Abstract
We construct a general equilibrium model with private information in which borrowers and lenders enter into long-term dynamic credit relationships. Each new generation of ex ante identical individuals is divided in equilibrium into workers and entrepreneurs. Workers save through financial intermediaries (or banks) in the form of interest-bearing deposits and supply labor to entrepreneurs in a competitive labor market. Entrepreneurs borrow from banks to finance projects which produce unobservable sequences of random returns. Each bank holds deposits from a large number of workers and operates a portfolio of dynamic contracts with different credit positions. Although in equilibrium banks make zero profits because of free entry, they hold positive amounts of assets in the form of banking capital. We calibrate our model to the U.S. economy. We use our model economy to evaluate the effects of private information on aggregate economic activity and on individual welfare. We find that dynamic contracting is very effective at mitigating the effects of private information. We also find that the optimal provision of intertemporal incentives in our calibrated model economy leads to increasing consumption inequality over time within generational cohorts as in U.S. data.
1 Introduction

A typical neoclassical growth model assumes, in the spirit of Arrow-Debreu, perfect information and complete markets, and treats all commodities as indexed by all possible states of nature. In practice, however, economic arrangements are often complicated by moral hazard and other information asymmetries which can break down the premises of the Arrow-Debreu model, and hence some of its basic predictions such as full risk sharing. In the presence of private information, contracting among economic agents becomes vitally important for mitigating private information and attaining efficiency in production and risk sharing.

In this paper, we construct a general equilibrium model with private information in which borrowers and lenders enter into long-term dynamic credit relationships. We build on the existing literature on dynamic private information (including Townsend 1982, Green 1987, Spear and Srivastava 1987, Thomas and Worrall 1990, Taub 1990, Phelan and Townsend 1991, Atkeson and Lucas 1992, Wang 1995, Khan and Ravikumar 1997a, b) which suggests that long-term contracts are more efficient at mitigating the effects of informational asymmetries and are important for understanding consumption inequality across individuals. The existing literature on dynamic contracting focuses mainly on normative models (social planning problems) and falls short of providing a quantitative general equilibrium model that can match data. The model we develop in this paper allows us to explore both qualitatively and quantitatively the implications of private information and dynamic contracting for the operation of the macroeconomy. Specifically, we calibrate the model to the U.S. economy and use it to address a set of questions including: How does dynamic contracting operate in general equilibrium? How does dynamic contracting affect inequality through the provision of intertemporal incentives? What role does private information play in the determination of the general equilibrium levels of aggregate quantities such as output and the capital stock? What are the implications of dynamic contracting for individual welfare (relative to static contracting under private information and full risk-sharing under complete information)?

The last twenty years have seen many efforts in modeling explicitly private information and the financial contracting process in the benchmark neoclassical growth model. In particular, a substantial part of the literature addresses the question of how the presence of private information contributes to the propagation of aggregate economic uncertainties. Bernanke and Gertler (1989), for example, argue that with private information, swings in the firm's balance sheet are a potential source of persistent output dynamics. In Williamson (1987b), monitoring costs of financial intermediaries are important for the propagation of aggregate disturbances. Kiyotaki and Moore (1997) also study business cycle dynamics propagated through the financial contracting process between entrepreneurs and investors, although their story is based on limited commitment instead of private information. Carlstrom and Fuerst (1997) develop a computable general equilibrium model based on Bernanke and Gertler (1989) to address quantitatively the importance of agency costs for the propagation of aggregate shocks. Cooley and Nam (1995) incorporate a problem of debt contract-
ing with asymmetric information into a quantitative monetary business cycle model to generate a persistent liquidity effect induced by monetary disturbances.

An obvious yet serious limitation of most of the existing literature on financial contracting and business cycles is that the financial lending and borrowing process is modeled as an one-shot game. In practice, banks often engage in long-term relationships, rather than interact only once with their borrowers. Modeling the financial lending process as a one-period contract necessarily misses the important dynamic features of the optimal contracting process, and has implications also for how successfully the contract can be used in a dynamic macroeconomic setting as an explicit description of the financial lending process. In a standard dynamic general equilibrium macroeconomic model, borrowers and lenders are all infinitely-lived agents, and very special assumptions have to be made in order to fit the static contracting relationship into the rest of the economy (which is fully dynamic).

The above limitation of the literature was first pointed out by Gertler (1992), who developed a model in which lenders and borrowers can enter into long-term but finite contractual relationships and used his model to show that shifts in aggregate economic fundamentals can be amplified through the process of long-term contracting. Yet, as Gertler himself pointed out, “a major limitation of this model is that it lies well short of a fully dynamic framework that can be matched to data. While allowing for multi-period contracts ... is a helpful step in this important direction, there is still a long way to go (p. 470).”

Thus what this paper attempts to undertake can also be viewed as just another step in the long way that Gertler (1992) pointed out. Instead of being ambitious in providing a theory that explains business cycles dynamics, this paper constructs a quantitative dynamic general equilibrium model with no aggregate uncertainty but in which long-lived economic agents can enter into fully dynamic financial lending contracts. We then use the model to examine the implications of dynamic contracting for the operation of the macroeconomy, especially for the equilibrium dynamics of the balance sheets of financial intermediaries.

In our model, the economy is populated by a sequence of overlapping generations of agents who potentially can live forever. Each new generation of ex ante identical individuals is divided, in equilibrium, into workers and entrepreneurs. Workers save through financial intermediaries (or banks) in the form of interest-bearing deposits and supply labor to entrepreneurs in a competitive labor market. Entrepreneurs borrow from financial intermediaries (banks) to finance projects which produce idiosyncratic sequences of random returns. The entrepreneur’s returns are not observed by any other parties. In our model, banks arise as institutions to facilitate financial lending and borrowing by providing risk-sharing for entrepreneurs and workers.

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1 See Carlstrom and Fuerst (1997) and Cooley and Nam (1995). Note that the static contracting relationship can be more comfortably embedded in an OLG framework (Boyd and Smith 1997). This perhaps is the main reason why most of the theoretical contributions in the literature have used an OLG structure with two-period-lived agents.

2 In an earlier version of this paper, we use the standard model of costly state verification (Townsend 1979, Wang 1999) to motivate financial intermediation. Specifically, individual lenders do not directly observe entrepreneurs'
from a large number of workers and operates a portfolio of dynamic contracts with different credit positions.

A distinctive feature of our model is that, in equilibrium, banks hold positive amounts of capital assets which we refer to as banking capital. Banking capital arises even though we assume a competitive banking industry, so that banks earn zero profits. The existence of banking capital in our model is an equilibrium outcome associated with the dynamic lending process. In the model here, the equilibrium provision of intertemporal incentives in the dynamic contract implies that the entrepreneurs make more repayments in earlier stages rather than later stages during the credit contracting process. Given that there are new contracts signed each period, in equilibrium the total net cash flow from the bank’s portfolio is negative at a point in time; interest earned on the bank’s capital exactly balances this deficit. As we will show in the paper, the equilibrium size of banking capital depends critically on the rate of interest.

The importance of banking capital is well documented in the literature. Holmstrom and Tirole (1997) argue that banking capital is important for the explanation of credit crunches. In Holmstrom and Tirole (1997), the amount of banking capital is exogenously given, unlike in our model in which it is endogenously determined.

Another distinctive feature of our model is that we model occupational choice as an equilibrium phenomenon. In a typical growth model, firms are not modeled explicitly as economic agents with independent identities, and the equilibrium stock of capital is determined essentially by the consumer’s decisions on savings. In the model here, ex ante identical economic agents choose to become entrepreneurs and workers, and the two types of agents play different roles. In our model, the economy’s capital stock is not pinned down by the equilibrium marginal efficiency of capital as in a typical growth model. Instead, it is determined by the amount of investment projects (or firms) that people choose to operate in equilibrium.

One key finding of our paper is that dynamic contracting is very effective at mitigating the effects of private information: even when informational asymmetries are large, the economy can achieve close to a first-best outcome. We also show that moving from dynamic to static contracting (i.e., restricting the ability of contracts to use intertemporal incentives) has dramatic, adverse affects both on the level of aggregate economic activity and on the welfare of individuals.

Finally, we show that our model economy provides one explanation for increasing consumption inequality over time within generational cohorts, as documented in U.S. data by Deaton and Paxson (1994).

Section 2 presents the model, Section 3 discusses computation of the model, Section 4 explains how we calibrate the model, Section 5 presents the results, and Section 6 concludes.

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3 Williams (1987b) also models financial intermediation in a competitive market but the banks in his model do not hold assets in equilibrium.
2 The Model Economy

Time is discrete and lasts forever. The economy consists of a sequence of overlapping generations, each of which contains a continuum of individuals. Each individual faces a time-invariant probability \( \Delta \) of surviving into the next period. The total measure of individuals in the economy is equal to one. We assume that each new generation has measure \( 1 - \Delta \), so that the number of births and the number of deaths are equal at any point in time. There is one good per period in the economy; this good can be used both for consumption and investment. Each individual, when alive, is endowed with one unit of time in each time period. An individual’s preferences over streams of consumption and leisure are given by:

\[
E_0 \sum_{t=0}^{\infty} (\beta \Delta)^t U(c_t, \ell_t),
\]

where \( c_t \) denotes period \( t \) consumption, \( \ell_t \) denotes period \( t \) leisure, and \( \beta \in [0,1) \) is the discount rate.

Each newly born individual decides to become either a worker or an entrepreneur. Each individual’s “career” choice is irreversible. Workers save through financial intermediaries (or banks) in the form of interest-bearing deposits and supply labor to entrepreneurs in a competitive labor market. Entrepreneurs use fixed amounts of capital and labor to operate risky projects. In order to finance their projects, entrepreneurs sign dynamic lending contracts with banks. Banks compete for new contracts, so that new contracts earn zero profits in equilibrium.

2.1 Workers

Let \( r \) be the interest rate paid by the bank on deposits and let \( \omega \) be the wage rate per unit of time. In our stationary equilibrium (to be described in Section 2.5), the prices \( r \) and \( \omega \) do not vary. Workers take these prices as given when making their decisions. Following Blanchard (1985), workers take part in an annuities market which distributes the assets of workers who die to surviving workers in proportion to the size of their deposits. Specifically, the gross rate of return on deposits for a surviving worker is \( R/\Delta \), where \( R = 1 + r \). Workers begin life with zero assets and seek to maximize expected lifetime utility subject to a sequence of period-by-period budget constraints.

The worker’s dynamic problem takes the recursive form:

\[
S(d; r, \omega) = \max_{d', h} [U(c, 1 - h) + \beta \Delta S(d'; r, \omega)]
\]

subject to:

\[
c + d' = \frac{R}{\Delta} d + \omega h,
\]

\( d' \geq 0 \), and \( h \geq 0 \), where \( d \) is deposits at the beginning of the period, \( d' \) is deposits at the beginning of next period, \( h \) is the amount of time spent working, and \( S(d; r, \omega) \) is the worker’s lifetime expected
utility given that he currently has deposits (or savings) equal to $d$ and that he faces prices $r$ and $\omega$.

Let $d' = f(d; r, \omega)$ and $h = g(d; r, \omega)$ be the optimal decision rules associated with this problem. In the steady state, these decision rules and the birth/death rate $1 - \Delta$ give rise to a stationary distribution of workers across deposit holdings. In each period, fraction $1 - \Delta$ of workers die and are replaced with an equal number of new workers, each of whom begins life with zero assets. Consequently, fraction $1 - \Delta$ of workers have zero deposits, fraction $(1 - \Delta)\Delta$ have deposits equal to $d_1(r, \omega) = f(0; r, \omega)$, fraction $(1 - \Delta)\Delta^2$ have deposits equal to $d_2(r, \omega) = f(d_1(r, \omega); r, \omega)$, and, in general, fraction $(1 - \Delta)\Delta^i$ have deposits equal to $d_i(r, \omega)$. Suppose for the moment that the population (or measure) of workers is $1$. The total amount of deposits $D(r, \omega)$ held by workers is then:

$$D(r, \omega) = (1 - \Delta) \sum_{i=0}^{\infty} \Delta^i d_i(r, \omega),$$

where $d_0(r, \omega) = 0$. Similarly, the total amount of labor supplied by workers is given by:

$$H(r, \omega) = (1 - \Delta) \sum_{i=0}^{\infty} \Delta^i g(d_i; r, \omega).$$

Note that $D(r, \omega)$ and $H(r, \omega)$ can also be viewed as per capita quantities; i.e., $D(r, \omega)$ is deposits per worker and $H(r, \omega)$ is hours per worker in the steady-state equilibrium of our economy.

2.2 Entrepreneurs

Each entrepreneur operates a long-lived project that produces output in each period of the life of the project. Each entrepreneurial project requires in each period $K$ units of capital and $L$ units of labor (in addition to the entrepreneur's own labor). Project returns are random and idiosyncratic. Specifically, let $\bar{\theta}$ be the amount of output produced by a project in any given period. We assume that $\bar{\theta}$ is equal to $\theta_L = (1 - \sigma)\bar{\theta}$ with probability $\pi$ and is equal to $\theta_H = (1 + \sigma)\bar{\theta}$ with probability $1 - \pi$, where $\sigma > 0$ and $\pi \in (0, 1)$. In any given period, therefore, the expected value of an entrepreneur's output is $\bar{\theta}$ and, if $\pi$ is equal to one-half (as it is in Section 4), its standard deviation is $\sigma$. Project returns are independent and identically distributed across time. We assume that the entrepreneur alone directly observes the level of his output in any given period. In other words, outside parties cannot observe the entrepreneur's output.

A new entrepreneur, like a new worker, has no wealth. In order to finance his project, therefore, the entrepreneur must borrow in the credit market. Although each entrepreneur has the same preferences as a typical worker, entrepreneurs, unlike workers, do not make a labor-leisure decision; instead, each entrepreneur spends $\bar{h}$ units of his time endowment working and $\bar{\ell} = 1 - \bar{h}$ units of his

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4We do not allow workers to borrow since we restrict $d'$ to be nonnegative. In the calibrated version of our economy, it turns out that this restriction is not binding.

5In Section 2.5, where we describe the equilibrium conditions of our stationary economy, the measure of workers is determined endogenously.
time endowment in leisure. Entrepreneurs (and their projects) survive into the next period with (time-invariant) probability \( \Delta \).

### 2.3 The Credit Market and Financial Intermediation

Financial intermediaries (banks) arise in our model as institutions to provide risk sharing for the risk-averse workers and entrepreneurs. In each period, banks raise funds from a short-term credit market (by accepting deposits from the workers) in which all agents can participate. In addition, they provide funds for entrepreneurs. Since banks can hold a portfolio that consists of a large number of depositors and entrepreneurs, in the steady state equilibrium on which we will focus our attention in this paper, standard deposit contracts are optimal and there is no default risk for the depositors. That is, in each period, for each unit of savings that the worker deposits in the bank, the bank pays \( 1 + r \) units at the end of the period, where \( r \) is the endogenously determined interest rate on deposits.

Since a typical entrepreneur's project is long-lived and since the returns on this project are private information, it is optimal for banks and entrepreneurs to enter into long-term dynamic credit relationships, instead of entering into a sequence of short-term contracts, or having the entrepreneur borrow directly from the short-term credit market each period. The structure of the optimal dynamic contract will be discussed in the following section, but each dynamic contract specifies a history-dependent repayment scheme to which the entrepreneur is committed, and in exchange for which the bank promises to provide capital that is needed by the entrepreneur throughout his productive life.

There is free entry into the competitive banking industry. As a result, in equilibrium all banks make zero profits. In principle, any coalition of agents can establish a bank. Since banks make zero profits, however, who owns banks is immaterial. Moreover, it is without loss of generality to view the banking industry as consisting of a single representative bank as we will do in the remainder of the paper.

### 2.4 The Dynamic Lending Contract

As discussed in the previous section, banks lend capital to entrepreneurs so that investment and production can take place. The informational asymmetry between banks and entrepreneurs and the long-lived nature of entrepreneurs' projects imply that the optimal credit relationship between banks and entrepreneurs is itself long-lived. This section describes the dynamic lending contract governing the credit relationship between a typical bank and a typical entrepreneur.

Specifically, the bank seeks to maximize the net present value of the stream of payments from the entrepreneur to the bank subject to incentive compatibility (truth-telling) and promise-keeping constraints. The bank discounts the future at the interest rate \( r \), which it takes as given. In addition, when discounting future payments the bank recognizes that, in any given period, a project will die with time-invariant probability \( 1 - \Delta \).
Following Spear and Srivastava (1987) and others, the state variable in the bank’s dynamic problem is \( w \), the amount of future utility that the bank promises to deliver to the entrepreneur. The solution to the bank’s problem consists of a pair of functions \( m(w, \theta) \) and \( w'(w, \theta) \), where \( m(w, \theta) \) specifies the entrepreneur’s payment if his current promised utility is \( w \) and he reports that his output is \( \theta \) and \( w'(w, \theta) \) specifies the entrepreneur’s promised utility at the beginning of the next period if his current promised utility is \( w \) and he reports that his output is \( \theta \).

Let \( v(w) \) be the value function associated with the bank’s dynamic contracting problem; \( v(w) \) is the net present value of the stream of payments from the entrepreneur to the bank given that the entrepreneur’s current level of promised utility is \( w \). Promised utilities take on values in the state space \( W = ((1 - \beta \delta)^{-1}U(0, \bar{\theta}), \infty) \). The following recursive dynamic programming problem determines the value function \( v \):

\[
v(w) = \max_{m_L, m_H, w_L, w_H} \{ \pi m_L + (1 - \pi)m_H + R^{-1} \Delta [\pi v(w_L) + (1 - \pi)v(w_H)] \}
\]

subject to:

\[
U(\theta_L - m_L, \bar{\theta}) + \beta \Delta w_L \geq U(\theta_L - m_H, \bar{\theta}) + \beta \Delta w_H \tag{6}
\]

\[
U(\theta_H - m_H, \bar{\theta}) + \beta \Delta w_H \geq U(\theta_H - m_L, \bar{\theta}) + \beta \Delta w_L \tag{7}
\]

\[
\pi[U(\theta_L - m_L, \bar{\theta}) + \beta \Delta w_L] + (1 - \pi)[U(\theta_H - m_H, \bar{\theta}) + \beta \Delta w_H] = w \tag{8}
\]

\[
m_i \leq \theta_i, \quad i = L, H
\]

\[
w_i \in W, \quad i = L, H \tag{10}
\]

For \( i = L, H, m_i \) is the entrepreneur’s payment to the bank if he reports that his output is equal to \( \theta_i \) and \( w_i \) is the entrepreneur’s promised utility at the beginning of the next period if he reports that his output is equal to \( \theta_i \). Equations (6) and (7) are the truth-telling constraints: equation (6) gives the entrepreneur an incentive not to falsely report that his output is \( \theta_H \), while equation (7) gives the entrepreneur an incentive not to falsely report that his output is \( \theta_L \). Equation (8) is the promise-keeping constraint; it ensures that the amount of utility delivered to the entrepreneur (the left-hand side) is equal to the amount that has been promised (the right-hand side). Finally, equation (9) states that the entrepreneur’s payment to the bank cannot exceed his output and equation (10) states that tomorrow’s promised utility must lie in the state space \( W \).

### 2.5 Equilibrium

Let \( \lambda \) be the fraction of newly born individuals who choose to become entrepreneurs; \( \lambda \) is an endogenous variable in our stationary equilibrium. Since the total measure of individuals in the economy is equal to one, \( \lambda \) is, in addition, the measure of entrepreneurs in the economy and \( 1 - \lambda \) is the measure of workers in the economy.

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The left endpoint of this interval is the lifetime utility of the entrepreneur if the entrepreneur gives all of his output to the bank in every period.
In equilibrium, each new individual must be indifferent between becoming a worker and becoming an entrepreneur. Let $w_0$ be the promised expected lifetime utility of a new entrepreneur. Indifference between becoming a worker and becoming an entrepreneur requires that the following condition hold:

$$S(0; r, \omega) = w_0. \tag{11}$$

In other words, a new worker’s lifetime utility is equal to the amount of utility that a bank promises to deliver to a new entrepreneur.

Market-clearing in the labor market requires that the following condition hold:

$$\lambda L = (1 - \lambda) H(r, \omega), \tag{12}$$

where the left-hand side is the aggregate demand for labor by entrepreneurs (recall that each entrepreneurial project requires $K$ units of capital and $L$ units of labor) and the right-hand side is the aggregate supply of labor by workers.

As noted in Section 2.3, we allow free entry into the banking sector. In equilibrium, therefore, the value of a new contract to a bank is equal to zero. The cost associated with a new contract is the net present value of the resources that the bank must commit to the entrepreneur. We assume that capital used in production depreciates at the rate $\delta$, so the cost of providing capital to an entrepreneur in any given period is $(r + \delta)K$. In addition, an entrepreneur’s labor costs in any given period are equal to $\omega L$. The benefit associated with a new contract is the net present value of the stream of payments from the entrepreneur to the bank. The bank discounts future cash flows at the rate $R^{-1}\Delta$. The zero-profit condition, therefore, reads:

$$v(w_0) = \frac{(r + \delta)K + \omega L}{1 - R^{-1}\Delta}. \tag{13}$$

In our stationary equilibrium, the representative bank holds a stationary (time-invariant) portfolio of contracts. Each contract in the portfolio is indexed by the current level of promised utility associated with the contract. Let the stationary distribution of promised utilities in the bank’s portfolio be denoted $\Gamma$. Formally, $\Gamma$ is the invariant distribution determined by the law of motion (or decision rule) $u'(w, \theta)$, the initial promised utility $w_0$, the birth/death rate $1 - \Delta$, and the stochastic process governing the evolution of $\theta$.

In each period, the bank receives total payments equal to $M$ from the entrepreneurs in its portfolio. Formally,

$$M = \int \left[ \pi m(w, \theta_L) + (1 - \pi) m(w, \theta_H) \right] \Gamma(dw). \tag{14}$$

In addition, the bank makes total payments equal to $E$ to the entrepreneurs in its portfolio. These payments are defined by:

$$E = \int [(r + \delta)K + \omega L] \Gamma(dw) = (r + \delta)K + \omega L, \tag{15}$$
In general, $M$ and $E$ are not equal to each other. This difference is accounted for by what we call banking capital. In particular, a typical bank holds a constant amount of capital $B$ in the stationary equilibrium of our economy. As we show in Appendix 1, the zero-profit condition for new contracts implies that $B$ satisfies:

$$rB + M - E = 0.$$  

(16)

In other words, at any point in time, the interest earned on the bank’s capital exactly balances the difference between payments to and from the entrepreneurs in the bank’s portfolio. Note that at this stage, $B$ can be positive, negative, or zero, depending on the difference between $M$ and $E$. In the calibrated version of the model, it turns out that $B$ is positive.

Market-clearing in the asset market, then, requires that the following condition hold:

$$\lambda K = (1 - \lambda) D(r, \omega) + \lambda B,$$

(17)

where $B$ is given by equation (16). Note that, in this context, $B$ can be interpreted as banking capital per entrepreneur (or project). The left-hand side of this equation is the aggregate demand for capital (assets) by entrepreneurs, while the right-hand side is the aggregate supply of capital by workers and banks.

We can now summarize the equilibrium conditions in the following formal definition of a steady-state equilibrium:

**Definition of Equilibrium**

A steady-state equilibrium consists of value functions $S$ and $v$, decision rules $f$, $g$, $w'$, and $m$, prices $r$ and $\omega$, an initial promised utility $w_0$, a distribution of promised utilities $\Gamma$, deposits $D$, labor supply $H$, payments to banks from entrepreneurs $M$, payments to entrepreneurs from banks $E$, banking capital $B$, and a fraction of entrepreneurs $\lambda$ satisfying:

1. Given $r$ and $\omega$, $S$, $f$, and $g$ solve the worker’s problem (2).

2. Given $r$ and $\omega$, $D$ and $H$ are consistent with $f$, $g$, and the birth/death rate $1 - \Delta$ (i.e., $D$ and $H$ are determined by equations (3) and (4)).

3. Given $r$, $v$, $w'$, and $m$ solve the dynamic contracting problem (5).

4. New individuals are indifferent between becoming a worker and becoming an entrepreneur (i.e., equation (11) holds).

5. The distribution of promised utilities $\Gamma$ is the invariant distribution determined by the initial promised utility $w_0$, the decision rule $w'$, the birth/death rate $1 - \Delta$, and the stochastic process for $\theta$. 

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6. $M$ and $E$ are defined as in equations (14) and (15), respectively, and $B$ is determined by equation (16).

7. The value of a new contract is equal to zero (i.e., equation (13) holds).

8. The labor market clears (i.e., equation (12) holds).

9. The asset market clears (i.e., equation (17) holds).

Finally, we note that when the equilibrium conditions are satisfied, the goods market also clears in each period, i.e., aggregate output is divided between aggregate consumption of workers, aggregate consumption of entrepreneurs, and aggregate investment. To see this, let $Y$ denote output per entrepreneur, let $C_W$ denote consumption per worker, and let $C_E$ denote consumption per entrepreneur. By definition, $Y = C_E + M$, i.e., entrepreneurial output is divided between entrepreneurial consumption and payments from entrepreneurs to banks. Inserting the definition of $E$ (see equation (15)) into equation (16), we can, therefore, obtain the following expression for banking capital:

$$B = \frac{(r + \delta)K + \omega L - (Y - C_E)}{r},$$

which implies that

$$rB = (r + \delta)K + \omega L - Y + C_E.$$ (18)

Now multiply both sides of the market-clearing condition (17) for the asset market by $r$ and replace the term $rB$ with the right-hand side of (18) to obtain:

$$\lambda r K = (1 - \lambda)r D + \lambda[(r + \delta)K + \omega L - Y + C_E].$$

Rearranging this expression and using the market-clearing condition for the labor market (12) to substitute for $\lambda L$ yields:

$$\lambda Y = (1 - \lambda)(r D + \omega H) + \lambda C_E + \lambda \delta K.$$  

Finally, note that the workers’ budget constraints imply that $C_W = r D + \omega H$ in a steady-state equilibrium. Thus, aggregate output $\lambda Y$ is divided between aggregate consumption of workers (i.e., $(1 - \lambda)C_W$), aggregate consumption of entrepreneurs (i.e., $\lambda C_E$), and aggregate investment (i.e., $\lambda \delta K$).

3 Computation

The steady-state equilibrium can be computed by finding the prices $r$ and $\omega$ that solve the pair of equations $F_1(r, \omega) = 0$ and $F_2(r, \omega) = 0$. The function $F_1$ corresponds to the zero-profit condition (13), while the function $F_2$ corresponds to market-clearing in the asset market (see equation (17)). To evaluate these functions, we proceed as follows:

1. Choose $r$ and $\omega$. 
2. Solve the worker’s problem (2) and calculate $D$ and $H$ according to equations (3) and (4).

3. Solve the dynamic contracting problem (5).

4. Use the market-clearing condition in the labor market (see equation (12)) to solve for $\lambda$:

$$\lambda = \frac{H}{H + L}.$$  \hspace{1cm} (19)

5. Use Monte Carlo simulation to compute an approximation to $M$ as defined in equation (14). Specifically, use a (pseudo)random number generator to generate a sequence $\{\theta_t\}_{t=0}^N$ and a sequence $\{u_t\}_{t=0}^N$, where the $u_t$’s are independent and identically distributed and each $u_t$ is uniformly distributed on the $[0,1]$ interval.\footnote{In practice, we use antithetic variates when conducting the Monte Carlo simulations. For our problem, this approach increases numerical accuracy without increasing computational cost.} Use these simulated sequences to generate a simulated sequence $\{w_t\}_{t=0}^N$, where $w_t$ is promised utility in period $t$. In particular, proceed as follows:

(a) Set $w_0 = S(0; r, \omega)$ and $t = 0$.

(b) If $u_t \leq 1 - \Delta$, then the entrepreneur dies at the beginning of the next period and is replaced by a new entrepreneur. In this case, set $w_{t+1} = w_0$. Otherwise, set $w_{t+1} = w'(w_t, \theta_t)$.

(c) Increment $t$. Iterate on steps (b)-(c) until $t = N$.

Given $\{w_t\}_{t=0}^N$ and $\{\theta_t\}_{t=0}^N$, the approximation to $M$ is computed as follows:

$$\hat{M} = N^{-1} \sum_{t=0}^N m(w_t, \theta_t).$$

6. Evaluate $F_1(\omega, \omega)$ and $F_2(\omega, \omega)$, defined as follows:

$$F_1(\omega, \omega) = v(S(0; r, \omega); r) - (1 - R^{-1} \Delta)^{-1} E$$

$$F_2(\omega, \omega) = \lambda K - (1 - \lambda) D - \lambda \hat{B},$$

where $E = (r + \delta) K + \omega L$, $\hat{B} = (E - \hat{M})/r$, and $\lambda$ is given by equation (19).\footnote{Note that we have emphasized the dependence of $v$ on $r$ by introducing $r$ as an explicit argument of $v$.}

7. Use Newton’s method with numerical (one-sided) derivatives to find the $r$ and $\omega$ that satisfy $F_1(r, \omega) = 0$ and $F_2(r, \omega) = 0$.

We compute the solution to the consumer’s problem using methods similar to those described in the Appendix to Krusell and Smith (1998). We also adapt these methods to compute the solution to the dynamic contracting problem.\footnote{When $\sigma = 0$, the dynamic contracting problem has an analytical (closed-form) solution. Appendix 3 derives this solution.} We describe these methods in detail in Appendix 2. In the simulation of promised utilities described in Section 3, we use 500,000 antithetic pairs (i.e., in terms of computation time, $N = 1,000,000$).
4 Calibration

We let the instantaneous utility function $U(c, \ell) = \log(c) + \eta \log(\ell)$. Given this parameterization, we must choose values for the following model parameters: $\beta$ (discount rate), $\Delta$ (survival rate), $\eta$ (relative weight on leisure in the utility function), $\pi$ (the probability that the entrepreneur has high output), $\bar{\theta}$ (expected value of an entrepreneur’s output), $\sigma$ (parameter governing the variation of an entrepreneur’s output), $K$ (units of capital per entrepreneurial project), $L$ (units of labor per entrepreneurial project), $\delta$ (rate of depreciation of capital), and $\bar{h}$ (hours of work supplied by an entrepreneur in each period).

We let a period in the model correspond to one year. We let $\Delta = 0.98$, so that the expected value of the length of an individual’s (working) life is 50 years. We set $\beta = 0.9633$; this choice implies that the equilibrium interest rate is approximately 4% (a standard number in the existing literature). In general, we can choose $\beta$ to match the observed real interest rate.

Given the specification of the utility function, $\bar{\theta}$ is simply a scaling parameter: without loss of generality, we set $\bar{\theta} = 1$. The capital-output ratio in our economy is given by:

$$\frac{\lambda K}{\lambda Y} = \frac{K}{Y} = \frac{K}{\bar{\theta}} = K,$$

where, as in Section 2.5, $Y$ is output per entrepreneur. To roughly match the capital-output ratio in the U.S. economy, therefore, we set $K = 2.65$.

We let $\delta = 0.1$, implying a quarterly depreciation rate of approximately 2.5% as in many existing macroeconomic studies. This choice for $\delta$ implies that the steady-state investment-to-output ratio is $(\lambda \delta K)/(\lambda Y) = 0.265$.

We set $\pi = 0.5$, in which case $\sigma$ can be interpreted as the standard deviation of an entrepreneur’s output. The value of $\sigma$ can be calibrated in one of two ways, according to the two interpretations of this parameter. First, at any point in time, $\sigma$ is the cross-sectional standard deviation of output across firms (entrepreneurial projects). Second, for any given firm, $\sigma$ is the standard deviation of its output across time. In future work, we plan to explore both of these approaches to calibrating $\sigma$. For the moment, we experiment with different values of $\sigma$.

The final three parameters for which we must choose parameters are $L$ (amount of labor per project), $\eta$ (the weight on leisure in the utility function), and $\bar{h}$ (hours of work supplied by an entrepreneur in each period). We choose $L$ and $\eta$ parameters by imposing two conditions on the model economy: one, average hours of work per worker (i.e., $H$) is equal to one-third; two, labor’s share of income is equal to 0.64 (this is a typical number from existing macroeconomic studies). We interpret entrepreneurial consumption (i.e., $C_E$) as labor income, so that labor’s share of income is given by:

$$\frac{(1 - \lambda) \omega H + \lambda C_E}{\lambda Y} = \frac{\lambda \omega L + \lambda C_E}{\lambda Y} = \frac{\omega L + C_E}{Y}.$$

Note that when $H = 1/3$, the expected amount of time that a new worker spends working over the course of his lifetime is equal to one-third of his lifetime time endowment. Accordingly, we set $\bar{h}$
equal to one-third as well.

To compute the calibrated values of $L$ and $\eta$, we use various schemes for searching over $(L, \eta)$ pairs. For each pair that we check, we compute the prices $r$ and $w$ by setting the functions $F_1$ and $F_2$ defined in (20) and (20) equal to zero.

5 Results

In our baseline model, we set $\sigma = 0.25$. In this case, we must set $L = 1.4$ and $\eta = 1.67$ in order to ensure that hours per worker is equal to one-third and labor’s share of income is equal to 0.64. We also computed the equilibrium of our economy for three additional values of $\sigma$: 0, 0.1, and 0.25. When $\sigma = 0$, an entrepreneur’s output is constant over time, thereby eliminating problems of moral hazard. Alternatively, one could imagine that an entrepreneur’s output does vary over time but can be freely observed by the bank. In this case, the optimal dynamic contract implements the same allocation that obtains when $\sigma = 0$. We refer to this allocation, which features full risk-sharing, as the first-best allocation. Table 1 summarizes the equilibrium values of key endogenous variables for each of the three parameterizations of the model that we consider.

<table>
<thead>
<tr>
<th>Table 1</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma = 0.5$ &amp; $\sigma = 0.25$ &amp; $\sigma = 0.1$ &amp; $\sigma = 0$</td>
</tr>
<tr>
<td>$r$ &amp; 0.04003 &amp; 0.04002 &amp; 0.04001 &amp; 0.04001</td>
</tr>
<tr>
<td>$\omega$ &amp; 0.3596 &amp; 0.3608 &amp; 0.3616 &amp; 0.3621</td>
</tr>
<tr>
<td>$\lambda$ &amp; 0.19211 &amp; 0.19218 &amp; 0.19224 &amp; 0.19228</td>
</tr>
<tr>
<td>$B/K$ &amp; 0.0735 &amp; 0.0742 &amp; 0.0751 &amp; 0.0761</td>
</tr>
<tr>
<td>$w_0$ &amp; -37.702 &amp; -37.646 &amp; -37.603 &amp; -37.580</td>
</tr>
<tr>
<td>$H$ &amp; 0.3329 &amp; 0.3331 &amp; 0.3332 &amp; 0.3333</td>
</tr>
<tr>
<td>$C_W$ &amp; 0.1431 &amp; 0.1435 &amp; 0.1438 &amp; 0.1440</td>
</tr>
<tr>
<td>$C_E$ &amp; 0.1332 &amp; 0.1317 &amp; 0.1306 &amp; 0.1301</td>
</tr>
<tr>
<td>Var(log($c_E$)) (age 0) &amp; 0.0012 &amp; 0.0008 &amp; 0.0004 &amp; 0</td>
</tr>
<tr>
<td>Var(log($c_E$)) (age 30) &amp; 0.128 &amp; 0.069 &amp; 0.024 &amp; 0</td>
</tr>
</tbody>
</table>

Notes: $\sigma$ is the standard deviation of an entrepreneur’s output, $r$ is the interest rate, $\omega$ is the wage rate, $\lambda$ is the fraction of entrepreneurs, $B/K$ is the fraction of aggregate capital that is held by banks (recall that aggregate banking capital is given by $\lambda B$ and aggregate capital is given by $\lambda K$), $w_0$ is the expected lifetime utility of a new individual\(\textsuperscript{10}\), $H$ is hours of work per worker, $C_W$ is consumption per worker, $C_E$ is consumption per entrepreneur, and Var(log($c_E$)) is the cross-sectional variance of the logarithm of consumption within a cohort of entrepreneurs.

Table 1 contains several important results. First, banks hold positive amounts of capital in equilibrium: depending on the value of $\sigma$, 7% to 8% of the aggregate capital stock is held by banks. As we noted in Section 2.5, banking capital arises because a bank’s total payments to entrepreneurs

\(\textsuperscript{10}\)To be precise, the numbers in the row labelled ‘$w_0$’ are actually equal to $w_0$ minus a typical entrepreneur’s expected lifetime utility of leisure (which is a constant in our model).
at a point in time (i.e., \( E \)) are not necessarily equal to its total payments from entrepreneurs (i.e., \( M \)). In particular, if \( M < E \), then \( B = (M - E)/r > 0 \).

Using the zero-profit condition for a new contract, it is straightforward to show that a sufficient condition for \( M \) to be smaller than \( E \) is that the payments to the bank of a typical entrepreneur decline, on average, as the entrepreneur ages. Put differently, if the average payments of a cohort of entrepreneurs decline over time, then \( M \) will be less than \( E \) and in steady state banks will hold positive amounts of capital. In our calibrated economies, we find that the average payments to banks of a cohort of entrepreneurs do decline over time. This result stems from several mechanisms at work in our economy.

First, for reasons that we discuss below, the equilibrium gross interest rate (i.e., \( R = 1 + r \)) is larger than \( \beta^{-1} \). This difference implies that the bank, which discounts future cash flows at rate \( R^{-1} \Delta \), is less patient than the entrepreneur, who discounts future utility at rate \( \beta \Delta \). The bank therefore prefers to receive payments from the entrepreneur relatively early. In order to achieve this objective, the entrepreneur must be induced to sacrifice consumption today for higher promised utility in the future. This mechanism, therefore, tends to push payments down over time because payments from the entrepreneur to the bank decrease as promised utility increases. (For the case \( \sigma = 0.25 \), Figure 1 graphs the payment schedule \( m(w, \theta) \) for each value of \( \theta \). Note that \( m(w, \theta) \) is a decreasing function of \( w \) for each \( \theta \).) Wang (1999) provides a theoretical account of this “interest rate effect.”

Second, because of private information, full intertemporal risk sharing is not achievable, and the optimal dynamic contract implies that the entrepreneur, when in a low output state, borrows against future income. Consequently, relative to full risk-sharing, the entrepreneur’s expected utility falls over time, other things equal. Much of the dynamic contracting literature has focused on the implications of this mechanism for the long-run behavior of expected utilities (Green 1987, Thomas and Worrall 1990, Atkeson and Lucas 1992). This “incentive effect” counteracts the interest rate effect discussed above. We evaluate quantitatively the net effect of both mechanisms in our calibrated general equilibrium model. We find that the interest rate effect dominates for low values of \( \sigma \) but that the incentive effect dominates for higher values of \( \sigma \) (in our experiments, the incentive effect dominates when \( \sigma = 0.5 \), but the interest effect dominates for the other values of \( \sigma \) that we examined). Figure 2, which graphs the law of motion for promised utility for the case \( \sigma = 0.25 \), summarizes the net quantitative effect of the two offsetting mechanisms. Notice that promised utility rises when the entrepreneur reports a high level of output and falls when the entrepreneur reports a low level of output (the middle line in Figure 2 is the 45-degree line). Although it is difficult to discern in the graph, the average of the two decision rules (one for each value of \( \theta \)) lies above the 45-degree line. In other words, the equilibrium law of motion \( \bar{w}'(w, \theta) \) implies that the stochastic process for the entrepreneur’s promised utility is a submartingale. Thus promised utility tends to rise over time. For the case \( \sigma = 0.5 \), the counterpart of Figure 2 is qualitatively similar, but the average of the two decision rules lies below the 45-degree line, and the entrepreneur’s promised
utility is a supermartingale.

A third and final mechanism at work in our economy tends to push the average payments of a cohort of entrepreneurs down over time. In particular, as shown in Figure 1, the payment schedule \( m(w, \theta) \) is a concave function of \( w \) for each value of \( \theta \). Thus, as the promised utility levels of a cohort of entrepreneurs spread out over time, Jensen's inequality implies that average payments to banks of this cohort will fall over time even if the law of motion for promised utility is an exact random walk with no drift. As discussed above, promised utility levels in fact tend to drift (either up or down) over time. When promised utility levels drift up over time, the third mechanism provides another force that tends to push payments down over time. When promised utility levels drift down over time, the third mechanism counteracts the effects of this downward drift. Quantitatively, we find that, when \( \sigma = 0.5 \), although the entrepreneur's promised utility drifts down over time, the third mechanism (due to the concavity of the function \( m \)) dominates, so that the average payments of a cohort of entrepreneurs again fall over time.

For the case \( \sigma = 0.25 \), Figure 3 graphs the expected value (as of the beginning of the entrepreneur's life) of the payments to the bank that a typical entrepreneur makes at each stage of his life. As discussed above, average payments decline as the entrepreneur ages. (The horizontal line in Figure 3 is the value of the resources that the bank commits to the entrepreneur at each stage of the entrepreneur's life.)

We turn now to explain the finding that \( R > \beta^{-1} \) in equilibrium. It is clear from the worker's optimization problem that \( R \) must be greater than \( \beta^{-1} \) in order to induce workers to save.\(^{11}\) For example, if \( R = \beta^{-1} \), a new worker would not accumulate assets, preferring instead simply to consume his labor income in each period. This outcome cannot be an equilibrium since aggregate savings would be zero and production would not take place.

Figure 4 displays (an approximation to) the equilibrium distribution of promised utilities \( \Gamma \) for the case \( \sigma = 0.25 \).\(^{12}\) The distribution has a large variance, reflecting the fact that the promised utility of unlucky entrepreneurs (those with streaks of low output) falls over time, while the promised utility of lucky entrepreneurs (those with streaks of high output) rises over time. As discussed above, however, promised utility tends to rise on average when \( \sigma = 0.25 \). This fact is reflected in the mean of \( \Gamma \), which is higher than the initial promised utility \( w_0 \) of a new entrepreneur.

A second important finding contained in Table 1 is that private information has very small effects on aggregate economic activity, even when \( \sigma \) is large. Although increases in \( \sigma \) tend to depress aggregate output and aggregate capital (\( \lambda \) falls as \( \sigma \) increases), the quantitative effects are tiny. The effects on the welfare of a newly born individual are somewhat larger: when \( \sigma = 0.25 \), for example, a new individual is worse off, relative to the case \( \sigma = 0 \), by the equivalent of 0.4% of per period consumption. Nonetheless, the welfare effects of private information in an economy

\(^{11}\) Blanchard and Fischer (1989) make a similar point in Chapter 3.3.

\(^{12}\) Figure 4 is a histogram based on the first 10,000 simulated values for promised utility; see Section 3 for details concerning this simulation.
with dynamic contracting are small in an absolute sense. In other words, the optimal provision of intertemporal incentives by means of the dynamic contract is very effective at mitigating the effects of private information.\textsuperscript{13} Even when \( \sigma \) is large, the equilibrium allocation is close to a first-best allocation (i.e., one that would obtain in the absence of private information).

The effectiveness of dynamic contracting can be illustrated in a dramatic manner by restricting the bank to use static contracts, that is, contracts that are contingent only on current promised utility and not on the current realization of an entrepreneur’s output. These contracts are static in the sense that they do not use intertemporal incentives to elicit truth-telling. Formally, these contracts solve the following dynamic programming problem:

\[
v(w) = \max_{m,w'} \left[ m + R^{-1} \Delta v(w') \right]
\]

subject to \( m < \theta_L \), \( w \in \mathcal{W} \), and

\[
\pi [U(\theta_L - m, \tilde{e}) + \beta \Delta w'] + (1 - \pi) [U(\theta_H - m, \tilde{e}) + \beta \Delta w'] = w.
\]

We embed these static contracts in a model economy identical to the one described in Section 2.5. Table 2 summarizes the quantitative results for a variety of values of \( \sigma \) (when \( \sigma = 0 \), the results are identical to those in the last column of Table 1).

<table>
<thead>
<tr>
<th></th>
<th>( \sigma = 0.5 )</th>
<th>( \sigma = 0.25 )</th>
<th>( \sigma = 0.1 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( r )</td>
<td>0.0453</td>
<td>0.0408</td>
<td>0.0402</td>
</tr>
<tr>
<td>( \omega )</td>
<td>0.082</td>
<td>0.259</td>
<td>0.340</td>
</tr>
<tr>
<td>( \lambda )</td>
<td>0.149</td>
<td>0.184</td>
<td>0.191</td>
</tr>
<tr>
<td>( B/K )</td>
<td>0.002</td>
<td>0.014</td>
<td>0.042</td>
</tr>
<tr>
<td>( w_0 )</td>
<td>-63.92</td>
<td>-43.57</td>
<td>-38.69</td>
</tr>
<tr>
<td>( H )</td>
<td>0.245</td>
<td>0.316</td>
<td>0.329</td>
</tr>
<tr>
<td>( C_W )</td>
<td>0.041</td>
<td>0.106</td>
<td>0.136</td>
</tr>
<tr>
<td>( C_E )</td>
<td>0.501</td>
<td>0.266</td>
<td>0.157</td>
</tr>
<tr>
<td>( \text{Var}(\log(c_E)) ) (age 0)</td>
<td>13.23</td>
<td>3.23</td>
<td>0.65</td>
</tr>
<tr>
<td>( \text{Var}(\log(c_E)) ) (age 30)</td>
<td>12.48</td>
<td>3.09</td>
<td>0.60</td>
</tr>
</tbody>
</table>

Notes: See Table 1.

To assess the effect of restricting banks to use static contracts, the results in Table 2 can be compared to the corresponding results for dynamic contracts in Table 1. When \( \sigma = 0.25 \), for example, introducing static contracts reduces steady state aggregate output by roughly 4% and reduces the initial promised utility of a newly born individual by the equivalent of 38% of per period consumption (largely because risk sharing is so ineffective with static contracts). When \( \sigma = 0.5 \),

\textsuperscript{13}Khan and Ravikumar (1997b) make a similar point.
the effects of moving from dynamic contracting to static contracting are even more striking: output drops by 22% and individual welfare falls by the equivalent of 77% of per period consumption.

A third important finding that emerges from Tables 1 and 2 concerns heterogeneity in consumption within a cohort of entrepreneurs. Deaton and Paxson (1994) document that in U.S. data the variance of the logarithm of consumption within a cohort of individuals increases dramatically as the cohort ages (from 0.25 at age 25 to 0.47 at age 55). Table 1 shows that our model economy exhibits the same pattern: for example, when $\sigma = 0.5$, the variance of log consumption within a cohort of entrepreneurs increases from roughly zero at birth (since entrepreneurs are ex ante identical) to 0.13 at age 30. Although this increase is not as large as the one observed in the data, it nonetheless indicates that the optimal provision of intertemporal incentives could be one force explaining increasing dispersion of consumption over time within a cohort of ex ante identical individuals.\footnote{See Storesletten, Telmer, and Yaron (1997) for an alternative explanation.} Figure 5 graphs the cross-sectional variance of log consumption against the age of the cohort for the case $\sigma = 0.25$. This figure shows that the cross-sectional variance increases linearly with the cohort’s age, a shape which is roughly consistent with the facts documented in Deaton and Paxson (1994). Finally, note that static contracting is at odds with the observed data: inequality within a cohort decreases over time as the cohort ages.

As a final point, note that since dynamic contracts are very effective in an environment in which banks cannot observe an entrepreneur’s output, it is clear that the effects of introducing costly state verification (i.e., allowing banks to observe an entrepreneur’s output by paying a fixed cost) would be small.

\section{Conclusion and Extensions}

This paper develops and implements a general equilibrium model with private information in which borrowers and lenders enter into long-term credit relationships. As far as we know, this is the first general equilibrium model in which a fully dynamic banking sector is explicitly modelled. We calibrate the model to U.S. aggregate data and use the model to analyze the impact of informational frictions on the behavior of the macroeconomic aggregates and on the welfare of individuals.

One line along which future research can be undertaken is to conduct policy experiments for which our model is suited. For example, our model can be used to evaluate the impact of banking regulation. Suppose financial intermediaries are required to hold reserves for the loans they make. This would increase the cost of financial intermediation, which in turn would have an impact on the performance of the aggregate economy. Our model could also be used to evaluate the impact of various tax policies on the performance of the macroeconomy, particularly through their effects on the equilibrium quantities of financial transactions and on the agents’ occupational choices.

A key assumption in our model is that there are no aggregate risks, so that the macroeconomic aggregates do not vary over time. An important extension to our model is to introduce aggregate

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14 See Storesletten, Telmer, and Yaron (1997) for an alternative explanation.
uncertainty in the form of common shocks to the productive opportunities of entrepreneurs. This extension will allow us to examine how financial intermediation affects the propagation of aggregate shocks. Introducing aggregate uncertainty is a challenging computational problem since agents in the economy then need to keep track of the dynamic behavior of the distribution of asset holdings across workers and of the distribution of promised utilities across entrepreneurs. This is the subject of ongoing research.\footnote{In particular, we are currently exploring whether the “approximate aggregation” findings of Krusell and Smith (1998) extend to the economic environment in this paper.}
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Appendix 1

This appendix provides a statement and proof of the equation of banking capital. Let \( w^t \in \mathcal{W} \) denote the expected utility of a \( t \)-year-old entrepreneur.

Make the following definitions:

\[
A \equiv \delta K + \omega L
\]

\[
E \equiv \sum_{t=1}^{\infty} (1 - \Delta) \Delta^{t-1} A
\]

\[
\overline{m}(w^t) \equiv \pi m_L(w^t) + (1 - \pi) m_H(w^t)
\]

\[
M \equiv \sum_{t=1}^{\infty} (1 - \Delta) \Delta^{t-1} \int \overline{m}(w^t) dw^t
\]

\[
X_t \equiv \int [\overline{m}(w^t) - A] dw^t
\]

\[
B_t \equiv \sum_{\tau=1}^{t} R^\tau \Delta^{\tau-1} X_\tau
\]

\[
B \equiv \sum_{t=1}^{\infty} (1 - \Delta) B_t
\]

where \( \{m_L, m_H\} \) is the optimal payment scheme of the dynamic contract.

**Theorem** \( (R - 1) B + M = E \).

Proof. The fact that ex ante the bank makes zero profits on each contract implies

\[
0 = X_1 + \frac{\Delta}{R} X_2 + \frac{\Delta^2}{R^2} X_3 + ...
\]

which by simple manipulation gives rise to the following equations

\[
0 = B_t + \frac{\Delta^t}{R} Z_{t+1}, \quad t = 1, 2, ...
\]

(20)

where

\[
Z_{t+1} = \sum_{i=1}^{\infty} \left( \frac{\Delta}{R} \right)^{i-1} X_{t+i},
\]

and we have:

\[
B = -(1 - \Delta) \sum_{t=1}^{\infty} \frac{\Delta^t}{R} Z_{t+1}.
\]

Thus to show that the theorem holds we need only show that

\[
-(R - 1)(1 - \Delta) \sum_{t=1}^{\infty} \frac{\Delta^t}{R} Z_{t+1} + \sum_{t=1}^{\infty} (1 - \Delta) \Delta^{t-1} \int \overline{m}(w^t) dw^t = \sum_{t=1}^{\infty} (1 - \Delta) \Delta^{t-1} A
\]

21
or

\[ \sum_{t=1}^{\infty} \Delta^{t-1} X_t = \frac{R-1}{R} \sum_{t=1}^{\infty} \Delta^t Z_{t+1} \]

(21)

Now by the definition of \( B_t \), it holds that

\[ B_t = R B_{t-1} + \Delta^{t-1} X_t, \quad t = 1, 2, ..., B_0 = 0 \]

which, combined with (20), yields:

\[ \Delta^{t-1} X_t = -R B_{t-1} - \frac{\Delta^t}{R} Z_{t+1}, \quad t = 1, 2, ... \]

Therefore to show that (21) holds, we need only show that

\[ -R \sum_{t=1}^{\infty} B_{t-1} - \frac{1}{R} \sum_{t=1}^{\infty} \Delta^t Z_{t+1} = \frac{R-1}{R} \sum_{t=1}^{\infty} \Delta^t Z_{t+1} \]

But this is equivalent to

\[ \sum_{t=1}^{\infty} B_{t-1} = - \sum_{t=1}^{\infty} \frac{\Delta^t}{R} Z_{t+1} \]

which holds because \( \sum_{t=1}^{\infty} B_{t-1} = \sum_{t=1}^{\infty} B_t \), given \( B_0 = 0 \).

**Appendices 2 and 3**

To be added.
Figure 1: Entrepreneur's Payment to Bank (sigma = 0.25)
(upper line = high output, lower line = low output)
Figure 2: Law of Motion for Promised Utility ($\sigma = 0.25$) (upper line = high output, lower line = low output)
Figure 3: Average Payment Schedule of a Typical Entrepreneur (sigma = 0.25)
Figure 4: Equilibrium Distribution of Promised Utilities (sigma = 0.25)
Figure 5: Cross-Sectional Variance of Log Consumption
(sigma = 0.25)