = \{ ta^1 + (1-t)a^3 | t \in [0,1] \}.
\] (6)

In particular, continuity of preferences implies that \( \alpha^1 + \epsilon(1, \ldots, 1) \prec \alpha^2 \prec \alpha^3 + \epsilon(1, \ldots, 1) \) for small \( \epsilon \). Thus, if \( \alpha^2 = t\alpha^1 + (1-t)\alpha^3 \) for some \( t \in [0,1] \) then \( \alpha^2 \neq t(\alpha^1 + \epsilon(1, \ldots, 1)) + (1-t)(\alpha^3 + \epsilon(1, \ldots, 1)) \) for all \( t \in [0,1] \), i.e., replacing \( \alpha^1 \) and \( \alpha^3 \) by \( \alpha^1 + \epsilon(1, \ldots, 1) \) and \( \alpha^3 + \epsilon(1, \ldots, 1) \) we can ensure that (6) is satisfied. Next, let

\[
O^1 = \{ a | \alpha \succ \alpha^2 \}, \quad O^2 = \{ a | \alpha \prec \alpha^2 \}.
\] (7)

Then \( O^1 \cup O^2 \cup \{ a^2 \} = \mathfrak{L} \), as by assumption there does not exist \( \alpha \neq \alpha^2 \) with \( \alpha \sim \alpha^2 \). Because \( \alpha^2 \notin L \) it follows that \( O^1 \cup O^2 \supset L \). Since \( O^1 \cap O^2 = \emptyset \) this implies that \( L \) is disconnected, a contradiction. Hence there exists \( \alpha \) such that \( \alpha \sim \alpha' \) for some \( \alpha' \neq \alpha \).

**Step 2** Let \( I_\alpha \) be the indiffERENCE curve through \( \alpha \) and let \( \alpha' \in I_\alpha \) with \( \alpha' \neq \alpha \). Then \( pa+(1-p)\alpha' \in I_\alpha \) for all \( p \in [0,1] \).

This follows immediately from independence of irrelevant alternatives.

**Step 3** Suppose there exists \( t \notin [0,1] \) with \( \hat{\alpha} = ta + (1-t)a' \notin \mathfrak{L} \). Then \( \hat{\alpha} \in I_\alpha \).

Suppose by way of contradiction that \( \hat{\alpha} \succ \alpha \). If \( t > 1 \), then let \( p = 1/t \). Since \( \alpha \sim \alpha' \) it follows that \( \hat{\alpha} \succ \alpha' \). Thus, the definition of \( p \) and independence of irrelevant alternatives implies \( \alpha = p\hat{\alpha} + (1-p)\alpha' \succ \alpha' \), a contradiction. If \( t < 0 \) then define \( p = 1/(1-t) \). Then \( \alpha' = p\hat{\alpha} + (1-p)\alpha \succ \alpha \), again a contradiction. The contradiction when \( \hat{\alpha} \prec \alpha \) is derived similarly.

**Step 4** Indifference curves are parallel hyperplanes. That is, let \( \beta = \alpha' - \alpha \) and \( \hat{\alpha} \notin \mathfrak{L} \). Suppose that \( \hat{\alpha} + t\beta \in \mathfrak{L} \) for some \( t \in \mathbb{R} \). Then \( \hat{\alpha} + t\beta \sim \hat{\alpha} \).

Suppose by way of contradiction that \( \hat{\alpha} + t\beta \succ \hat{\alpha} \). Let \( \tilde{\alpha} = 0.5\hat{\alpha} + 0.5\alpha \). Then independence of irrelevant alternatives implies \( \tilde{\alpha} + 0.5\beta \hat{\alpha} + 0.5\beta \alpha \succ \alpha \). Further, because \( \alpha \sim \alpha' \), independence of irrelevant alternatives implies \( \hat{\alpha} = 0.5\alpha + 0.5\alpha' \sim 0.5\alpha' + 0.5\tilde{\alpha} \). Thus, the fact that \( \alpha' = \alpha + \beta \) implies \( \alpha \sim \alpha + 0.5\beta \). Step 3 implies that \( \hat{\alpha} + 0.5\beta = t(\hat{\alpha} + 0.5\beta) + (1-t)\hat{\alpha} \sim \hat{\alpha} \), a contradiction. In a similar way we derive a contradiction when \( \hat{\alpha} + \beta \prec \hat{\alpha} \).

**Step 5** There exists a utility function that describes preferences which is of the weighted issue type.

Let \( \bar{\lambda} \) be the vector that is orthogonal to the indifference curves, i.e., \( (\alpha - \alpha') \cdot \bar{\lambda} = 0 \) for all \( \alpha \sim \alpha' \). We normalize \( \bar{\lambda} \) such that \( (\alpha - \alpha') \cdot \bar{\lambda} > 0 \) for all \( \alpha \succ \alpha' \). Then \( u(\alpha) = \alpha \cdot \bar{\lambda} \) is a utility function that
describes the preferences. Let \( \theta \in A \) be defined by

\[
\theta_i = \begin{cases} 
0 & \text{if } \lambda_i \leq 0; \\
1 & \text{otherwise.} 
\end{cases}
\]

Then for \( a \in A \) we have \( u(a) = \sum_{i=1}^{\infty} \lambda_i a_i = -\sum_{i=1}^{\infty} |\lambda_i| a_i - \sum_{i \geq 0} \lambda_i \). Since, the last summand is a constant, it solely rescales the utility function and we can therefore drop this term. Thus, utility if of the weighted-issue form.

**Proof of Proposition 2.** The “only if” direction is obvious. Suppose that the “if” statement is false. Since \( a^0 \) is majority-efficient, there must be \( \bar{a} \in A^1 \) such that \( \bar{a} \succ a^0 \). But then, Candidate 1 could win by playing \( \bar{a} \), which cannot be true in equilibrium.

**Proof of Proposition 3.** We first show show that if \( \tau' \in C(\tau) \) then voter \( \tau' \) prefers \( a^j \) to \( a' \) whenever voter \( \tau \) does, i.e., \( a^j \succeq \tau \ a' \) implies \( a^j \succeq \tau' \ a' \).

Let \( D \) be the set of issues for which \( a^j_i \neq a'_i \), i.e. \( D = \{i | a^j_i \neq a'_i \} \). Similarly, let \( E = \{i | \theta_i \neq \theta'_i \} \) be the set of issues where the ideal points of \( \tau \) and \( \tau' \) differ. Let \( X_i \) be defined by

\[
X_i(\theta, a) = \begin{cases} 
1 & \text{if } |\theta_i - a_i| = 0; \\
0 & \text{if } |\theta_i - a_i| = 1. 
\end{cases}
\]

Suppose, by contradiction, that \( a^j \succeq \tau \ a' \), but \( a' \succ \tau' \ a^j \). This is equivalent to

\[
\sum_{i \in D} \lambda_i X_i(\theta, a^j) \geq \sum_{i \in D} \lambda_i X_i(\theta, a') \tag{8}
\]

\[
\sum_{i \in E} \lambda_i X_i(\theta', a') > \sum_{i \in E} \lambda_i X_i(\theta', a^j) \tag{9}
\]

Summing up these two inequalities and rearranging gives

\[
\sum_{i \in \overline{D \cup E}} \lambda_i [(X_i(\theta', a') - X_i(\theta', a^j)) - (X_i(\theta, a') - X_i(\theta, a^j))] > 0. \tag{10}
\]

It is easy to see that, if either \( i \notin D \) or \( i \notin E \), the corresponding term in square brackets on the left-hand side of (11) must be zero. Thus, it must be true that

\[
\sum_{i \in \overline{D \cup E}} \lambda_i [(X_i(\theta', a') - X_i(\theta', a^j)) - (X_i(\theta, a') - X_i(\theta, a^j))] > 0. \tag{11}
\]

Note that, for all \( i \in E \), \( X_i(\theta, a^j) = 0 \) and \( X_i(\theta', a^j) = 1 \). (Clearly, \( X_i(\theta, a^j) \neq X_i(\theta', a^j) \) for all \( i \in E \), and \( X_i(\theta', a^j) = 1 \) and \( X_i(\theta', a^j) = 0 \) would contradict \( \theta_i = a^j_i \Rightarrow \theta'_i = a^j_i \)). Furthermore, since \( X_i(\theta', a') - X_i(\theta, a') \leq 1 \), (11) cannot hold, we get a contradiction.
Let \( \kappa: T \rightarrow \{0, 1\} \) be defined by \( \kappa(\tau) = 1 \) if \( a^l \geq_\tau a^j \) and \( \kappa(\tau) = 0 \) if \( a^l <_\tau a^j \). Then the first part of the proof implies that \( \kappa(\tau') \geq \kappa(\tau) \) for all \( \tau' \in C(\tau) \). Thus, the fact that \( \nu(\tau, C(\tau)) = 1 \) implies

\[
\mu'(\{\tau\mid a^l \geq_\tau a^j\}) = \int \kappa(\tau) \, d\mu'(\tau) = \int \int \kappa(\tau') \, \nu(\tau, d\tau') \, d\mu(\tau) \\
\geq \int \int \kappa(\tau) \, \nu(\tau, d\tau') \, d\mu(\tau) = \int \kappa(\tau) \, d\mu(\tau) = \mu(\{\tau\mid a^l \geq_\tau a^j\}).
\] (12)

Replacing \( \kappa \) by \( 1 - \kappa \) in (12) implies that \( \mu(\{\tau\mid a^l \leq_\tau a^j\}) \geq \mu'(\{\tau\mid a^l \leq_\tau a^j\}) \). Thus, \( \mu'(\{\tau\mid a^l \geq_\tau a^j\}) \geq \mu'(\{\tau\mid a^l \leq_\tau a^j\}) \) which means that \( a^l \) is majority preferred to \( a^j \). Since \( a^j \) was chosen arbitrarily, this concludes the proof.

**Proof of Corollary 2.** If \( |S| = 1 \), the result is immediate. Thus, without loss of generality, let \( S^j = \{1, 2\} \), i.e., the first two positions can be chosen freely. Let \( a = \mu((0, 0) \times (0, 1)^l), b = \mu((1, 0) \times (0, 1)^l), c = \mu((0, 1) \times (0, 1)^l), \) and \( d = \mu((1, 1) \times (0, 1)^l) \). Clearly, \( a + b + c + d = 1 \). Further, without loss of generality, let assume that \( c + d \geq a + b \) and \( b + d \geq a + c \). This immediately implies that \( d \geq a \). Now consider a distribution \( \tilde{\mu} \) with \( a = \tilde{\mu}((0, 0) \times (0, 1)^l) = \tilde{\mu}((1, 1) \times (0, 1)^l), \) and \( 0.5 - a = \tilde{\mu}((0, 1) \times (0, 1)^l) = \tilde{\mu}((1, 0) \times (0, 1)^l) \). Because of symmetry of \( \tilde{\mu} \), any policy is majority-efficient, for example a policy with \( a_1 = a_2 = 1 \). In order to get distribution \( \mu \) from \( \tilde{\mu} \) we must move weight from \( (0, 1)^j \times (0, 1)^l \) to \( (1, 1)^l \times (0, 1)^l \). This is possible as long as \( 0.5 - a \geq c \) and \( 0.5 - a \geq b \). Suppose by way of contradiction that \( 0.5 - a < c \). Then \( a + c > 0.5 \). Because \( a + b + c + d = 1 \) this implies \( a + c > b + d \), a contradiction. Similarly, \( 0.5 - a < b \) contradicts \( c + d \geq a + b \).

**Proof of Corollary 3.** See text.

**Proof of Proposition 4.** Suppose that \( (a^0, a^l) \) is an equilibrium. Then, each candidate wins with probability 0.5. (Suppose, to the contrary, that Candidate 0 (say), always loses. However, because \( A^0 = A^1 \), he could improve by choosing \( a^0 = a^1 \), a contradiction). Let \( \hat{a} \) be an arbitrary feasible policy. If \( \hat{a} \geq^* a^0 \), then Candidate 1 wins (with probability 1) if he offers policy \( \hat{a} \). Since \( a^0 \) is an equilibrium strategy, \( a^0 \geq^* \hat{a} \) for all \( \hat{a} \). Similarly, \( a^1 \geq^* \hat{a} \) for all \( \hat{a} \). Hence, both \( a^0 \) and \( a^1 \) are majority-efficient.

Now suppose that \( a^0 \) and \( a^l \) are majority-efficient. We have to show that \( (a^0, a^l) \) is an equilibrium. Since \( a^0 \geq^* a^1 \) and \( a^l \geq^* a^0 \) (by majority-efficiency), each candidate gets 50% of the votes and thus wins with probability 0.5. Furthermore, by majority-efficiency of \( a^0 \) and \( a^l \), \( a^0 \geq^* \hat{a} \) and \( a^l \geq^* \hat{a} \), for all \( \hat{a} \). Hence, there is no profitable deviation, so that \( (a^0, a^l) \) is an equilibrium.

Now consider a mixed strategy equilibrium \( (\sigma^0, \sigma^l) \). Each candidate must win with probability 0.5 (otherwise, the candidate who wins with the lower probability could deviate to the strategy of his opponent, thereby increasing his winning probability to 0.5). Furthermore, in order for mixing to be optimal, every policy in the support of \( \sigma^j \) must give agent \( j \) a winning probability of 0.5. Now, assume by way of contradiction that the support of \( \sigma^j \) contains a set \( B \) of policies that are not majority-efficient.
Because the set of policies is finite, $B$ must occur with strictly positive probability. Then policies in $B$ only win if Candidate $-j$ also selects a non-majority-efficient policy. Because the winning probability must be 0.5, this implies that the opponent uses a non-majority-efficient strategy with strictly positive probability. Let $\tilde{a}^j$ be a majority-efficient policy. Suppose that Candidate $j$ uses the alternative strategy $\tilde{\sigma}^j$ which uses $\tilde{a}^j$ whenever a policy in $B$ is selected under $\sigma^j$ and corresponds to $\sigma^j$, otherwise. Then $\tilde{a}^j$ wins whenever the opponent selects a non-majority-efficient policy and ties whenever the opponent uses a majority-efficient policy. Thus, Candidate $j$'s winning probability strictly increases, a contradiction. Hence, every policy in the support of $\sigma^j$ is majority-efficient. ■

**Proof of Proposition 5.** For $(0, 0)$ to be an equilibrium, it must be true that

$$SV_0 \geq (1 - \xi_0)SV_0 + (1 - \xi_1)SV_1$$
$$SV_1 \geq (1 - \xi_0)SV_0 + (1 - \xi_1)SV_1,$$

which can be rearranged to give (3). Similarly, for $(1, 1)$ to be an equilibrium, it must be true that

$$SV_0 \geq \xi_0SV_0 + \xi_1SV_1$$
$$SV_1 \geq \xi_0SV_0 + \xi_1SV_1,$$

which is equivalent to (4). For $(0, 1)$ to be an equilibrium, it must be true that

$$SV_0 \leq (1 - \xi_0)SV_0 + (1 - \xi_1)SV_1$$
$$SV_1 \leq (1 - \xi_0)SV_0 + (1 - \xi_1)SV_1.$$

This implies $SV_1 / SV_0 = \frac{\xi_0}{1 - \xi_1} = \frac{1 - \xi_0}{\xi_1}$. Cross-multiplying the last equality implies $\xi_0 + \xi_1 = 1$, and using this implies $SV_1 / SV_0 = 1$. ■

**Proof of Proposition 6.** First, note that we can renumber issues such that $F = \{1, \ldots, m\}$ is the set of fixed issues. Further, we can assume without loss of generality that $\mu(\{\tau|\theta_i = a_i^0\}) > 0.5$ for all $i \in \{1, \ldots, k\}$, $\mu(\{\tau|\theta_i = a_i^1\}) \geq 0.5$ for all $i \in \{k + 1, \ldots, k'\}$, and that $a_i^0 = a_j^1$ for all $i \in \{k' + 1, \ldots, m\}$. Since $\phi$ is one-to-one it follows that $k' \leq 2k$. We first assume that $k' = 2k$.

For all $i \in 3$, let $\tilde{a}_i = 1$ if $\mu(\{\tau|\theta_i = 1\}) > 0.5$ and $\tilde{a}_i = 0$ if $\mu(\{\tau|\theta_i = 1\}) < 0.5$. Define the random variable $X_i$ by

$$X_i(\theta) = \begin{cases} 1 & \text{if } \theta_i = \tilde{a}_i \\ 0 & \text{if } \theta_i \neq \tilde{a}_i \end{cases}$$

Note that voters of type $\theta$ strictly prefer candidate 0 or are indifferent between the candidates if

$$\sum_{i=1}^k \lambda_i X_i(\theta) + \sum_{i=k+1}^{2k} \lambda_i (1 - X_i(\theta)) + \sum_{k=m+1}^l \lambda_i X_i(\theta) \geq \sum_{i=1}^k \lambda_i (1 - X_i(\theta)) + \sum_{i=k+1}^{2k} \lambda_i X_i(\theta) + \sum_{k=m+1}^l \lambda_i (1 - X_i(\theta)),$$
which is equivalent to

\[
\sum_{i=1}^{k} (\lambda_{i}X_{i}(\theta) - \lambda_{i+k}X_{i+k}(\theta)) + \sum_{k=m+1}^{l} \lambda_{i}X_{i}(\theta) \geq 0.5 \sum_{i=1}^{k} (\lambda_{i} - \lambda_{i+k}) + 0.5 \sum_{k=m+1}^{l} \lambda_{i}.
\]  

(13)

Let \( p_{i} = \mu(\theta_{i} = \bar{a}_{i}) \). Define \( \hat{X}_{i} = \lambda_{i}X_{i} - \lambda_{i+k}X_{i+k} \). Then \( \hat{X}_{i} \) has the four realizations \(-\lambda_{i+k}, 0, \lambda_{i} - \lambda_{i+k}, \) and \( \lambda_{i} \), which occur with probabilities \((1 - p_{i})p_{i+k}, (1 - p_{i})(1 - p_{i+k}), p_{i}p_{i+k}, \) and \( p_{i}(1 - p_{i+k}) \). Let \( \hat{Y}_{i} \) be a random variable which has realizations \(-\lambda_{i+k} \) and \( \lambda_{i} \) with the same probability of \( q = 0.5((1 - p_{i})p_{i+k} + p_{i}(1 - p_{i+k})) \), and the remaining two realizations \( 0 \) and \( \lambda_{i} - \lambda_{i+k} \) with the same probability of \( q' = 0.5((1 - p_{i})(1 - p_{i+k}) + p_{i}p_{i+k}) \). Since \( p_{i} > p_{i+k} \) it follows immediately that \( \hat{X}_{i} \) first order stochastically dominates \( \hat{Y}_{i} \). Thus, there exists a random variable \( \hat{Z}_{i} \geq 0 \) such that \( \hat{X}_{i} = \hat{Y}_{i} + \hat{Z}_{i} \). Furthermore, note that \( E[\hat{Y}] = (q + q')(\lambda_{i} - \lambda_{i+k}) = 0.5(5\lambda_{i} - \lambda_{i+k}) \).

Next, note that \( X_{i} \) first order stochastically dominates a random variable \( Y_{i} \) which pays \( 1 \) with probability \( 0.5 \) and \( 0 \) with probability \( 0.5 \). Thus, for \( i > m \) we can find random variables \( Z_{i} \) with \( Z_{i} \geq 0 \) and \( X_{i} = Y_{i} + Z_{i} \). Clearly, \( E[Y_{i}] = 0.5\lambda_{i} \) Thus,

\[
\mu \left( \tau \left| \sum_{i=1}^{k} (\lambda_{i}X_{i}(\theta) - \lambda_{i+k}X_{i+k}(\theta)) + \sum_{k=m+1}^{l} \lambda_{i}X_{i}(\theta) \geq 0.5 \sum_{i=1}^{k} (\lambda_{i} - \lambda_{i+k}) + 0.5 \sum_{k=m+1}^{l} \lambda_{i} \right) \right) \\
= \mu \left( \tau \left| \sum_{i=1}^{k} (\hat{Y}_{i}(\theta) + \hat{Z}_{i}(\theta)) + \sum_{k=m+1}^{l} \lambda_{i}(Y_{i}(\theta) + Z_{i}(\theta)) \geq \sum_{i=1}^{k} E[\hat{Y}_{i}] + \sum_{k=m+1}^{l} E[Y_{i}] \right) \right) \\
> \mu \left( \tau \left| \sum_{i=1}^{k} \hat{Z}_{i}(\theta) + \sum_{k=m+1}^{l} \lambda_{i}Y_{i}(\theta) \geq \sum_{i=1}^{k} E[\hat{Y}_{i}] + \sum_{k=m+1}^{l} E[Y_{i}] \right) \right) = 0.5,
\]

where the last equality follows because both \( Y_{i} \) and \( \hat{Y}_{i} \) are symmetrically distributed. In view of (13) this means that candidate 0 wins by receiving more than 50% of the vote share. This proves the first statement for \( k' = 2k \).

Now suppose that \( k' < 2k \). Let \( p > 0.5 \) such that \( p < \mu(\tau|\theta_{i} = a_{i}^{0}) \) for all \( i < k \). Then define \( 2k - k' \) artificial issues such that \( 0.5 < \mu(\tau|\theta_{i} = a_{i}^{1}) < p \) and \( \lambda_{i} = 0 \) for these issues. Now both candidates \( j = 0, 1 \) have the same number of issues with \( \mu(\tau|\theta_{i} = a_{i}^{1}) > 0.5 \), and as a consequence the first part of the argument implies. This concludes the proof of the first statement.

To prove the second statement, first set \( \lambda_{i} = 0 \) for all \( i \in \{1, k+1, l\} \). We now show that there exist weights \( \lambda_{1}, \lambda_{k} \) and \( \lambda_{l} \) such that candidate 1 wins by choosing the majority inefficient position on issue \( I \). In view of the first part of the proof, we must have \( \lambda_{1} > \lambda_{1+k} \), else candidate 0 always wins.

If candidate 0 chooses the majority inefficient position on issue \( I \), then in view of (13) voter \( \theta \) votes for candidate 0 if

\[
\lambda_{1}X_{1} - \lambda_{k+1}X_{k} - \lambda_{1}X_{l} \geq 0.5(\lambda_{1} - \lambda_{1+k} - \lambda_{l}).
\]  

(14)

If candidate 1 chooses the same position as candidate 0 on issue \( I \) then voter \( \theta \) votes for candidate 0 if

\[
\lambda_{1}X_{1} - \lambda_{k+1}X_{k} \geq 0.5(\lambda_{1} - \lambda_{1+k}).
\]  

(15)

Since \( \lambda_{1} < \lambda_{k+1} \) it follows that (15) is satisfied only for types \( \theta \) with \( \theta_{i+k} \neq \bar{a}_{i+k} \). Since \( \mu(\theta_{i+k} \neq \bar{a}_{i+k}) < 0.5 \), this implies that candidate 0 loses.
Now suppose that $\lambda_{k+1} - \lambda_1 < \lambda_I < \lambda_1 + \lambda_{k+1}$. Then (15) is satisfied (with a strict inequality) for all $\theta \in A = \{\theta | \theta_1 = \theta_{k+1} = \theta_I = 0, or \theta_1 = 1, \theta_{k+1} = \theta_I = 0, or \theta_1 = \theta_{k+1} = 1, \theta_I = 0, or \theta_1 = \theta_I = 1, \theta_{k+1} = 0\}$. Clearly, 
\[
\mu(A) = \mu(\{\theta_1 = a_1^0\}) + \mu(\{\theta_{k+1} = a_{k+1}^0\}) + \mu(\{\theta_1 = a_1^1\}) \mu(\{\theta_{k+1} = a_{k+1}^1\}) \mu(\{\theta_I = \bar{a}_I\}) 
\]

Since $\mu(\{\theta_1 = a_1^0\}) > \mu(\{\theta_{k+1} = a_{k+1}^0\})$, it follows immediately that $\mu(A) > 0.5$ if $\mu(\{\theta_I = \bar{a}_I\})$ is close to 0.5. Thus, there exists an $\epsilon > 0$ such that candidate 0 wins by choosing the majority inefficient position on issue $I$ if $\mu(\{\theta_I = \bar{a}_I\}) < 0.5 + \epsilon$. The same argument applies for any $i > m$. Finally, note that the candidates’ vote share are continuous in a neighborhood of $\Lambda$ of $A$. This proves the second statement of the proposition.

**Proof of Proposition 7.** The first statement is proved in the text. To show the second statement, note that all $n$ candidates can play $a^*$ in equilibrium, as one candidate could deviate to some $a' \in \bar{A}$ and win. Also, in a mixed strategy equilibrium, the probability of winning with a majority-inefficient position must be strictly positive, as otherwise, a candidate who is supposed to play some other policy than $a^*$ would benefit by deviating to $a^*$. ■

**Proof of Proposition 8.** Clearly, $a_n$ cannot be an outcome under runoff rule: Candidate $n$ cannot win in the second round, as any opponent is majority-preferred to Candidate $n$. Furthermore, winning in the first round would require that those voters who rank $a_n$ highest form an outright majority, which is impossible (because this would imply $a_n >^* a_i$ for all $i \neq n$).

Under runoff rule, the plurality rule winner receives the most votes in the first round and either wins outright or proceeds into a runoff. If he is ranked higher than his opponent by a majority, then the outcome is the same under plurality and runoff rule. If his opponent is ranked higher by a majority, his opponent wins and the runoff outcome is strictly preferred to the plurality outcome by a majority. Thus, the outcome under runoff rule is weakly majority-preferred to the one under plurality rule. ■

**Proof of Proposition 9.** See text. ■
References


