Question 1 A utility function is given by \( u(x_1, x_2) = \sqrt{x_1} + 4 \sqrt{x_2} \). Suppose that prices are \( p_1 = 1, p_2 = 3 \).

The partial derivatives are \( \frac{1}{2} \frac{1}{\sqrt{x_1}} \) and \( 4 \frac{1}{2 \sqrt{x_2}} \). Thus, \( \text{MRS} = \frac{\sqrt{x_2}}{4 \sqrt{x_1}} = \frac{1}{9} \).

Then the equation of the income offer curve is

\[
x_2 = \frac{36}{81} x_1
\]

Question 2 The indirect utility function is given by

\[
v(p_1, p_2, I) = \frac{I}{p_1^{0.2} p_2^{0.8}}
\]

The expenditure function is

\[
e(p_1, p_2, u) = u \frac{p_1^{0.2}}{p_2^{0.8}}
\]

and the Hicksean demand functions are

\[
h_1(p_1, p_2, u) = 0.2 u \left( \frac{p_1}{p_2} \right)^{0.8}, h_2(p_1, p_2, u) = 0.8 u \left( \frac{p_1}{p_2} \right)^{0.2}
\]

Question 3 A utility function is given by \( u(x_1, x_2) = x_1 + \ln x_2 \). Suppose that prices are \( p_1 = 2, p_2 = 1 \), and the person wants to get a utility of \( \bar{u} \). Specify the expenditure minimization problem in the box below

\[
\min_{x_1, x_2} 2x_1 + x_2, \quad \text{subject to (i) } x_1 + \ln x_2 \geq \bar{u}, \text{ (ii) } x_1 \geq 0, \text{ (iii) } x_2 \geq 0.
\]

Thus, the Lagrangean is given by (specify all constraints for the Lagrangean. Also, the Lagrangean must be written in such a way that the multipliers are all greater or equal to zero).

\[
\mathcal{L} = 2x_1 + x_2 - \lambda_1 (x_1 + \ln x_2 - \bar{u}) - \lambda_2 x_1 - \lambda_3 x_2
\]

The first order conditions are

\[
\frac{\partial \mathcal{L}}{\partial x_1} : 2 - \lambda_1 - \lambda_2 = 0
\]

\[
\frac{\partial \mathcal{L}}{\partial x_1} : 1 - \lambda_1 / x_2 - \lambda_3 = 0
\]

Question 4 Income offer curves and indifference curves are depicted below. Originally price are \( p_1 = 1, p_2 = 1 \) and income is \( I = 20 \). Then the price of good 2 increases to \( p_2 = 4 \). Determine graphically the Hicks and the Slutsky substitution and income effect.
The Slutzky substitution effect for goods 1 and 2 is \[ \Delta'x_1 = 15, \Delta'x_2 = -3.75 \]
The Slutzky income effect for goods 1 and 2 is \[ \Delta'x_1 = -15, \Delta'x_2 = -3.75 \]
The Hicks substitution effect for goods 1 and 2 is \[ \Delta'x_1 = 10, \Delta'x_2 = -5 \]
The Hicks income effect for goods 1 and 2 is \[ \Delta'Ix_1 = -10, \Delta'Ix_2 = -2.5 \]

**Question 5** A person's utility function is given by \( u(L, c) = L^2c \), where \( L \) is leisure and \( c \) is consumption. The person's income is derived from work.

(a) Suppose the hourly wage is \( w = 10 \). Thus, MRS = \( 2c/L = 10 \). Hence \( c = 5L \).

\[ c = 10\ell, \text{ where } \ell = 24 - L. \] Thus, \( c = 240 - 10L \), i.e., \( c + 10L = 240 \). Thus, \( 15L = 240 \), which implies \( L = 16 \) and \( c = 80 \). Thus, utility is
Then, the person’s maximum utility is 20,480.

(b) At the new prices we have $2c/L = 6$, i.e., $c = 3L$. In addition, $L^2c = 20,480$, i.e., $3L^3 = 20,480$. Hence, $L = 18.97$, and $c = 56.91$. $\ell = 24 - L$ implies $\ell = 5.03$. Since the after-tax wage is 6, the total wage income is 30.18. However, to afford 56.91 the person needs a lump sum subsidy of 26.73. The tax revenue is $4\ell = 20.12$. Thus, the deadweight loss is 6.61.

| Tax revenue is: 20.12 |
| The lump sum subsidy is: 26.73 |
| The deadweight loss is: 6.61 |

**Question 6** Suppose preferences are given by $u(x_1, x_2) = 2\sqrt{x_1x_2}$, which yields the indirect utility function $v(p_1, p_2, I) = I / \sqrt{p_1p_2}$, and the expenditure function $e(p_1, p_2, u) = u \sqrt{p_1p_2}$. Suppose prices are $p_1 = 1$, $p_2 = 25$ and income is $I = 600$. Then the price increases of good 1 increases to $p_1 = 9$. Determine the compensating and the equivalent variation associated with the price change.

The compensating variation is $600 - e(9, 25, v(1, 25, 600))$. Note that $v(1, 25, 600) = 120$, and $e(9, 25, 120) = 1, 800$.

**The compensating variation is: -1, 200.**

The equivalent variation is $e(1, 25, v(9, 25, 600)) - 600$. Note that $v(9, 25, 600) = 40$, and $e(1, 25, 40) = 200$.

**The equivalent variation is: -400**

**Question 7** Suppose a person’s Bernoulli utility function is given by $u(x) = \ln(x)$. The person has 100 Dollars to invest. There are two investments available: (a) A riskless asset that pays an interest rate of 0%. A risky assets that that either pays 40% with probability 0.6 or -40% with probability 0.4.

Suppose the person invests $\alpha$ Dollars into the risky asset $100 - \alpha$ Dollars into the riskless asset. The person’s expected utility is

$$0.6 \ln((100 - \alpha) + 1.4\alpha) + 0.4 \ln((100 - \alpha) + 0.6\alpha)$$

The first order conditions is

$$\frac{0.6(-1 + 1.4)}{(100 - \alpha) + 1.4\alpha} + \frac{0.4(-1 + 0.6)}{(100 - \alpha) + 0.6\alpha} = 0.$$

Thus, $4((100 - \alpha) + 1.4\alpha) = 6((100 - \alpha) + 0.6\alpha)$, which implies

$$\alpha = 50$$
**Question 8** A person has mean variance preferences of the form $10E[X] - 2\text{Var}[X]$, where $X$ is the random variable that describes the portfolio return.

Suppose the person has 100 Dollars. He invests $\alpha$ Dollars in a risky asset with mean return 1.7 and a variance of 0.1. The remainder $(100 - \alpha)$ is invested in a riskless asset with return 1.1.

Thus, the person solves

$$\max_{\alpha} 10(1.6\alpha + 1.1(100 - \alpha)) - 0.1\alpha^2.$$ 

The first order condition is $17 - 11 - 0.4\alpha = 0$.

**The optimal $\alpha = 15$**

**Question 9** A person has 10,000 Dollars of wealth. He/she has an opportunity to make an investment which has an initial cost of 6,000 Dollars, but has a payoff of 20,000 Dollars with probability 0.2, 10,000 with probability 0.6, and zero otherwise.

(a) The expected utility is

$$-0.2/24,000 - 0.6/14,000 - 0.2/4,000.$$ 

**The expected utility of making the investment is $-0.0001012$**

**The expected utility of not making the investment is $-0.0001$**

Therefore the person should (circle the correct answer) **not make the investment**.

(b) Now suppose that highest payoff is $m$ instead of 20,000 Dollars. Determine the value of $m$ such that the person is just indifferent between making and not making the investment. The expected utility is

$$-0.2/(m + 4000) - 0.6/14,000 - 0.2/4,000.$$ 

$m = 24,000$