Question 1 A utility function is given by \( u(x_1, x_2) = 4 \sqrt{x_1} + \sqrt{x_2} \). Suppose that prices are \( p_1 = 3, \ p_2 = 2 \).

The partial derivatives are \( 4/2 \sqrt{x_1} \) and \( 1/(2 \sqrt{x_2}) \). Thus, MRS = \( 4 \sqrt{x_2}/(\sqrt{x_1}) = 3/2 \).

Then the equation of the income offer curve is

\[
x_2 = (9/64)x_1
\]

Question 2 The indirect utility function is given by

\[
v(p_1, p_2, I) = \frac{I}{p_1^{0.9}p_2^{0.1}}
\]

The expenditure function is

\[
e(p_1, p_2, u) = up_1^{0.9}p_2^{0.1}
\]

and the Hicksean demand functions are

\[
h_1(p_1, p_2, u) = 0.9u \left( \frac{p_2}{p_1} \right)^{0.1}, \ h_2(p_1, p_2, u) = 0.1u \left( \frac{p_1}{p_2} \right)^{0.9}
\]

Question 3 A utility function is given by \( u(x_1, x_2) = \ln(x_1) + x_2 \). Suppose that prices are \( p_1 = 1, \ p_2 = 4, \) and the person wants to get a utility of \( \bar{u} \). Specify the expenditure minimization problem in the box below

\[
\min_{x_1, x_2} x_1 + 4x_2, \ \text{subject to} \ (i) \ \ln(x_1) + x_2 \geq \bar{u}, \ (ii) \ x_1 \geq 0, \ (iii) \ x_2 \geq 0.
\]

Thus, the Lagrangean is given by (specify all constraints for the Lagrangean. Also, the Lagrangean must be written in such a way that the multipliers are all greater or equal to zero).

\[
\mathcal{L} = x_1 + 4x_2 - \lambda_1(\ln(x_1) + x_2 - \bar{u}) - \lambda_2 x_1 - \lambda_3 x_2
\]

The first order conditions are

\[
\frac{\partial \mathcal{L}}{\partial x_1} : \ 1 - \lambda_1/x_1 - \lambda_2 = 0
\]

\[
\frac{\partial \mathcal{L}}{\partial x_1} : \ 4 - \lambda_1 - \lambda_3 = 0
\]

Question 4 Income offer curves and indifference curves are depicted below. Originally price are \( p_1 = 1, \ p_2 = 1 \) and income is \( I = 20 \). Then the price of good 1 increases to \( p_1 = 4 \). Determine graphically the Hicks and the Slutzky substitution and income effect.
The Slutzky substitution effect for goods 1 and 2 is $\Delta x_1 = -3.75, \Delta x_2 = 15$

The Slutzky income effect for goods 1 and 2 is $\Delta' x_1 = -3.75, \Delta' x_2 = -15$

The Hicks substitution effect for goods 1 and 2 is $\Delta s x_1 = -5, \Delta s x_2 = 10$

The Hicks income effect for goods 1 and 2 is $\Delta' x_1 = -2.5, \Delta' x_2 = 10$

**Question 5** A person’s utility function is given by $u(L, c) = L^2c$, where $L$ is leisure and $c$ is consumption. The person’s income is derived from work.

(a) Suppose the hourly wage is $w = 8$. Thus, $MRS = 2c/L = 8$. Hence $c = 4L$. 
$c = 8\ell$, where $\ell = 24 - L$. Thus, $c = 192 - 8L$, i.e., $c + 8L = 192$. Thus, $12L = 192$, which implies $L = 16$ and $c = 64$. Thus, utility is
Then, the person’s maximum utility is $16,384$.

(b) At the new prices we have $2c/L = 6$, i.e., $c = 3L$. In addition, $L^2c = 16,384$, i.e., $3L^3 = 16,384$. Hence, $L = 17.61$, and $c = 52.83$. $\ell = 24 - L$ implies $\ell = 6.39$. Since the after-tax wage is 6, the total wage income is 38.34. However, to afford 52.83 the person needs a lump sum subsidy of 14.49. The tax revenue is $4\ell = 12.78$. Thus, the deadweight loss is 1.71.

**Tax revenue is: 12.78**

**The lump sum subsidy is: 14.49**

**The deadweight loss is: 1.71**

**Question 6** Suppose preferences are given by $u(x_1, x_2) = 2\sqrt{x_1 x_2}$, which yields the indirect utility function $v(p_1, p_2, I) = I/\sqrt{p_1 p_2}$, and the expenditure function $e(p_1, p_2, u) = u \sqrt{p_1 p_2}$. Suppose prices are $p_1 = 1$, $p_2 = 25$ and income is $I = 150$. Then the price increases of good 1 increases to $p_1 = 25$. Determine the compensating and the equivalent variation associated with the price change.

The compensating variation is $150 - e(25, 25, v(1, 25, 150))$. Note that $v(1, 25, 150) = 30$, and $e(25, 25, 30) = 750$.

**The compensating variation is: -600.**

The equivalent variation is $e(1, 25, v(25, 25, 150)) - 150$. Note that $v(25, 25, 150) = 6$, and $e(1, 25, 6) = 30$.

**The equivalent variation is: -120**

**Question 7** Suppose a person’s Bernouli utility function is given by $u(x) = \ln(x)$. The person has 1,000 Dollars to invest. There are two investments available: (a) A riskless asset that pays an interest rate of 0%. A risky assets that that either pays 100% with probability 0.4 or -40% with probability 0.6.

Suppose the person invests $\alpha$ Dollars into the risky asset $1,000 - \alpha$ Dollars into the riskless asset. The person’s expected utility is

$$0.4 \ln((1,000 - \alpha) + 2\alpha) + 0.6 \ln((1,000 - \alpha) + 0.4\alpha)$$

The first order conditions is

$$\frac{0.4(-1 + 2)}{(100 - \alpha) + 2\alpha} + \frac{0.6(-1 + 0.6)}{(1,000 - \alpha) + 0.6\alpha} = 0.$$ 

Thus, $24((1,000 - \alpha) + 2\alpha) = 40((1,000 - \alpha) + 0.6\alpha)$, which implies

$\alpha = 400$
**Question 8** A person has mean variance preferences of the form $20E[X] - 2\text{Var}[X]$, where $X$ is the random variable that describes the portfolio return.

Suppose the person has 200 Dollars. He invests $\alpha$ Dollars in a risky asset with mean return 1.8 and a variance of 0.1. The remainder $(200 - \alpha)$ is invested in a riskless asset with return 1.1.

Thus, the person solves

$$\max_{\alpha} 20(1.8\alpha + 1.1(200 - \alpha)) - 0.2\alpha^2.$$ 

The first order condition is $36 - 22 - 0.4\alpha = 0$.

**The optimal $\alpha = 35$**

**Question 9** A person has 1,000 Dollars of wealth. He/she has to opportunity to make an investment which has an initial cost of 400 Dollars, but has a payoff of 3,000 Dollars with probability 0.2, 1,000 with probability 0.4, and zero otherwise.

(a) The expected utility is

$$-0.2/3, 600 - 0.4/1, 600 - 0.4/600.$$ 

**The expected utility of making the investment is $-0.000972$**

**The expected utility of not making the investment is $-0.001$**

Therefore the person should (circle the correct answer) **make the investment**

(b) Now suppose that highest payoff is $m$ instead of 20,000 Dollars. Determine the value of $m$ such that the person is just indifferent between making and not making the investment. The expected utility is

$$-0.2/(m + 4000) - 0.4/14,000 - 0.2/4,000.$$ 

$m = 1,800$