Question 1 A utility function is given by $u(x_1, x_2) = \sqrt{x_1} + 4\sqrt{x_2}$. Suppose that prices are $p_1 = 1$, $p_2 = 3$.

The partial derivatives are $1/2\sqrt{x_1}$ and $4/(2\sqrt{x_2})$. Thus, MRS $= \sqrt{x_2}/(4\sqrt{x_1}) = 1/9$.

Then the equation of the income offer curve is

$$\boxed{\mathbf{x_2 = (36/81)x_1}}$$

Question 2 The indirect utility function is given by

$$\boxed{\mathbf{v(p_1, p_2, I) = \frac{I}{p_1^{0.2} p_2^{0.8}}}}$$

The expenditure function is

$$\boxed{\mathbf{e(p_1, p_2, u) = u p_1^{0.2} p_2^{0.8}}}$$

and the Hicksean demand functions are

$$\boxed{\mathbf{h_1(p_1, p_2, u) = 0.2u\left(\frac{p_2}{p_1}\right)^{0.8}, h_2(p_1, p_2, u) = 0.8u\left(\frac{p_1}{p_2}\right)^{0.2}}}$$

Question 3 A utility function is given by $u(x_1, x_2) = x_1 + \ln x_2$. Suppose that prices are $p_1 = 2$, $p_2 = 1$, and the person wants to get a utility of $\bar{u}$. Specify the expenditure minimization problem in the box below *3 points*

$\min_{x_1, x_2} 2x_1 + x_2$, subject to (i) $x_1 + \ln x_2 \geq \bar{u}$, (ii) $x_1 \geq 0$, (iii) $x_2 \geq 0$.

Thus, the Lagrangean is given by (specify *all* constraints for the Lagrangean. Also, the Lagrangean must be written in such a way that the multipliers are all greater or equal to zero).

$\mathfrak{L} = 2x_1 + x_2 - \lambda_1(x_1 + \ln x_2 - \bar{u}) - \lambda_2 x_1 - \lambda_3 x_2$

The first order conditions are

$$\boxed{\mathbf{\frac{\partial \mathfrak{L}}{\partial x_1} : \quad 2 - \lambda_1 - \lambda_2 = 0}}$$

$$\boxed{\mathbf{\frac{\partial \mathfrak{L}}{\partial x_1} : \quad 1 - \lambda_1/x_2 - \lambda_3 = 0}}$$

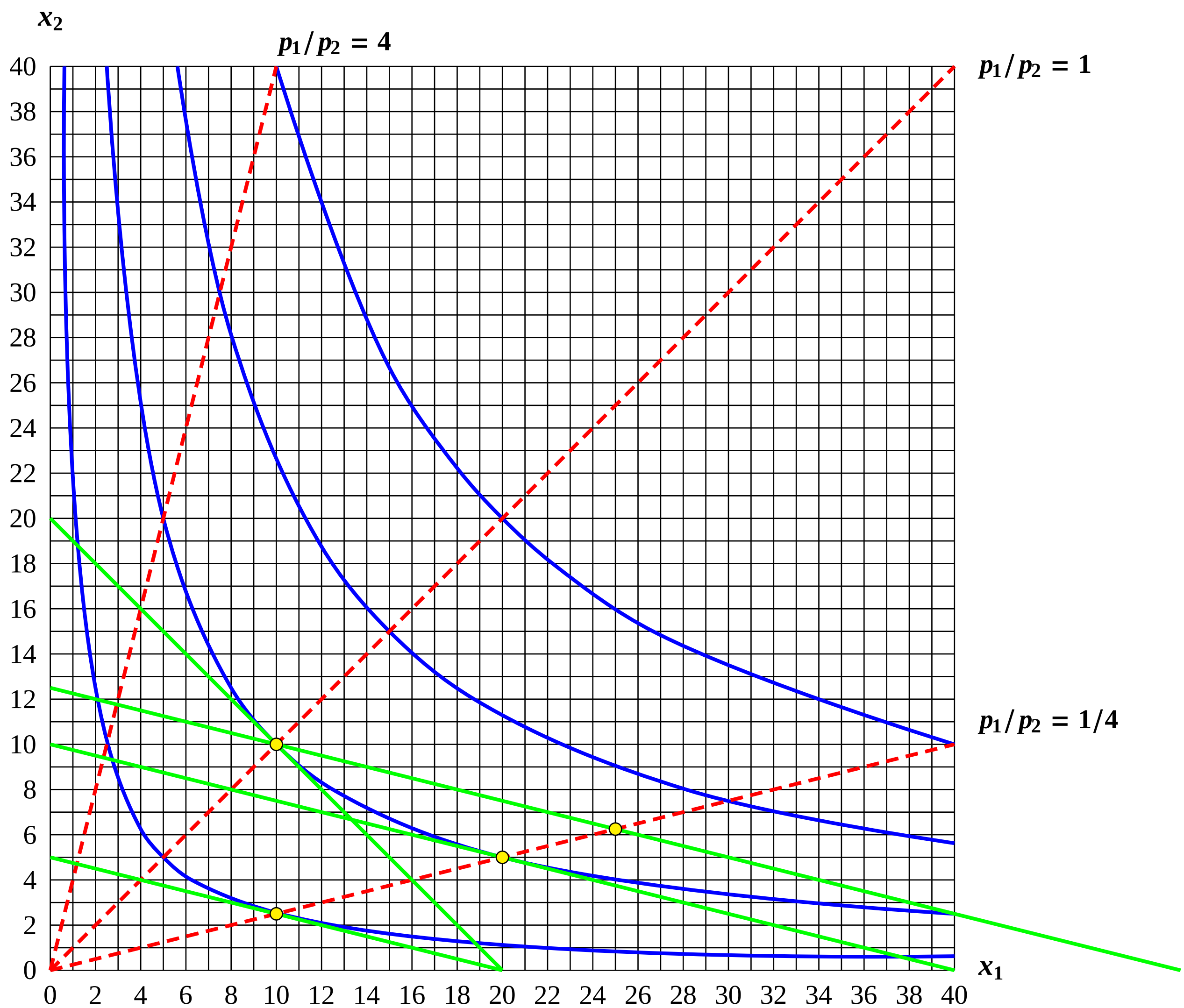
Question 4 Income offer curves and indifference curves are depicted below. Originally price are $p_1 = 1$, $p_2 = 1$ and income is $I = 20$. Then the price of good 2 increases to $p_2 = 4$. Determine graphically the Hicks and the Slutzky substituion and income effect.

The Slutzky substitution effect for goods 1 and 2 is $\boxed{\Delta^s x_1 = 15, \Delta^s x_2 = -3.75}$

The Slutzky income effect for goods 1 and 2 is $\boxed{\Delta^I x_1 = -15, \Delta^I x_2 = -3.75}$

The Hicks substitution effect for goods 1 and 2 is $\boxed{\Delta^s x_1 = 10, \Delta^s x_2 = -5}$

The Hicks income effect for goods 1 and 2 is $\boxed{\Delta^I x_1 = -10, \Delta^I x_2 = -2.5}$



Question 5 A person's utility function is given by $u(L, c) = L^2 c$, where L is leisure and c is consumption. The person's income is derived from work.

(a) Suppose the hourly wage is $w = 10$. Thus, MRS $= 2c/L = 10$. Hence $c = 5L$. $c = 10\ell$, where $\ell = 24 - L$. Thus, $c = 240 - 10L$, i.e., $c + 10L = 240$. Thus, $15L = 240$, which implies $L = 16$ and $c = 80$. Thus, utility is

Then, the person's maximum utility is $\boxed{\textbf{20,480}}$.

(b) At the new prices we have $2c/L = 6$, i.e., $c = 3L$. In addition, $L^2c = 20,480$, i.e., $3L^3 = 20,480$. Hence, $L = 18.97$, and $c = 56.91$. $\ell = 24 - L$ implies $\ell = 5.03$. Since the after-tax wage is 6, the total wage income is 30.18. However, to afford 56.91 the person needs a lump sum subsidy of 26.73. The tax revenue is $4\ell = 20.12$. Thus, the deadweight loss is 6.61.

Tax revenue is: 20.12

The lump sum subsidy is: 26.73

The deadweight loss is: 6.61

Question 6 Suppose preferences are given by $u(x_1, x_2) = 2\sqrt{x_1x_2}$, which yields the indirect utility function $v(p_1, p_2, I) = I/\sqrt{p_1p_2}$, and the expenditure function $e(p_1, p_2, u) = u\sqrt{p_1p_2}$. Suppose prices are $p_1 = 1$, $p_2 = 25$ and income is $I = 600$. Then the price increases of good 1 increases to $p_1 = 9$. Determine the compensating and the equivalent variation associated with the price change.

The compensating variation is $600 - e(9, 25, v(1, 25, 600))$. Note that $v(1, 25, 600) = 120$, and $e(9, 25, 120) = 1,800$.

The compensating variation is: $-1,200$.

The equivalent variation is $e(1, 25, v(9, 25, 600)) - 600$. Note that $v(9, 25, 600) = 40$, and $e(1, 25, 40) = 200$.

The equivalent variation is: -400

Question 7 Suppose a person's Bernoulli utility function is given by $u(x) = ln(x)$. The person has 100 Dollars to invest. There are two investments available: (a) A riskless asset that pays an interest rate of 0%. A risky assets that that either pays 40% with probability 0.6 or -40% with probability 0.4.

Suppose the person invests α Dollars into the risky asset $100 - \alpha$ Dollars into the riskless asset. The person's expected utility is

$$\boxed{0.6\ln((100-\alpha)+1.4\alpha)+0.4\ln((100-\alpha)+0.6\alpha)}$$

The first order conditions is

$$\frac{0.6(-1+1.4)}{(100-\alpha)+1.4\alpha} + \frac{0.4(-1+0.6)}{(100-\alpha)+0.6\alpha} = 0.$$

Thus, $4((100 - \alpha) + 1.4\alpha) = 6((100 - \alpha) + 0.6\alpha)$, which implies

$$\boxed{\alpha = 50}$$

Question 8 A person has mean variance preferences of the form $10E[X] - 2\text{Var}[X]$, where X is the random variable that describes the portfolio return.

Suppose the person has 100 Dollars. He invests α Dollars in a risky asset with mean return 1.7 and a variance of 0.1. The remainder $(100 - \alpha)$ is invested in a riskless asset with return 1.1.

Thus, the person solves

$$\max_{\alpha} 10(1.6\alpha + 1.1(100 - \alpha)) - 0.1\alpha^2.$$

The first order condition is $17 - 11 - 0.4\alpha = 0$.

$\boxed{\textbf{The optimal } \boldsymbol{\alpha} \textbf{ = 15}}$

Question 9 A person has 10,000 Dollars of wealth. He/she has to opportunity to make an investment which has an initial cost of 6,000 Dollars, but has a payoff of 20,000 Dollars with probability 0.2, 10,000 with probability 0.6, and zero otherwise.

(a) The expected utility is

$$-0.2/24,000 - 0.6/14,000 - 0.2/4,000.$$

$\boxed{\textbf{The expected utility of making the investment is } \mathbf{-0.0001012}}$

$\boxed{\textbf{The expected utility of \textit{not} making the investment is } \mathbf{-0.0001}}$

Therefore the person should (*circle the correct answer*) $\boxed{\textbf{not make the investment}}$.

(b) Now suppose that highest payoff is m instead of 20,000 Dollars. Determine the value of m such that the person is just indifferent between making and not making the investment. The expected utility is

$$-0.2/(m + 4000) - 0.6/14,000 - 0.2/4,000.$$

$\boxed{\boldsymbol{m} \mathbf{= 24,000}}$